FRAMES AND ARCHES

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A panoramic view of the Pasadena Pioneers Bridge in Pasadena, California, during the final stage of construction. It illustrates an imaginative reinforced concrete bridge design, through the use of towering columns and graceful arches. The surrounding subtropical flora adds to the inherent beauty of the bridge. *(Courtesy of the Truck Mixer Manufacturers Bureau of Washington, D.C.)*

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FRAMES AND ARCHES

CONDENSED SOLUTIONS FOR STRUCTURAL ANALYSIS

VALERIAN LEONTOVICH, M.S.

Structural Engineer of International Engineering Company, Inc.
Subsidiary of Morrison-Knudsen Co., Inc.
Formerly Principal Designer of San Francisco Office of Sverdrup and Parcel, Inc.
Member, American Society of Civil Engineers
Member, American Concrete Institute
Registered Professional Engineer

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FOREWORD

by S. P. TIMOSHENKO
Professor Emeritus of Engineering Mechanics
Stanford University

The appearance of this book is a considerable step forward in applied structural engineering. Its contents are selected to facilitate practical design of rigid frames and arches. For the first time, a solution for analysis of frames with members of variable cross section, based on the classical method, is presented in a convenient practical form. All computations with this method are confined to simple, short operations; the time required for frame analysis is greatly reduced.

Another important feature of the book is a new solution of frames with members of constant cross section. Since the author's method is substantially shorter than any other in current practice, his approach should, no doubt, be favored by designers.

In conclusion, one must mention that the book provides comprehensive tables and graphs for a wide range of straight and curved members of variable cross section. The impressive array of formulas and data presented in this book furnishes a complete and invaluable professional tool for solution of frames usually encountered in engineering practice.

PREFACE

Statically indeterminate frames and arches with members of variable cross section have found considerable application in modern construction. This development has placed a special emphasis upon speed and reliability of engineering design.

Rigidly connected members constitute a frame structure, and analysis of it should be predicated on the elastic behavior of the entire structure. Simplified or short-cut analyses based on assumptions of questionable validity can no longer be tolerated in engineering practice.

While many methods of frame analysis are now available, only a limited number are practical for engineering office use. Methods involving solution of several simultaneous equations are tedious, time-consuming, and, therefore, prohibitive for everyday application. Modern methods, no doubt, are conveniently applicable to simple frames, but become very cumbersome when applied to more complex frames. Slow progress in the use of gable and arched frames, known for their fundamental economy, can be somewhat attributed to the fact that so far it has not been possible to produce quickly an economical design of these frames.

The principal object of this book is to provide a convenient, scientific tool for the analysis of arches and frames with members of variable cross section. This is accomplished by providing condensed solutions for redundant quantities in a large variety of structures, based upon development by the author.

Another object is to present these results in the form of equations which can be solved by a slide rule, still yielding sufficient accuracy in the result. This task is accomplished by the systematic reduction of all expressions to their simplest form.

Incidental objectives are to demonstrate the inherent precision and brevity of the elastic center method, to emphasize utmost efficacy of classical methods of analysis, and to instigate further research in this field.

It appears to the author that the practical utility and universality of the modern methods have been considerably overemphasized during

the last two decades. Any method may be used for analysis of a structure, but the most judicious move is the selection of the simplest and shortest method with sufficient accuracy for the particular case. Many years ago, Professor L. E. Grinter wrote in his book an appraisal of the modern methods which bears repeating: "Despite much improvement over the classical methods, even the simplest modern methods of analysis are far too complicated and cumbersome. They waste too much time. . . ."

Certainly, there is no universal method in the sense of utility. Some problems may be solved easier and faster by one method, while others may be more conveniently treated by another. For the analysis of single span redundant arches and frames of variable section the elastic center method augmented by the concept of elastic parameters appears to offer the best solution. Through the use of the extended elastic center method the analysis of redundant frames with straight or curved members of variable section may be accomplished in the same simple manner as that of frames containing members of constant section alone. Even complex problems of the frames suffering "body swing" under load may be presented and solved with extreme simplicity.

During preparation of this text, it was suggested that the presentation would be made more comprehensive if frames with members of constant cross section were included. Although this subject has already been treated in some European and American publications,[1] it appeared advantageous to incorporate another version of some basic formulas better adapted to the present requirements. With these objectives in mind, the book is developed.

The first part of the book covers arches and frames with members of constant cross section. The second part is devoted to similar structures with members of variable cross section.

Expressions for redundant quantities as well as the bending moment and shearing and axial forces at any section of the structure are provided in the text for many vertical and horizontal loads. Solutions for complex structures are presented in such a manner that only algebraic computa-

[1] A. Kleinlogel, *Rahmenformeln*, W. Ernst und Sohn, Berlin, 1913; W. M. Wilson, F. E. Richart, and C. Weiss, "Analysis of Statically Indeterminate Structures by the Slope Deflection Method," *Univ. Ill. Eng. Exp. Sta. Bull.* 108, 1918; R. Saliger, *Praktische Statik*, Springer-Verlag OHG, Berlin, 1921; G. A. Hool and W. S. Kinne, *Stresses in Framed Structures*, McGraw-Hill Book Company, Inc., New York, 1923; M. S. Ketchum, *Structural Engineers' Handbook*, McGraw-Hill Book Company, Inc., New York, 1924; O. Belluzzi, *Formule Per Il Calcolo Dei Portali Incastrati*, Nicola Zanichelli, Bologna, 1930; K. Beyer, *Die Statik im Eisenbetonbau*, Springer-Verlag OHG, Berlin, 1933; and E. E. Amirikian, *Analysis of Rigid Frames*, U.S. Printing Office, Washington, D.C., 1942.

tions are required. No advance knowledge of structural engineering is presumed and solutions can be carried out successfully and with confidence by a practicing engineer or student.

Attention is given to the development of formulas in a convenient form for calculation, so that there will be no operations with very large or very small numbers. Generally, numbers used in computations will be not larger than 10,000 nor less than 0.001.

In the Appendix, numerical values of elastic parameters and load constants for straight and curved members of variable cross section are provided in charts and tables. These elastic parameters and constants are developed in such a manner that, besides representing certain characteristics of the members of variable cross section necessary for analyses of frames and arches, they might also be utilized in other valuable functions. As an example of such extended application, the use of the principle of superposition to obtain the elastic properties of members not covered by the charts or tables should be mentioned.

It is not intended that this book cover the analyses of all possible frames. On the contrary, only the more common types, shapes, and loadings are presented. If the reader, with the aid of this book, will be able to analyze any such frame within 30 minutes, then the author will consider his task accomplished.

To facilitate the preparation of Charts 1 to 16, inclusive, as presented on pages 418 through 433, the set of curves which appear in *Rahmentragwerke und Durchlauftrager*, 2d ed., by Richard Guldan, Springer-Verlag OHG, Vienna, 1943, has been partially utilized. Since copyright of the quoted book is vested in the Attorney General of the United States, the excerpts were used pursuant to License No. JA-1526 of the United States Attorney General.

The author is deeply indebted to Professor Emeritus S. P. Timoshenko for his encouragement and wise counsel and to Professor E. P. Popov, who read the entire manuscript and offered valuable suggestions. The author wishes to acknowledge his appreciation to Mark U. Viesselman and Stuart M. Alexander for their assistance in editing of the manuscript; and to the Directorates of the International Engineering Company, Inc., and Sverdrup and Parcel, Inc., for their pronounced interest and cooperation. In conclusion, he wishes to express thanks to Misses Frances Seger and Boe Gin for their care in typing the manuscript.

VALERIAN LEONTOVICH

CONTENTS

PART TWO: FRAMES AND ARCHES WITH MEMBERS OF VARIABLE CROSS SECTION

SYMBOLS AND ABBREVIATIONS

The following list of symbols has been adopted for this text. With few exceptions, no symbol is assigned a dual meaning.

ARABIC-NUMERAL SYMBOLS

1, 2, 3 . . .	Joints or support points in a frame or an arch

ROMAN-LETTER SYMBOLS

a, b	Lineal dimensions of members or their projections on vertical or horizontal axes
d	Depth of a member or thickness of an arch
d_r	Relative arch thickness
f, h	Lineal dimensions of members or their projections on vertical axis
k	Arch k value; arch characteristic defined in Art. 20-3
m	Moment arm of a force. Also the length of a distributed load on a member
n	Lineal dimension, or when used as a subscript denotes a numerical symbol assigned to a joint or a section of the structure
q	Lineal dimension
t	Temperature differential in degrees. Also, ratio $\left(\frac{\min d}{\max d}\right)^3$ of the member with variable depth
v	Haunch length to span ratio for a member with variable cross section
x	Horizontal coordinate, in general
x_1, x_2, . . .	Horizontal coordinates or distances from the joints defined by the subscripts
y	Vertical coordinate, in general

$y_1, y_2, \ldots$	Vertical coordinates or distances from the joints defined by the subscripts
A, B, C, D	Constants of a structure
E	Modulus of elasticity in tension or compression
F	Constant of a structure
G, J, K	Various mathematical expressions as defined by formulas in the text
H_n	Horizontal component of frame or arch reaction, at the section defined by subscript
I	Moment of inertia, in general
I_n	Moment of inertia of a member's cross section about its neutral axis; the subscript indicates the location of the section
$I_{(n)-(n+1)}$	Moment of inertia of a member's cross section about its neutral axis; the subscript identifies the member
L	Span of a frame or an arch between the center lines of the supports
M_n	Bending moment at the section defined by the subscript
N	Mathematical expression as defined by formula in the text
N_x	Axial force in an arched member, acting at the section defined by the horizontal coordinate x
P	Concentrated load
Q_x	Shearing force of an arch acting at the section defined by the horizontal coordinate x
R_n	Load constant for a straight member of variable cross section for the end of the member defined by subscript n
S, T, U	Load constants for arched members
V_n	Vertical component of frame or arch reaction, at the section defined by the subscript
W	Total distributed load
X, Y, Z	Various mathematical expressions as defined by formulas in the text

GREEK-LETTER SYMBOLS

α_n (alpha), β_n (beta)	Elastic parameters of an individual member for the end of the member defined by subscript n

γ (gamma), δ (delta)	Elastic parameters of an arched member
ε (epsilon)	Coefficient of thermal expansion
μ (mu)	Constant of the frame as defined in the text
τ (tau)	Arch constant; defined in Art. 20-8
ϕ (phi)	Frame constant, depending on the geometrical and physical properties of the frame
φ (phi)	Angle
ψ (psi)	Constant of the frame as defined in the text
Δ (delta)	Horizontal or vertical displacement of support
Θ (theta)	Elastic constant of a member

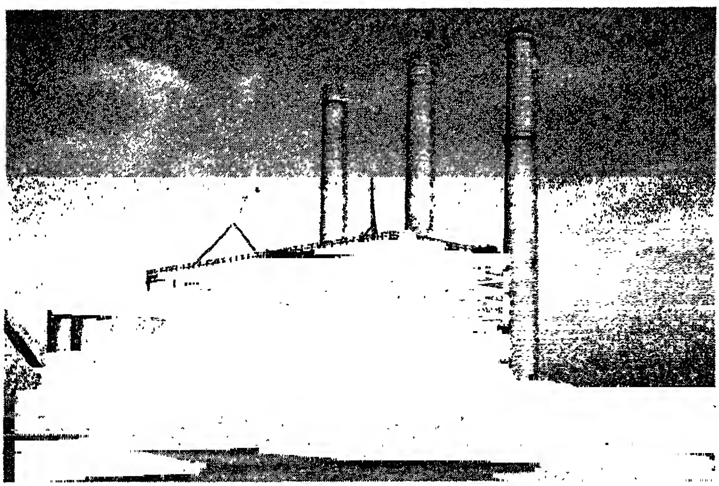
MATHEMATICAL SIGNS AND SYMBOLS

$+$	Plus (sign of addition)
$+$	Positive
$-$	Minus (sign of subtraction)
$-$	Negative
$\pm$ ($\mp$)	Plus or minus (minus or plus)
$\times$	Times, by (multiplication sign)
$/$	Divided by
$<$	Less than
$>$	Greater than
$=$	Equals
$\leqq$	Equal to or less than
$\geqq$	Equal to or greater than
$\cong$	Approximate to
$\therefore$	Therefore
( ) [ ] { }	Parentheses, brackets, and braces; quantities enclosed by them to be taken together in multiplying, dividing, etc.
$^\circ$	Angular or thermal degrees
$'$	Angular minutes

ABBREVIATIONS

k	kip = 1,000 lb
k′	ft-kip
k/′	kip per lin. ft
sin	sine
cos	cosine
sec	secant
var	variable

FRAMES AND ARCHES



Construction view of the Colbert Steam Power Plant, 800,000 kw capacity, built by the Tennessee Valley Authority on the shores of Pickwick Lake in the state of Alabama. This example of a modern industrial building illustrates the growing use of rigid frames in order to provide wide unobstructed areas for equipment and servicing facilities. At the left, the huge portal frames with their neat lines are distinctly recognizable. The frames span the generator room, which is 125 feet wide by 63 feet high. They shelter four turbogenerating units and support two 110-ton capacity overhead traveling cranes. Plant designed by the TVA staff. (Courtesy of the Tennessee Valley Authority.)

SECTION 1

INTRODUCTION

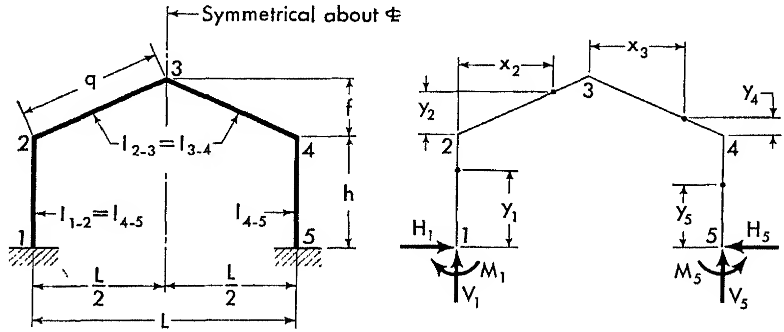
1-1. Contents. Condensed solutions for the analysis of twenty statically indeterminate frames and arches are presented in this book. These solutions provide final equations for redundant moments and forces for a number of loading conditions. Once the redundant quantities are determined, the moment, axial force, and shearing force at any section of a frame or an arch may be obtained without difficulty.

The first group of ten structures consists of frames and arches with members of constant cross section, while the second group of ten comprises frames and arches with members of variable cross section. For the latter group, tables and charts in the Appendix provide extensive facilities for solving a variety of problems.

A number of illustrative problems are presented in the text. It is suggested that these problems be studied thoroughly by the reader in order to comprehend all details of analysis as well as to gain familiarity with the tables and charts as sources of direct aid in the solution of problems.

1-2. Notations and Meaning of Terms. The definition and meaning of terms are comprehensively explained below to ensure correct interpretation and usage of the various terms. Because this text covers a wide range of structures with many variations of geometric and elastic properties, an expanded system of notations and designations is introduced, in order to avoid confusing duplication of symbols and terms.

Definition Sketch for General Notations. The solution of every frame or arch is preceded by a definition sketch explaining principal notations, coordinates, and the sign convention for moments and forces. Figure 1-1 is an example of the definition sketch explaining these terms for the gable frame with members of constant cross section.



The sketch appearing on the left, above, explains notations for a representative gable frame with members of constant cross section.

The sketch appearing on the right, above, shows positive directions of the moments and the vertical and horizontal components of the frame reactions. It also defines the coordinates for any section of the frame. Coordinates are to be considered only in the positive sense.

FIG. 1-1. Example of a definition sketch for general notations of gable frame

Joints of the frame are numbered consecutively in a clockwise direction starting from the left support with numeral 1. Subsequently, each member may be conveniently defined through the use of numerical symbols appearing at its joints. Generally, it is advisable to use these symbols in hyphenated form as, for example, 1-2, for the left column.

Individual coordinates are assigned to each member, as shown, and are measured only in the positive directions. The horizontal coordinate x, as a rule, carries a subscript corresponding to the joint from which it originates and is always measured positive to the right. The vertical coordinate y also, as a rule, carries a subscript corresponding to the joint from which it originates and is always measured positive upward. Some simplifications are made, however, in cases of simple structures. For example, in the case of an arch all horizontal coordinates have the same origin and therefore a subscript is unnecessary and is omitted. Some other simplifications of the same nature are introduced in the text. All these exceptions are clearly indicated on definition sketches for the specific structures.

Notations for Bending Moments and Forces. The bending moment, shearing and axial forces at the particular section of the frame or arch are designated, respectively, by the symbols M, Q, and N, with subscripts defining the section. When the section is taken at the joint of the frame, the subscript simply defines the joint. When the section is taken between the

joints, the subscript identifies the member and the distance from the preceding joint to the section. Thus $M_{1.5}$ denotes the bending moment at the section midway between joints 1 and 2. Similarly, $Q_{2.25}$ designates the shearing force at the section corresponding to the first quarter point of member 2-3.

In certain cases two moments of different sign and magnitude may exist at the same section of the frame or arch. In such cases the subscripts identifying these moments are composed of the related joint symbols, arranged by cross reference. Assume for the sake of example that this exists at joint 2 of the frame shown in Fig. 1-1. Then the moment acting on the upper end of member 1-2 is denoted as M_{21}, while the moment acting on the left end of the member 2-3 is denoted as M_{23}.

Notations for Moments of Inertia. The moment of inertia of a member's cross-sectional area about its neutral axis is generally denoted by symbol I with a subscript. The subscript defines the member, in accordance with the convention previously described. For a member of constant section, the moment of inertia of the cross-sectional area about its neutral axis is a constant value. Consequently, the subscript is sufficient identification. Thus, $I_{2\text{-}3}$ denotes the moment of inertia of the cross-sectional area about its neutral axis for member 2-3. For a member of variable cross section, the moment of inertia of the cross-sectional area about its neutral axis is a variable quantity and the above designation may serve only in a *general sense.* The abbreviated note (var) is added to emphasize the general meaning of the notation, as for example, $I_{2\text{-}3}$ (var).

When the moment of inertia of a member's cross-sectional area about its neutral axis refers to a particular section of the arch, it is then identified by the use of subscripts, in the same manner as described above for M, Q, and N.

Notations for Intermediate Sections of an Arch. In the analysis of the arched structures by the method presented in this text, the length of the curved member does not enter the calculations; instead, span and rise dimensions of the member are used. Similarly, the sections of the arched member are defined by the horizontal and vertical coordinates, which originate at the left end of the member. As a result, the adopted system for designating the sections of the member is based on span length rather than axis length. The numerical symbols which are assigned to the sections readily identify both the member and the horizontal coordinates of the sections. Thus, the section defined by the symbol 2.2 is between the joints 2 and 3 and corresponds to a horizontal coordinate of $0.2L$, where L is the span of the member. Similarly, section 2.6 corresponds

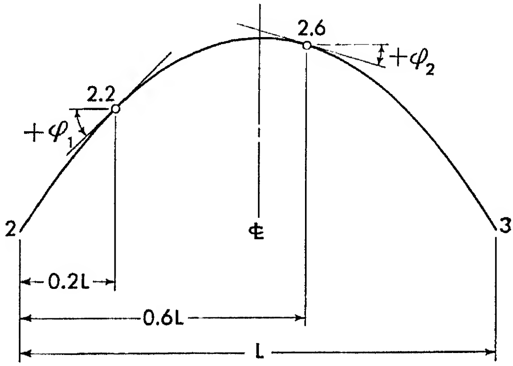


FIG. 1-2. Definition sketch for intermediate sections of the arch and angles of inclination of the arch axis

to a horizontal coordinate of 0.6L of the same member. The application of this system can be more readily understood from Fig. 1-2.

Arrangement of Notations. In regard to arrangement of notations it should be stated that only frames and arches which are symmetrical about their center lines are considered in this text; therefore, the notations on the sketches are generally provided for the members of the left half of the structure only.

1-3. Principles and Conventions. The general principles and conventions used in the text are set forth below. They should be carefully studied and used as general guides for the correct interpretation of the results of an analysis.

General Rule. The most important rule, which must be strictly observed, is that all calculations are to be performed algebraically. All quantities should be inserted into the equations with their proper sign, in order that results of the proper magnitude will be obtained mechanically with correct sign.

Loads. The direction and the nature of the load or impressed distortion are shown on the load diagram for each case. Moments and forces are determined for stated conditions. If a load is applied in the opposite direction to that shown on the diagram, the value of the load used in the equations should be given a negative sign.

Moments. The familiar beam convention for the signs of moments is used throughout this text. A bending moment is considered positive when it causes tension on the inner side of an arch or a frame and compression

on the outer side; the opposing bending moment is considered negative. In conformity with the stated sign convention, moment diagrams are drawn on the tension side of each member.

Reactions. Vertical reactions are positive when acting upward. Horizontal reactions are positive when acting toward the center of an arch or a frame.

Shearing Forces. The sign of shearing force is determined in accordance with standard beam convention. Thus, shearing force of a horizontal beam is positive if, at any considered section, the vertical component of the resultant of all loads and reactions to the left of the section is directed upward. Opposing force is considered negative.

The same convention, with minor modifications, is applied to curved members. Shearing force of an arched member is positive if, at any considered section, the normal component of the resultant of all loads and reactions to the left of the section is directed outward from the center of arch curvature.

Axial Forces. Axial force in an arched member is positive when it produces compression, and is negative when it produces tension. The sign of the axial force is positive if, at any considered section, the tangential component of the resultant of all loads and reactions to the left of the section is directed toward the right end of the arched member.

Moments of Inertia. Generally, the absolute values of the cross-sectional moment of inertia of members do not enter into calculations; instead, ratios are used. Only in the analyses of impressed distortions or temperature effects are the absolute values of the moments of inertia employed. There is no complication involved in determining and using cross-sectional moment of inertia of members of constant section. Certain doubt, however, may arise as to how to determine and apply cross-sectional moment of inertia of members with variable section. This apparent difficulty is solved by using a *reference moment of inertia of a member.* The reference moment of inertia of member's section for a straight member of variable cross section is *the I of the member's minimum cross section about its neutral axis.* The reference moment of inertia of any arched member is *the moment of inertia of the crown section about its neutral axis.* The latter convention permits the encompassing of haunched and tapered arched members in one classification.

Angles of Inclination of Arched Axis. The angle of inclination, with corresponding sign, is generally required at any section of the curved member for determination of axial and shearing forces. In this text, the sign of the angle, defining the direction of the slope, has been included

in the equations of the condensed solutions, so that only the *absolute* value of the angle need be considered. This one exception to the stated general rule for algebraic signs has been made in order to simplify the presentation of the analysis of arched structures.

The angle of inclination φ of the arch axis is measured as the acute angle between the horizontal and the tangent at the point under consideration, and, as mentioned above, is always considered positive. Figure 1-2 demonstrates the application of this convention.

1-4. Basic Assumptions. The condensed solutions of analysis given in this text have been derived using the theory of virtual work as commonly employed with the elastic center method. Generally, only energy of flexural deformation is considered in this analysis; the effect of shearing and axial deformations is neglected. However, for flat hingeless arches where the effect of axial deformations is appreciable, additional condensed solutions accounting for this effect are also presented. For all structures the supports are assumed to be unyielding except when the displacement of a support is specifically being considered.

The modulus of elasticity is considered to be the same for all members of a frame. Generally, the modulus of elasticity does not enter into calculations except for cases in which support displacement or temperature stresses are considered. In the latter case, it is assumed that all members of a frame are subject to the same temperature differential.

1-5. Condensed Solutions of Analysis. This text provides condensed solutions of analysis for common loadings on frames and arches with members of constant and variable cross section. In addition, a number of special loadings which have been used for calculation of maxima or minima of bending moments and redundant forces in arched structures are also considered. Furthermore, some important impressed distortions such as settlement and displacement of support, temperature effects, and applied moments are also included. The loadings considered are vertical and horizontal. It is apparent, of course, that other loadings can be resolved into vertical and horizontal components.

In performing the analyses of frames and arches by means of the condensed solutions, only simple algebraic operations are required. The results constitute the numerical values of the redundants. Once the redundants have been determined, the moments and forces may be obtained for any section of the structure. To facilitate even this stage of work, the expressions covering bending moment, at any section of the structure, are also given.

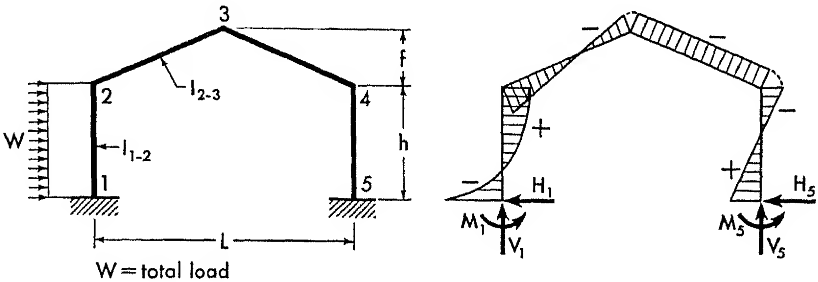


FIG. 1-3. Load, bending moment, and reaction diagrams

1-6. Bending Moment and Reaction Diagrams. The condensed solution of analysis for each loading on a frame or arch is preceded by load, bending moment, and reaction diagrams. The load diagram is presented on the left-hand side of the page, while the corresponding bending moment and reaction diagram is presented on the right-hand side, as illustrated in Fig. 1-3.

The load diagram illustrates direction and manner of loading, while the bending moment and reaction diagram illustrates directions of moments and forces that exist in the loaded structure of common proportion. In fact, each diagram is a representative diagram for a particular type of frame and load; however, it cannot be regarded as typical for all possible dimensional variations of the frame. The frame's dimensions may vary over wide ranges; as a result, the bending moments and forces in a specific frame may differ substantially in magnitude and even in sign from those shown. Thus, *the presented bending moment and reaction diagrams are not definitive, but rather illustrative.*

On these diagrams, in accordance with the fundamental principles of plane statics, letter symbols denoting moments and reactions are used without sign; the direction of a force or a moment signifies the sign of the force or moment. These directions have been obtained by application of standard sign conventions to the results of the analysis.

1-7. Dimensional Systems. Care should be exercised to introduce geometric and elastic properties of a structure into equations in consistent units of measure. Thus, for example, in determination of redundants due to the settlement of a support, the absolute value of the cross-sectional moment of inertia of a member (or the reference moment of inertia for a member of variable section) should be inserted into the equations in appropriate units

and corresponding units should be used for L, f, h, E, and Δ. If the moment of inertia is expressed in inches^4, length should be in inches, Δ in inches, and E (modulus of elasticity) should be in pounds per square inch (lb/sq in.); the resulting moment then will be obtained in inch-pounds (in.-lb) and reactions in pounds (lb). Another dimensional system may be selected, but all properties of a frame or an arch should be expressed in the same system.

1-8. Accuracy of Calculations. The condensed solutions of analysis as given in the text represent sets of precise equations reduced to their simplest form. Requiring only a minimal calculation, they provide very accurate results. For the majority of frames with members of constant section, results within half of 1 per cent accuracy are obtainable even by use of a slide rule. When charts are employed in the solution of frames with members of variable section, the accuracy may be affected by human errors in the selection of elastic constants.

For the analysis of arched structures it is advisable to use a calculating machine and to employ four or five significant figures in order to ensure optimum accuracy.

1-9. Equations. The moment expressions for two joints of the frame are generally combined in one equation. Thus, for example, the equation

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = \frac{PL\phi}{F} \pm \frac{K}{D}$$

represents in the combined form the following two equations:

$$M_2 = \frac{PL\phi}{F} + \frac{K}{D} \quad \text{and} \quad M_3 = \frac{PL\phi}{F} - \frac{K}{D}$$

The subscripts x and y of the moment symbol M, the axial force N, and shearing force Q are often used to indicate that the equation representing moment or force is applicable to any section of the member under consideration, within the boundary limits of the subscript.

1-10. Combining of Loads. Regardless of the number of loads considered in this text, it is obvious that they do not cover all practicable important cases. However, this apparent deficiency may be resolved in many cases by the use of the principle of superposition.

To illustrate this point, a frame under trapezoidal load, a case not

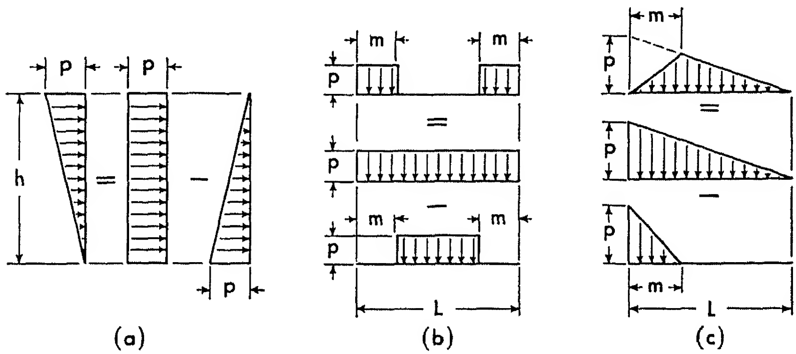


FIG. 1-4. Various examples of load superposition

covered in this text, is taken for the example. The solution may be obtained by combining, algebraically, the redundant quantities resulting from uniformly and triangularly distributed loads acting separately on the frame.

Similarly, the redundants of a frame having a triangular load on a column, with vertex at the base of the column, may be obtained by subtracting the redundant quantities due to a triangular load with vertex at the top from the redundant quantities due to a uniformly distributed load. A few additional typical examples of load superposition are shown graphically in Fig. 1-4.

1-11. Construction of Influence Lines. When a moving load on an arch or a frame is considered, the influence line is a very useful tool for the determination of that critical position of the load which produces a maximum value of a particular function.

An influence line, by definition, is a curve whose ordinate at any point represents the value of some function due to a unit load applied at that point. Therefore, the influence line may be easily constructed for any function of the frame or arch by application of the following method. A vertical or horizontal unit load is successively positioned along the length of the member, and the quantities of the function are computed for each position by means of condensed solutions of analysis. These quantities are then plotted as abscissas or ordinates for the points of load application; by drawing then a closing line, the influence line is obtained.

Since the load constants are given in the Appendix at intervals of $L/10$ or $L/12$, an influence line may be constructed accurately and expeditiously.

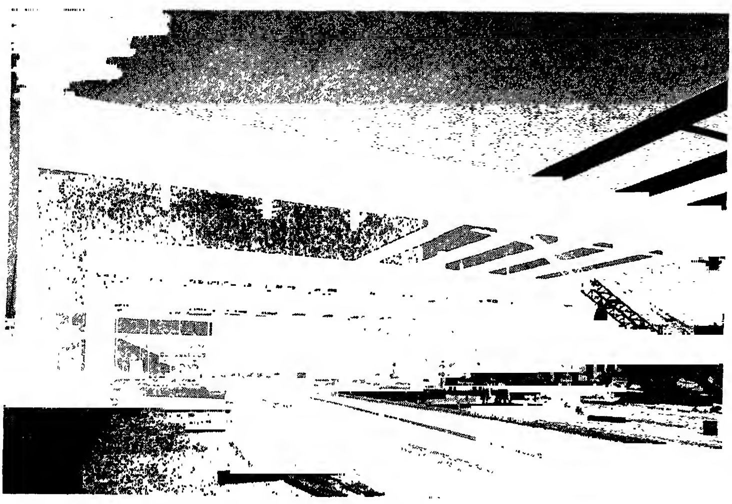
Finally, when the basic calculations for the influence line of any function are once accomplished, little additional work is required to obtain the influence line of any other function of the frame or arch under consideration.[1]

[1] A comprehensive description of influence lines and their construction for redundant structures may be found in textbooks. See, for example, J. B. Wilbur and C. H. Norris, *Elementary Structural Analysis*, McGraw-Hill Book Company, Inc., New York, 1948; also Clarence Dunham, *Theory and Practice of Reinforced Concrete*, 3d ed., McGraw-Hill Book Company, Inc., New York, 1953.

PART ONE

FRAMES AND ARCHES WITH MEMBERS OF CONSTANT CROSS SECTION

The S. R. Crown Hall at the Illinois Institute of Technology in Chicago houses the Departments of Architecture and the Institute of Design. Illustrating a new concept in architecture, this modern building is entirely enclosed in plate glass. The portal frames are attractively silhouetted against the sky. Designed by Ludwig Mies van der Rohe, director of the Department of Architecture of the Institute. (Courtesy of the Illinois Institute of Technology, Chicago, Ill.)

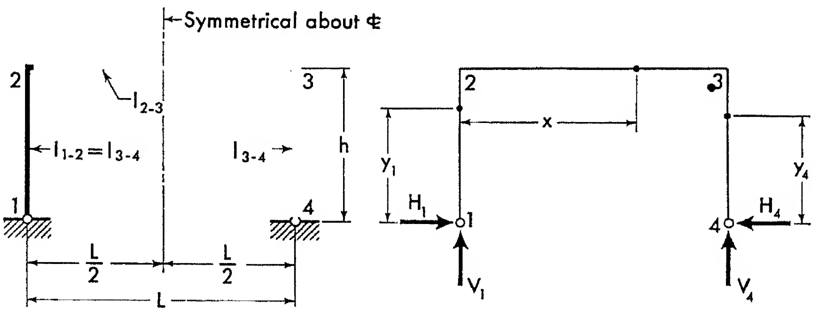


Close-up view of portal frames employed for the S. R. Crown Hall at the Illinois Institute of Technology. The all-welded frames are 120 feet in length and span the entire width of the building. In subsequent stages of construction they were integrated into a concrete ceiling slab forming an extremely rigid space structure. (Courtesy of the Illinois Institute of Technology, Chicago, Ill.)

SECTION 2

SYMMETRICAL PORTAL FRAMES WITH HINGED SUPPORTS

2-1. Notations, Coordinates, and Frame Constants



The sketch appearing on the left, above, explains notations for a representative portal frame with members of constant cross section.

The sketch appearing on the right, above, shows positive directions of the vertical and horizontal components of the frame reactions. It also defines the coordinates for any section of the frame. Coordinates are to be considered only in the positive sense.

Frame Constants: $\phi = \frac{I_{1-2}}{I_{2-3}} \cdot \frac{L}{h}$

$$A = 4\left(3 + \frac{2}{\phi}\right)$$

2-2. Equations of Frame Reactions and Moments. The equations for the vertical and the redundant horizontal components of frame reactions are

given on the following pages for a number of loading conditions. Corresponding expressions for the moment and forces at any section of a member with applied load are also provided.

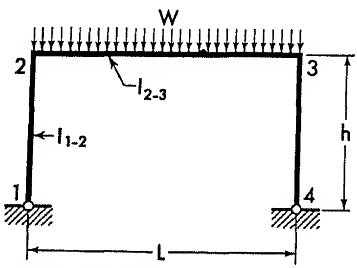
The equations for the moments of load-free members are listed below for reference.

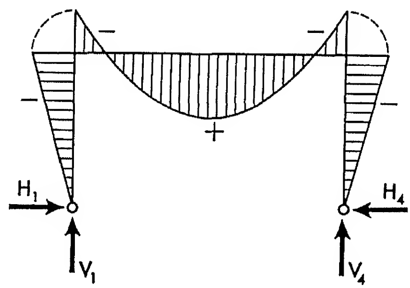
$$M_{y_1} = M_2 \frac{y_1}{h} \tag{2-1}$$

$$M_x = M_2 \left(1 - \frac{x}{L}\right) + M_3 \frac{x}{L} \tag{2-2}$$

$$M_{y_4} = M_3 \frac{y_4}{h} \tag{2-3}$$

2-3. Vertical Uniform Load on Girder





$$M_2 = M_3 = -\frac{WL}{A}$$

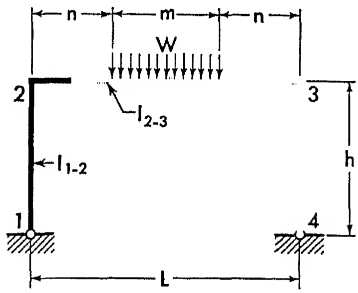
$$H_1 = H_4 = \frac{WL}{Ah}$$

$$V_1 = V_4 = \frac{W}{2}$$

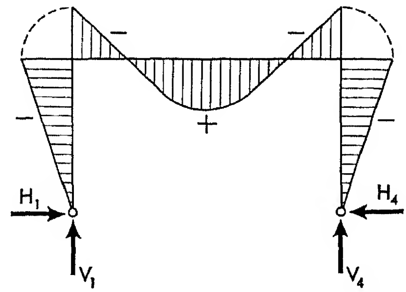
$$M_x = \frac{Wx}{2}\left(1 - \frac{x}{L}\right) + M_2$$

Apply Eqs. (2-1) and (2-3) to obtain the moment at any section of the frame columns.

2-4. Vertical Uniform Load over Center Part of Girder



W = total load



$$K = \frac{3L^2 - m^2}{2AL}$$

$$M_2 = M_3 = -WK \qquad H_1 = H_4 = \frac{WK}{h}$$

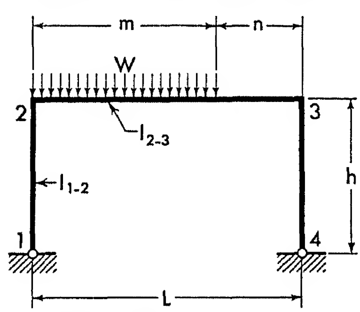
$$V_1 = V_4 = \frac{W}{2} \qquad M_{y_1} = M_2 \frac{y_1}{h}$$

When $x \leqq n$, $\qquad M_x = \frac{Wx}{2} + M_2$

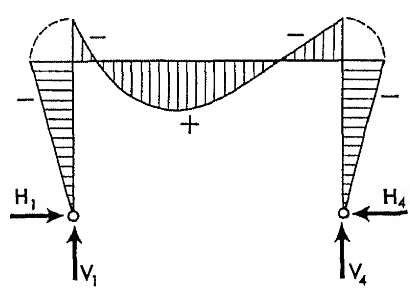
When $x > n$, but $\leqq \frac{L}{2}$, $\qquad M_x = \frac{W}{2}\left[x - \frac{(x-n)^2}{m}\right] + M_2$

Moments at corresponding sections in the right half of the frame are identical to those in the left half.

2-5. Vertical Uniform Load over Part of Girder



W = total load



$$K = \frac{m(L + 2n)}{AL}$$

For Notations and Constants, see Arts. 2-1 and 2-2

$$M_2 = M_3 = -WK \qquad H_1 = H_4 = \frac{WK}{h}$$

$$\left.\begin{matrix} V_1 \\ V_4 \end{matrix}\right\rangle = \frac{W}{2}\left(1 \pm \frac{n}{L}\right)$$

When $x \leqq m$

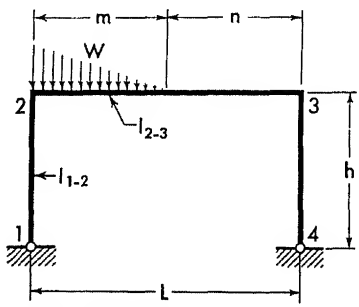
$$M_x = \frac{Wx}{2}\left(\frac{n}{L} + \frac{m-x}{m}\right) + M_2$$

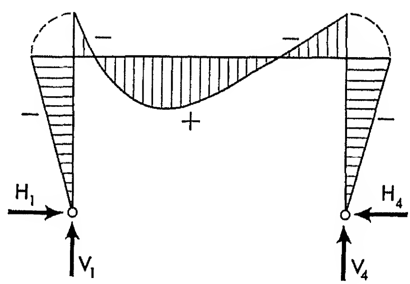
When $x > m$

$$M_x = \frac{Wm}{2}\left(1 - \frac{x}{L}\right) + M_2$$

Apply Eqs. (2-1) and (2-3) to obtain the moment at any section of the frame columns.

2-6. Vertical Triangular Load over Part of Girder





$$K = \frac{m(2n+m)}{AL} \qquad M_2 = M_3 = -WK$$

$$H_1 = H_4 = \frac{WK}{h} \qquad V_1 = W\left(1 - \frac{m}{3L}\right) \qquad V_4 = \frac{Wm}{3L}$$

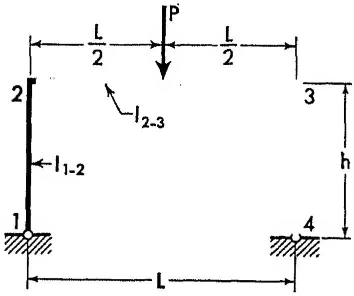
When $x \leqq m$

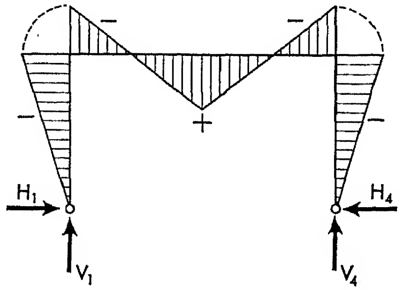
$$M_x = \frac{Wx}{3}\left[\frac{n}{L} + \frac{(m-x)(2m-x)}{m^2}\right] + M_2$$

When $x > m$, $\qquad M_x = \dfrac{Wm}{3}\left(1 - \dfrac{x}{L}\right) + M_2$

Apply Eqs. (2-1) and (2-3) to obtain the moment at any section of the frame columns.

2-7. Vertical Concentrated Load at Mid-point of Girder





$$M_2 = M_3 = -\frac{3PL}{2A} \qquad H_1 = H_4 = \frac{3PL}{2Ah}$$

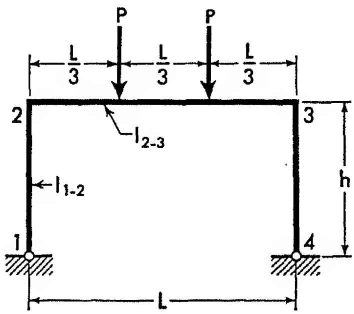
$$V_1 = V_4 = \frac{P}{2} \qquad M_{y_1} = M_2 \frac{y_1}{h}$$

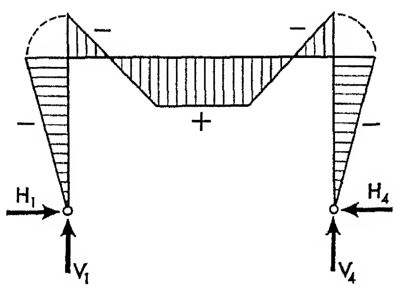
When $x \leqq \frac{L}{2}$

$$M_x = \frac{Px}{2} + M_2$$

Moments at corresponding sections in the right half of the frame are identical to those in the left half.

2-8. Two Equal Vertical Concentrated Loads on Girder





$$M_2 = M_3 = -\frac{8PL}{3A} \qquad H_1 = H_4 = \frac{8PL}{3Ah}$$

$$V_1 = V_4 = P$$

For Notations and Constants, see Arts. 2-1 and 2-2

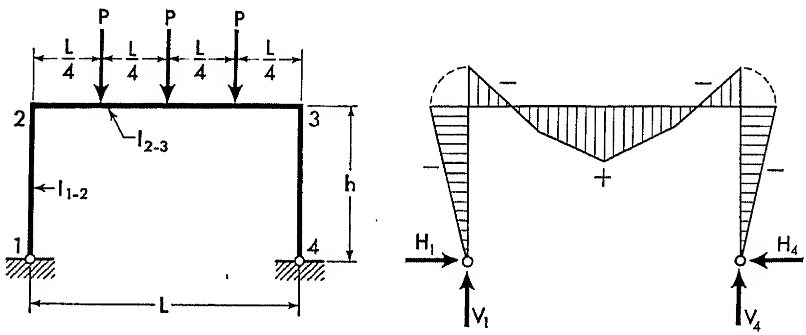
When $x \leqq \frac{L}{3}$, $\quad M_x = Px + M_2$

When $x > \frac{L}{3}$, but $\leqq \frac{L}{2}$

$$M_x = \frac{PL}{3} + M_2$$

Apply Eq. (2-1) to obtain the moment at any section of frame member 1-2. Moments at corresponding sections in the right half of the frame are identical to those in the left half.

2-9. Three Equal Vertical Concentrated Loads on Girder



$$M_2 = M_3 = -\frac{15PL}{4A}$$

$$H_1 = H_4 = \frac{15PL}{4Ah} \qquad V_1 = V_4 = \frac{3}{2}P$$

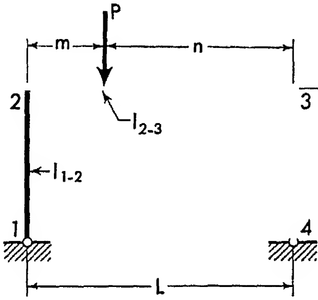
When $x \leqq \frac{L}{4}$, $\quad M_x = \frac{3Px}{2} + M_2$

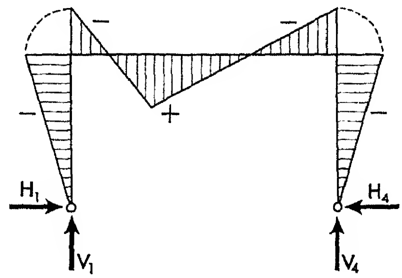
When $x > \frac{L}{4}$, but $\leqq \frac{L}{2}$

$$M_x = \frac{P(L + 2x)}{4} + M_2$$

Apply Eq. (2-1) to obtain the moment at any section of frame member 1-2. Moments at corresponding sections in the right half of the frame are identical to those in the left half.

2-10. Vertical Concentrated Load at Any Point of Girder





$$K = \frac{6mn}{AL} \qquad M_2 = M_3 = -PK$$

$$H_1 = H_4 = \frac{PK}{h} \qquad V_1 = P\left(1 - \frac{m}{L}\right)$$

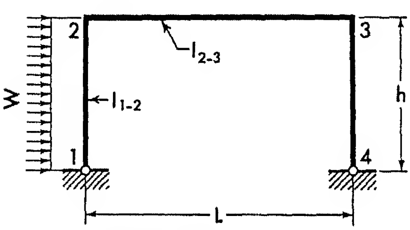
$$V_4 = \frac{Pm}{L}$$

When $x \leqq m$, $\qquad M_x = \dfrac{Pnx}{L} + M_2$

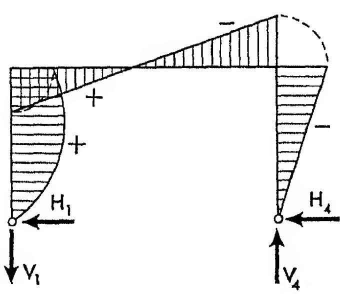
When $x > m$, $\qquad M_x = Pm\left(1 - \dfrac{x}{L}\right) + M_2$

Apply Eqs. (2-1) and (2-3) to obtain the moment at any section of the frame columns.

2-11. Horizontal Uniform Load on Column



W = total load



$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -\frac{Wh}{2}\left(\frac{1}{A\phi} \mp \frac{1}{2}\right)$$

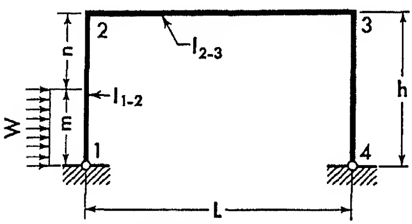
$$H_4 = \frac{W}{2}\left(\frac{1}{2} + \frac{1}{A\phi}\right) \qquad H_1 = -(W - H_4)$$

$$V_4 = \frac{Wh}{2L} \qquad V_1 = -V_4$$

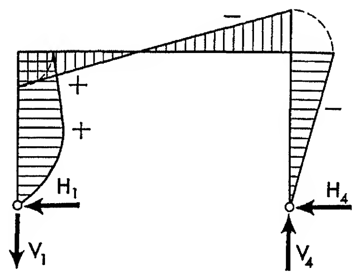
$$M_{y_1} = \frac{Wy_1}{2}\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (2-2) and (2-3) to obtain the moment at any section of frame members 2-3 and 3-4.

2-12. Horizontal Uniform Load over Part of Column



W = total load



$$g = \frac{m}{h} \qquad K = \frac{g}{A\phi}\left(1 - \frac{g^2}{2}\right)$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\} = -W\left(Kh \mp \frac{m}{4}\right)$$

$$H_4 = W\left(K + \frac{m}{4h}\right) \qquad H_1 = -(W - H_4)$$

$$V_4 = \frac{Wm}{2L} \qquad V_1 = -V_4$$

When $y_1 \leqq m$

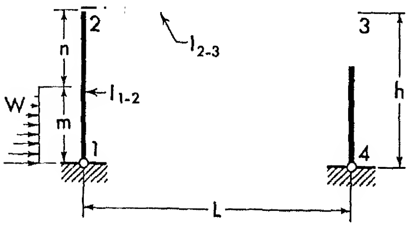
$$M_{y_1} = \frac{Wy_1}{2}\left(\frac{n}{h} + \frac{m - y_1}{m}\right) + M_2\frac{y_1}{h}$$

When $y_1 > m$

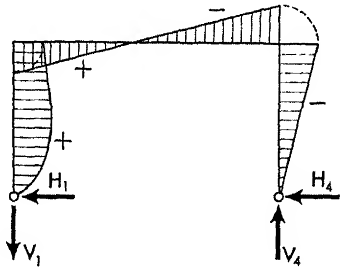
$$M_{y_1} = \frac{Wm}{2}\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (2-2) and (2-3) to obtain the moment at any section of frame members 2-3 and 3-4.

2-13. Horizontal Triangular Load over Part of Column



W = total load



$$g = \frac{m}{h} \qquad K = \frac{10 - 3g^2}{5A\phi} \qquad \left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -\frac{Wm}{3}\left(K \mp \frac{1}{2}\right)$$

$$H_4 = \frac{Wm}{3h}\left(\frac{1}{2} + K\right) \qquad H_1 = -(W - H_4)$$

$$V_4 = \frac{Wm}{3L} \qquad V_1 = -V_4$$

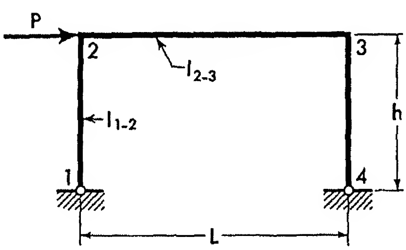
When $y_1 \leqq m$

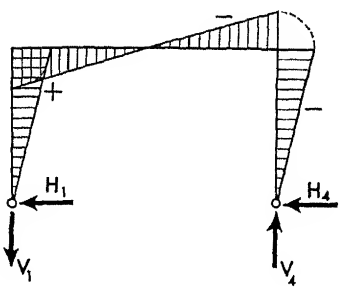
$$M_{y_1} = \frac{Wy_1}{3}\left[\frac{n}{h} + \frac{(m - y_1)(2m - y_1)}{m^2}\right] + M_2\frac{y_1}{h}$$

When $y_1 > m$, $\qquad M_{y_1} = \dfrac{Wm}{3}\left(1 - \dfrac{y_1}{h}\right) + M_2\dfrac{y_1}{h}$

Apply Eqs. (2-2) and (2-3) to obtain the moment at any section of frame members 2-3 and 3-4.

2-14. Horizontal Concentrated Load at Joint 2





$$M_2 = \frac{Ph}{2} \qquad M_3 = -\frac{Ph}{2}$$

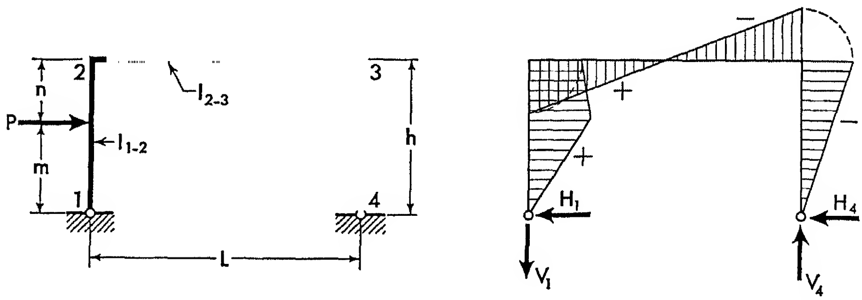
For Notations and Constants, see Arts. 2-1 and 2-2

$$H_1 = -\frac{P}{2} \qquad H_4 = \frac{P}{2}$$

$$V_4 = \frac{Ph}{L} \qquad V_1 = -V_4$$

Apply Eqs. (2-1) through (2-3) to obtain the moment at any section of the frame members.

2-15. Horizontal Concentrated Load at Any Point of Column



$$g = \frac{m}{h} \qquad K = \frac{4(1-g^2)}{A\phi}$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -\frac{Pm}{2}(K \mp 1)$$

$$H_4 = \frac{Pm}{2h}(1+K) \qquad H_1 = -(P - H_4)$$

$$V_4 = \frac{Pm}{L} \qquad V_1 = -V_4$$

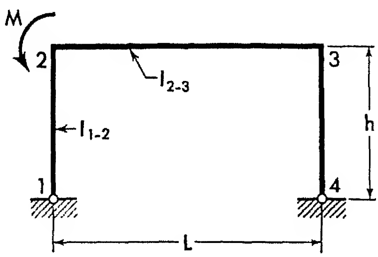
When $y_1 \leqq m$, $\qquad M_{y_1} = (M_2 + Pn)\frac{y_1}{h}$

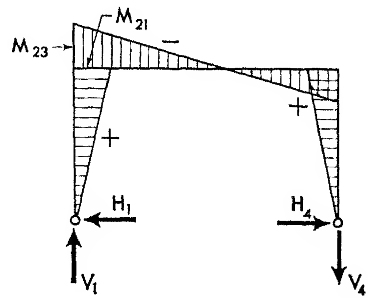
When $y_1 > m$

$$M_{y_1} = Pm\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (2-2) and (2-3) to obtain the moment at any section of frame members 2-3 and 3-4.

2-16. Moment Applied at Joint 2





$$M_{21} = M_3 = \frac{6M}{A}$$

$$M_{23} = -(M - M_{21})$$

$$H_1 = H_4 = -\frac{6M}{Ah}$$

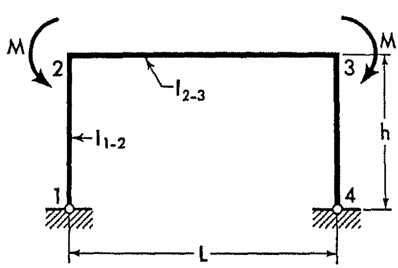
$$V_1 = \frac{M}{L} \qquad V_4 = -V_1$$

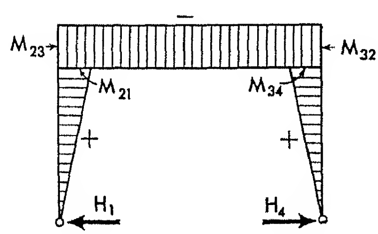
$$M_{y_1} = M_{21}\frac{y_1}{h}$$

$$M_x = M_{23}\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L}$$

Apply Eq. (2-3) to obtain the moment at any section of member 3-4.

2-17. Two Equal Moments Applied at Joints 2 and 3



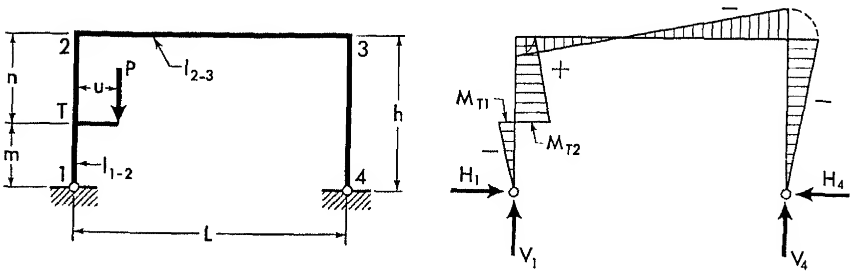


$$M_{21} = M_{34} = \frac{12M}{A} \qquad M_{23} = M_{32} = -(M - M_{21})$$

$$H_1 = H_4 = -\frac{12M}{Ah} \qquad V_1 = V_4 = 0$$

$$M_{y_1} = M_{21}\frac{y_1}{h} \qquad M_x = M_{23} \qquad M_{y_4} = M_{34}\frac{y_4}{h}$$

2-18. Vertical Concentrated Load Applied at Bracket



Bracket acts as a simple cantilever and its maximum moment is Pu at point T. The moment diagram of the cantilever is intentionally not shown so that the frame's bending moment diagram may be more clearly illustrated.

$$M = Pu$$

$$g = \frac{m}{h} \qquad K = \frac{2(1 - 3g^2)}{A\phi} \qquad \left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -\frac{M}{2}(2K \mp 1)$$

$$H_1 = H_4 = \frac{M}{2h}(1 + 2K) \qquad V_1 = P - \frac{M}{L} \qquad V_4 = \frac{M}{L}$$

When $y_1 < m$

$$M_{y_1} = -(M - M_2)\frac{y_1}{h}$$

When $y_1 = m$

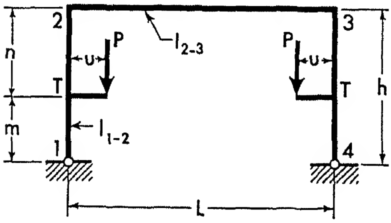
$$M_{T1} = -(M - M_2)\frac{m}{h} \quad \text{and} \quad M_{T2} = M\frac{n}{h} + M_2\frac{m}{h}$$

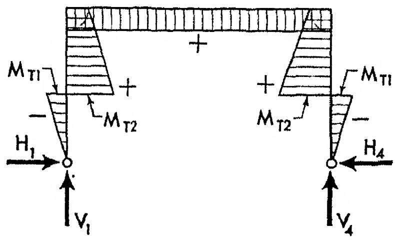
When $y_1 > m$

$$M_{y_1} = M\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (2-2) and (2-3) to obtain the moment at any section of frame members 2-3 and 3-4.

2-19. Two Equal Vertical Concentrated Loads Symmetrically Applied at Brackets





Brackets act as simple cantilevers with the maximum moments of Pu at points T. The moment diagrams of these cantilevers are intentionally not shown so that the frame's bending moment diagram may be more clearly illustrated.

$$M = Pu$$

$$g = \frac{m}{h} \qquad K = \frac{2(1 - 3g^2)}{A\phi}$$

$$M_2 = M_3 = -2MK$$

$$H_1 = H_4 = \frac{M}{h}(1 + 2K) \qquad V_1 = V_4 = P$$

When $y_1 < m$

$$M_{y_1} = -(M - M_2)\frac{y_1}{h}$$

When $y_1 = m$

$$M_{T1} = -(M - M_2)\frac{m}{h} \quad \text{and} \quad M_{T2} = M\frac{n}{h} + M_2\frac{m}{h}$$

When $y_1 > m$

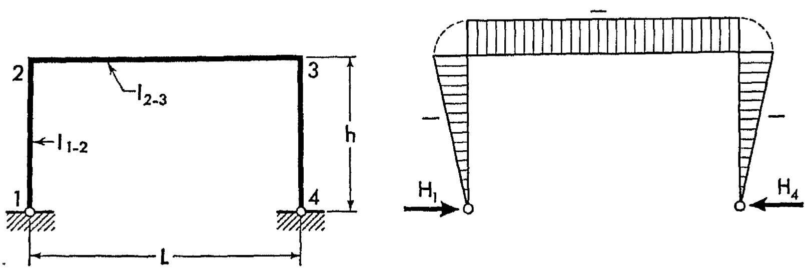
$$M_{y_1} = M\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

$$M_x = M_2$$

Moments at corresponding sections in the right half of the frame are identical to those in the left half.

For Notations and Constants, see Arts. 2-1 and 2-2

2-20. Effect of Temperature Rise. Range t° for entire frame.



$$H_1 = H_4 = \frac{3L\varepsilon t^\circ}{h^3(3\phi + 2)} EI_{1-2}$$

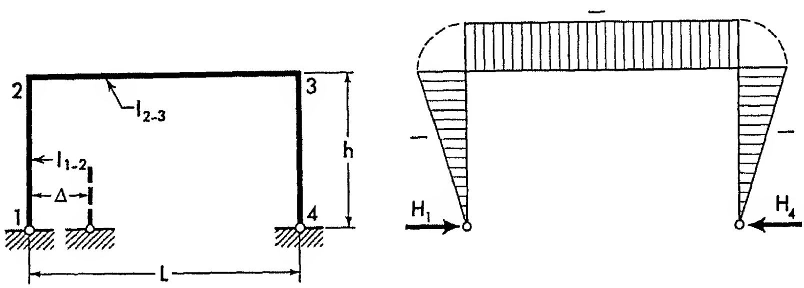
$$M_2 = M_3 = -H_4h$$

$$V_1 = V_4 = 0$$

Apply Eqs. (2-1) through (2-3) to obtain the moment at any section of the frame members.

Note: For temperature drop, introduce the value of t° with a negative sign.

2-21. Horizontal Displacement of One Support



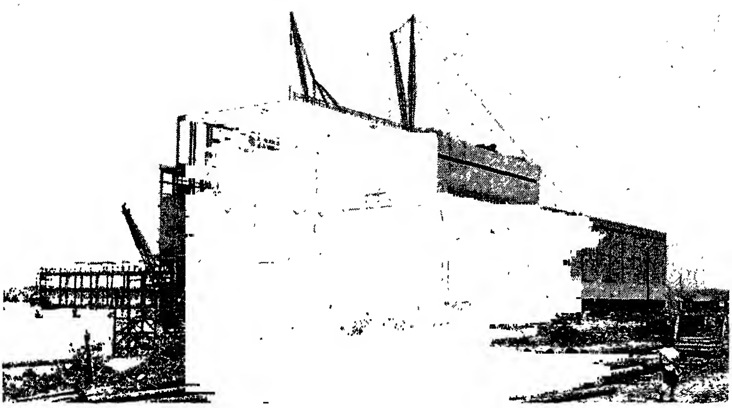
$$H_1 = H_4 = \frac{3\Delta}{h^3(3\phi + 2)} EI_{1-2}$$

$$M_2 = M_3 = -H_4h$$

$$V_1 = V_4 = 0$$

Apply Eqs. (2-1) through (2-3) to obtain the moment at any section of the frame members.

Note: If the direction of the frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.

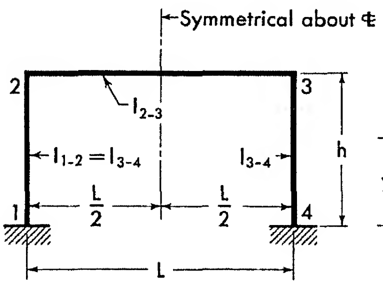


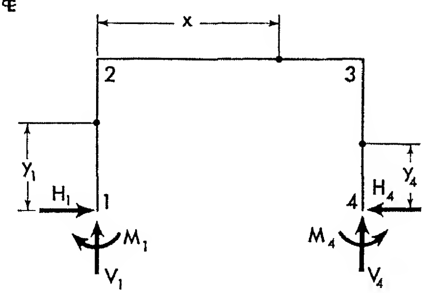
This construction view of the Johnsonville Steam Power Plant of the Tennessee Valley Authority, at Johnsonville, Tennessee, illustrates the use of frames as essential elements of modern industrial construction. The right lean-to of the building encloses the 24,000 square foot generator room, with a framework consisting of 17 welded portal frames with a span of 104 feet. Note that to provide a more pleasing overhead effect a slight camber has been imparted to the girders. (Courtesy of the Ingalls Iron Works Co., of Birmingham, Ala., steel fabricator.)

SECTION 3

SYMMETRICAL PORTAL FRAMES WITH FIXED SUPPORTS

3-1. Notations, Coordinates, and Frame Constants





The sketch appearing on the left, above, explains notations for a representative portal frame with members of constant cross section.

The sketch appearing on the right, above, shows positive directions of the moments and the vertical and horizontal components of frame reactions. It also defines the coordinates for any section of the frame. Coordinates are to be considered only in the positive sense.

Frame Constants: $\phi = \frac{I_{1-2}}{I_{2-3}} \cdot \frac{L}{h}$

$$D = 2\left(1 + \frac{6}{\phi}\right) \qquad F = 6\left(2 + \frac{1}{\phi}\right)$$

3-2. Equations of Frame Reactions and Moments. The equations for the redundant moments and the vertical and horizontal components of frame

reactions are given on the following pages for a number of loading conditions. Corresponding expressions for the moment and forces at any section of a member with applied load are also provided.

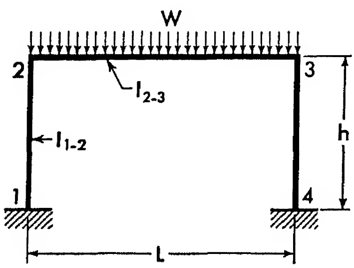
The equations for the moments of load-free members are listed below for reference.

$$M_{y_1} = M_2\frac{y_1}{h} + M_1\left(1 - \frac{y_1}{h}\right) \tag{3-1}$$

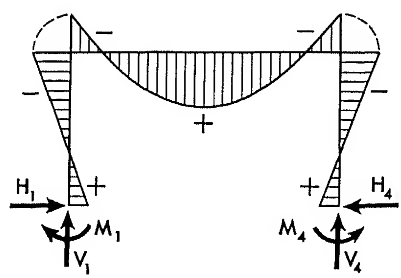
$$M_x = M_2\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L} \tag{3-2}$$

$$M_{y_4} = M_4\left(1 - \frac{y_4}{h}\right) + M_3\frac{y_4}{h} \tag{3-3}$$

3-3. Vertical Uniform Load on Girder



W = total load



$$M_1 = M_4 = \frac{WL}{2F}$$

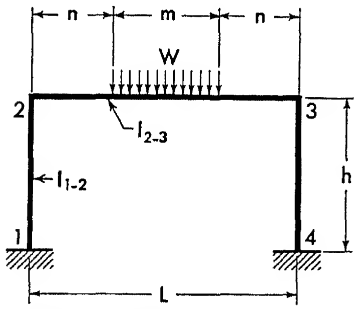
$$M_2 = M_3 = -\frac{WL}{F} = -2M_1$$

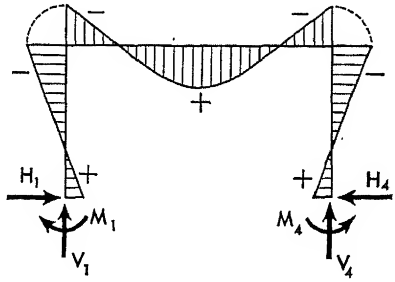
$$H_1 = H_4 = \frac{3M_1}{h} \qquad V_1 = V_4 = \frac{W}{2}$$

$$M_x = \frac{Wx}{2}\left(1 - \frac{x}{L}\right) + M_2$$

Apply Eq. (3-1) to obtain the moment at any section of the left column. Moments at corresponding sections in the right half of the frame are identical to those in the left half.

3-4. Vertical Uniform Load over Center Part of Girder





W = total load

$$K = \frac{3L^2 - m^2}{4FL} \qquad M_1 = M_4 = WK$$

$$M_2 = M_3 = -2WK \qquad H_1 = H_4 = \frac{3M_1}{h} \qquad V_1 = V_4 = \frac{W}{2}$$

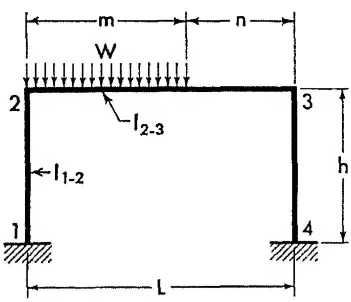
When $x \leqq n$, $\quad M_x = \dfrac{Wx}{2} + M_2$

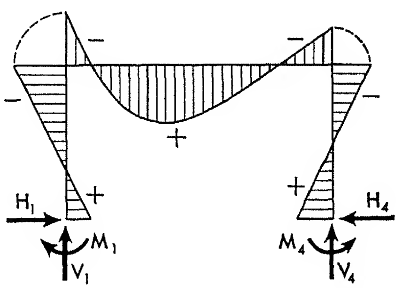
When $x > n$, but $\leqq n + m$ $\quad M_x = \dfrac{W}{2}\left[x - \dfrac{(x-n)^2}{m}\right] + M_2$

When $x > n + m$, $\quad M_x = \dfrac{W}{2}(L - x) + M_2$

Apply Eq. (3-1) to obtain the moment at any section of the left column. Moments at corresponding sections in the right half of the frame are identical to those in the left half.

3-5. Vertical Uniform Load over Part of Girder





W = total load

$$J = \frac{m(L + 2n)}{FL} \qquad K = \frac{mn^2}{2DL^2}$$

For Notations and Constants, see Arts. 3-1 and 3-2

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = \frac{W}{2}(J \mp 2K)$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -W(J \pm K) \qquad H_1 = H_4 = \frac{3WJ}{2h}$$

$$\left.\begin{matrix} V_1 \\ V_4 \end{matrix}\right\rangle = \frac{W}{2} \pm \frac{W}{2L}(n + 4K)$$

When $x \leqq m$

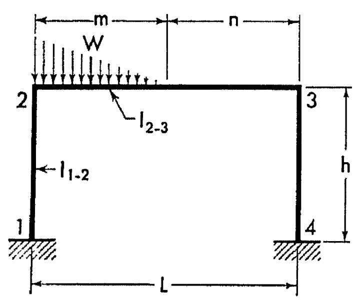
$$M_x = \frac{Wx}{2}\left(\frac{n}{L} + \frac{m - x}{m}\right) + M_2\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L}$$

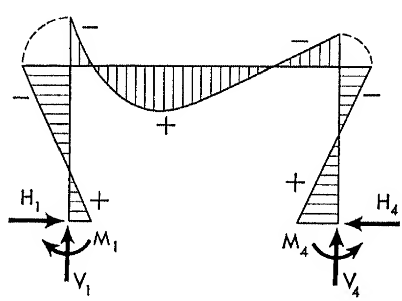
When $x > m$

$$M_x = \left(\frac{Wm}{2} + M_2\right)\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L}$$

Apply Eqs. (3-1) and (3-3) to obtain the moment at any section of the frame columns.

3-6. Vertical Triangular Load over Part of Girder





W = total load

$$g = \frac{m}{L} \qquad J = \frac{g}{15}(10 - 15g + 6g^2) \qquad K = g(2 - g)$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = \frac{WL}{2}\left(\frac{K}{F} \mp \frac{J}{D}\right)$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -WL\left(\frac{K}{F} \pm \frac{J}{2D}\right)$$

$$H_1 = H_4 = \frac{3WLK}{2Fh} \qquad V_4 = W\left(\frac{m}{3L} - \frac{J}{D}\right)$$

$$V_1 = W - V_4$$

When $x \leqq m$

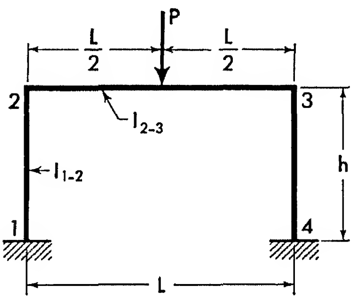
$$M_x = \frac{Wx}{3}\left[\frac{n}{L} + \frac{(m - x)(2m - x)}{m^2}\right] + M_2\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L}$$

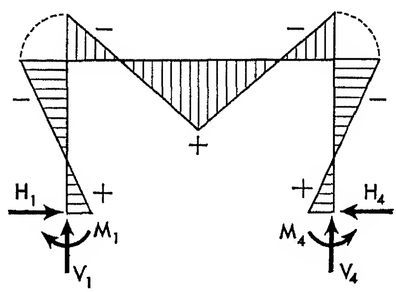
When $x > m$

$$M_x = \left(\frac{Wm}{3} + M_2\right)\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L}$$

Apply Eqs. (3-1) and (3-3) to obtain the moment at any section of the frame columns.

3-7. Vertical Concentrated Load at Mid-point of Girder





$$M_1 = M_4 = \frac{3PL}{4F}$$

$$M_2 = M_3 = -\frac{3PL}{2F}$$

$$H_1 = H_4 = \frac{3M_1}{h} \qquad V_1 = V_4 = \frac{P}{2}$$

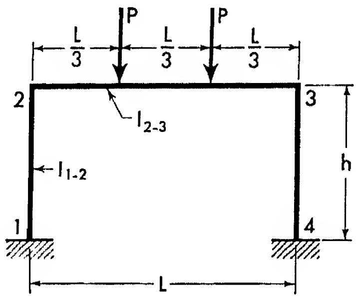
When $x \leqq \frac{L}{2}$

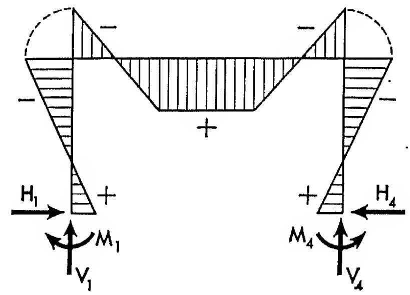
$$M_x = \frac{Px}{2} + M_2$$

Apply Eq. (3-1) to obtain the moment at any section of the left column. Moments at corresponding sections in the right half of the frame are identical to those in the left half.

For Notations and Constants, see Arts. 3-1 and 3-2

3-8. Two Equal Vertical Concentrated Loads on Girder





$$M_1 = M_4 = \frac{4PL}{3F} \qquad M_2 = M_3 = -\frac{8PL}{3F} = -2M_1$$

$$H_1 = H_4 = \frac{4PL}{Fh} \qquad V_1 = V_4 = P$$

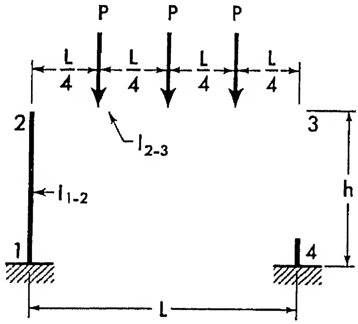
When $x \leqq \frac{L}{3}$, $\qquad M_x = Px + M_2$

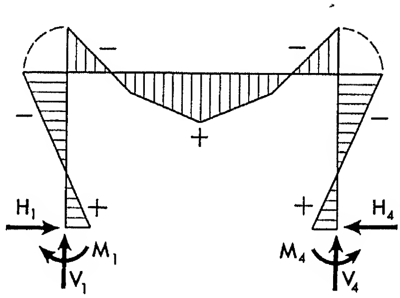
When $x > \frac{L}{3}$, but $\leqq \frac{L}{2}$

$$M_x = \frac{PL}{3} + M_2$$

Apply Eq. (3-1) to obtain the moment at any section of the left column. Moments at corresponding sections in the right half of the frame are identical to those in the left half.

3-9. Three Equal Vertical Concentrated Loads on Girder





$$M_1 = M_4 = \frac{15PL}{8F}$$

$$M_2 = M_3 = -\frac{15PL}{4F} = -2M_1$$

$$H_1 = H_4 = \frac{3M_1}{h} \qquad V_1 = V_4 = \frac{3P}{2}$$

When $x \leqq \frac{L}{4}$

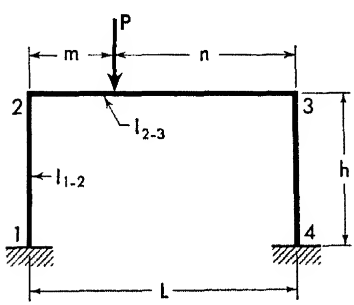
$$M_x = \frac{3Px}{2} + M_2$$

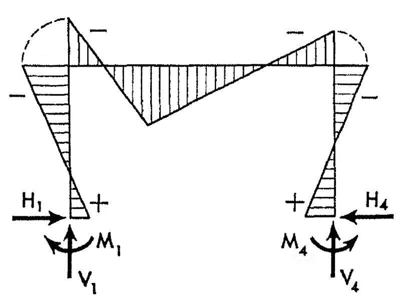
When $x > \frac{L}{4}$, but $\leqq \frac{L}{2}$

$$M_x = \frac{P(L + 2x)}{4} + M_2$$

Apply Eq. (3-1) to obtain the moment at any section of the left column. Moments at corresponding sections in the right half of the frame are identical to those in the left half.

3-10. Vertical Concentrated Load at Any Point of Girder





$$J = \frac{6mn}{L^2} \qquad K = \frac{J}{3}\left(1 - \frac{2m}{L}\right)$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = \frac{PL}{2}\left(\frac{J}{F} \mp \frac{K}{D}\right)$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -PL\left(\frac{J}{F} \pm \frac{K}{2D}\right)$$

$$H_1 = H_4 = \frac{3PLJ}{2Fh}$$

For Notations and Constants, see Arts. 3-1 and 3-2

$$V_1 = P\left(1 - \frac{m}{L} + \frac{K}{D}\right) \qquad V_4 = P\left(\frac{m}{L} - \frac{K}{D}\right)$$

When $x \leqq m$

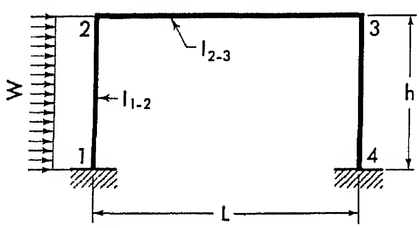
$$M_x = (Pn + M_3)\frac{x}{L} + M_2\left(1 - \frac{x}{L}\right)$$

When $x > m$

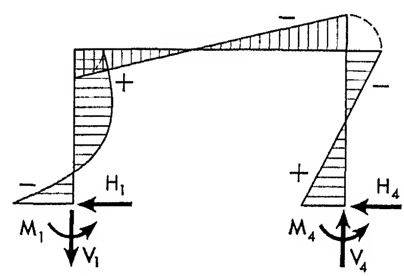
$$M_x = (Pm + M_2)\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L}$$

Apply Eqs. (3-1) and (3-3) to obtain the moment at any section of the frame columns.

3-11. Horizontal Uniform Load on Column



W = total load



$$J = \frac{3\phi}{4} + \frac{1}{4} \qquad K = \frac{D\phi - 4}{4D}$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = -\frac{Wh}{\phi}\left(\frac{J}{F} \pm K\right)$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -\frac{Wh}{4\phi}\left(\frac{1}{F} \mp \frac{4}{D}\right)$$

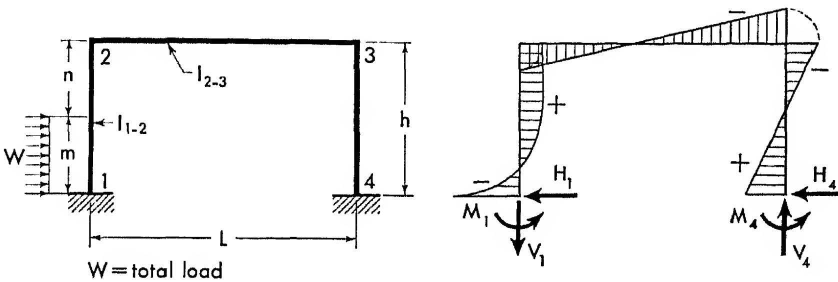
$$H_4 = \frac{W}{4}\left(1 - \frac{4J - 1}{F\phi}\right) \qquad H_1 = -(W - H_4)$$

$$V_4 = \frac{2Wh}{DL\phi} \qquad V_1 = -V_4$$

$$M_{y_1} = \left(\frac{Wy_1}{2} + M_1\right)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (3-2) and (3-3) to obtain the moment at any section of frame members 2-3 and 3-4.

3-12. Horizontal Uniform Load over Part of Column



$$g = \frac{m}{h} \qquad G = \frac{g}{4}(4 - 2g^2 - J)$$

$$J = (2 - g)^2$$

$$K = \frac{g}{4}[2 - g^2 - J(2 + 3\phi)]$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = \frac{Wh}{\phi}\left[\frac{K}{F} \mp \frac{g(D\phi - 4g)}{4D}\right]$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -\frac{Wh}{\phi}\left(\frac{G}{F} \mp \frac{g^2}{D}\right)$$

$$H_4 = W\left(\frac{g}{4} + \frac{G + K}{F\phi}\right) \qquad H_1 = -(W - H_4)$$

$$V_4 = \frac{2Whg^2}{DL\phi} \qquad V_1 = -V_4$$

When $y_1 \leqq m$

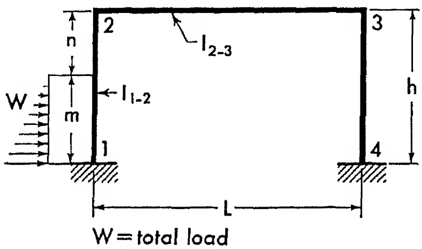
$$M_{y_1} = \frac{Wy_1}{2}\left(\frac{n}{h} + \frac{m - y_1}{m}\right) + M_1\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

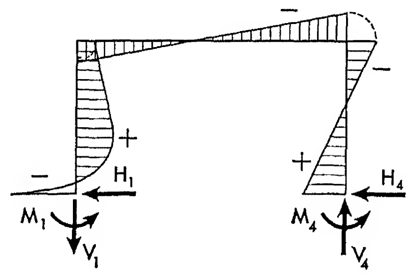
When $y_1 > m$

$$M_{y_1} = \left(\frac{Wm}{2} + M_1\right)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (3-2) and (3-3) to obtain the moment at any section of frame members 2-3 and 3-4.

3-13. Horizontal Triangular Load over Part of Column





$$g = \frac{m}{h} \qquad G = \frac{g^2}{10}(5 - 3g)$$

$$J = \frac{g}{15}(20 - 15g + 3g^2)$$

$$K = g\left(\frac{1}{3} - \frac{g^2}{10}\right) - J\left(1 + \frac{3\phi}{2}\right)$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = \frac{Wh}{\phi}\left[\frac{K}{F} \mp \frac{g}{6D}(D\phi - 3g)\right]$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -\frac{Wh}{\phi}\left(\frac{G}{F} \mp \frac{g^2}{2D}\right)$$

$$H_4 = W\left(\frac{G + K}{F\phi} + \frac{g}{6}\right) \qquad H_1 = -(W - H_4)$$

$$V_4 = \frac{Whg^2}{DL\phi} \qquad V_1 = -V_4$$

When $y_1 \leqq m$

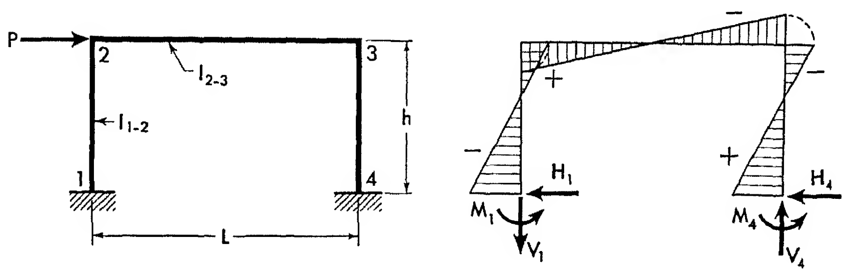
$$M_{y_1} = \frac{Wy_1}{3}\left[\frac{n}{h} + \frac{(m - y_1)(2m - y_1)}{m^2}\right] + M_1\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

When $y_1 > m$

$$M_{y_1} = \left(\frac{Wm}{3} + M_1\right)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (3-2) and (3-3) to obtain the moment at any section of frame members 2-3 and 3-4.

3-14. Horizontal Concentrated Load at Joint 2



$$K = \frac{3}{D\phi}$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = \mp Ph\left(\frac{1}{2} - K\right)$$

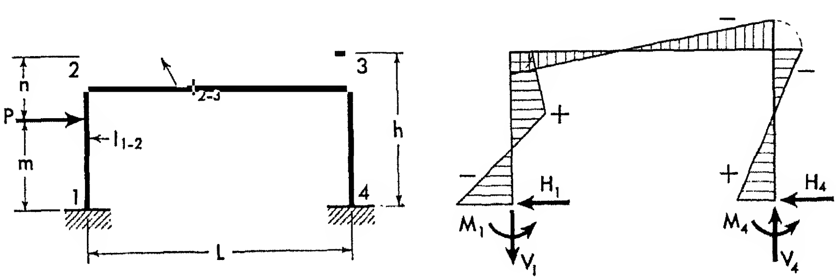
$$M_2 = PhK \qquad M_3 = -PhK$$

$$H_1 = -\frac{P}{2} \qquad H_4 = \frac{P}{2}$$

$$V_4 = \frac{2PhK}{L} \qquad V_1 = -V_4$$

Apply Eqs. (3-1) through (3-3) to obtain the moment at any section of the frame members.

3-15. Horizontal Concentrated Load at Any Point of Column



$$G = \frac{3m^2n}{h^2\phi} \qquad J = G\,\frac{n + \phi(h + n)}{m}$$

$$K = \frac{6m}{\phi}\left(1 - \frac{n}{h}\right)$$

For Notations and Constants, see Arts. 3-1 and 3-2

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = -P\left(\frac{J}{F} \pm \frac{Dm - K}{2D}\right)$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -P\left(\frac{G}{F} \mp \frac{K}{2D}\right)$$

$$H_4 = \frac{P}{h}\left(\frac{G - J}{F} + \frac{m}{2}\right) \qquad H_1 = -(P - H_4)$$

$$V_4 = \frac{PK}{DL} \qquad V_1 = -V_4$$

When $y_1 \leqq m$

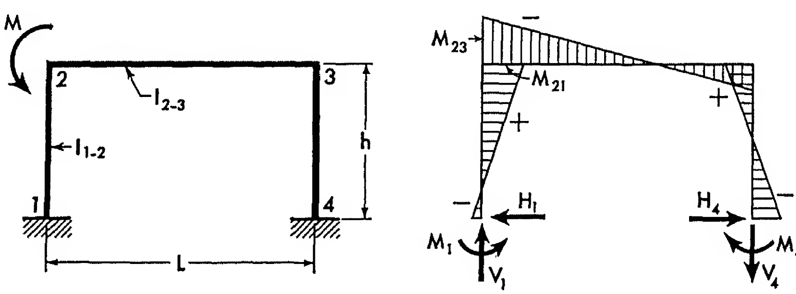
$$M_{y_1} = (M_2 + Pn)\frac{y_1}{h} + M_1\left(1 - \frac{y_1}{h}\right)$$

When $y_1 > m$

$$M_{y_1} = (M_1 + Pm)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (3-2) and (3-3) to obtain the moment at any section of frame members 2-3 and 3-4.

3-16. Moment Applied at Joint 2



$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = -M\left(\frac{3}{F} \mp \frac{1}{D}\right)$$

$$\left.\begin{matrix} M_{21} \\ M_3 \end{matrix}\right\rangle = M\left(\frac{6}{F} \pm \frac{1}{D}\right) \qquad M_{23} = -(M - M_{21})$$

$$H_1 = H_4 = -\frac{9M}{Fh}$$

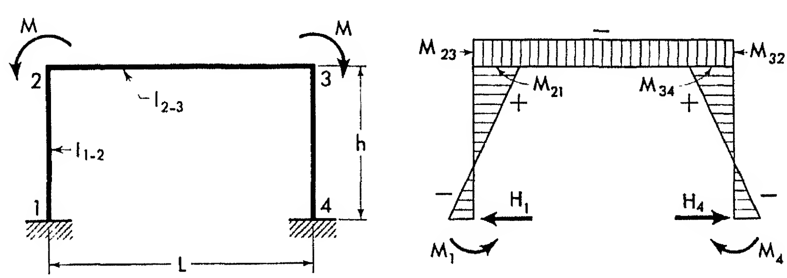
$$V_1 = \frac{M}{L}\left(1 - \frac{2}{D}\right) \qquad V_4 = -\frac{M}{L}\left(1 - \frac{2}{D}\right)$$

$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) + M_{21}\frac{y_1}{h}$$

$$M_x = M_{23}\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L}$$

$$M_{y_4} = M_4\left(1 - \frac{y_4}{h}\right) + M_3\frac{y_4}{h}$$

3-17. Two Equal Moments Applied at Joints 2 and 3



$$M_1 = M_4 = -\frac{6M}{F}$$

$$M_{21} = M_{34} = \frac{12M}{F}$$

$$M_{23} = M_{32} = -(M - M_{21})$$

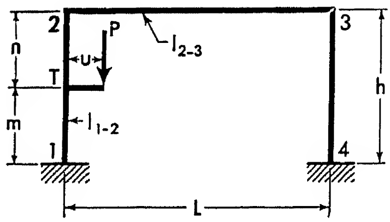
$$H_1 = H_4 = -\frac{18M}{Fh} \qquad V_1 = V_4 = 0$$

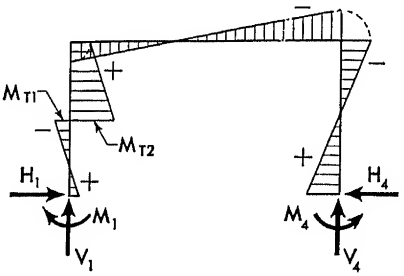
$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) + M_{21}\frac{y_1}{h}$$

$$M_x = M_{23}$$

$$M_{y_4} = M_4\left(1 - \frac{y_4}{h}\right) + M_{34}\frac{y_4}{h}$$

3-18. Vertical Concentrated Load Applied at Bracket





Bracket acts as a simple cantilever and its maximum moment is Pu at point T. The moment diagram of the cantilever is intentionally not shown so that the frame's bending moment diagram may be more clearly illustrated.

$$M = Pu \qquad g = \frac{m}{h} \qquad J = 2g - 3g^2$$

$$K = 1 - 2g + (1 + \phi)(J + 4g - 2)$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = \frac{M}{\phi}\left(\frac{3K}{F} \mp \frac{D\phi - 12g}{2D}\right)$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -\frac{3M}{\phi}\left(\frac{J}{F} \mp \frac{2g}{D}\right)$$

$$H_1 = H_4 = \frac{M}{h}\left[\frac{1}{2} + \frac{3(J+K)}{F\phi}\right]$$

$$V_4 = \frac{12Mg}{DL\phi} \qquad V_1 = P - V_4$$

When $y_1 < m$

$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) - (M - M_2)\frac{y_1}{h}$$

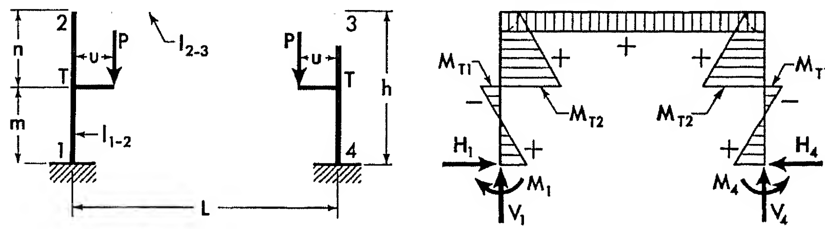
When $y_1 = m$

$$M_{T1} = M_1\frac{n}{h} - (M - M_2)\frac{m}{h} \qquad \text{and} \qquad M_{T2} = (M + M_1)\frac{n}{h} + M_2\frac{m}{h}$$

When $y_1 > m$, $\qquad M_{y_1} = (M + M_1)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$

Apply Eqs. (3-2) and 3-3) to obtain the moment at any section of frame members 2-3 and 3-4.

3-19. Two Equal Vertical Concentrated Loads Symmetrically Applied at Brackets



Brackets act as simple cantilevers with the maximum moments of Pu at points T. The moment diagrams of these cantilevers are intentionally not shown so that the frame's bending moment diagram may be more clearly illustrated.

$$M = Pu \qquad g = \frac{m}{h} \qquad J = 2g - 3g^2$$

$$K = 1 - 2g + (1 + \phi)(J + 4g - 2)$$

$$M_1 = M_4 = \frac{6MK}{F\phi}$$

$$M_2 = M_3 = -\frac{6MJ}{F\phi}$$

$$H_1 = H_4 = \frac{M}{h}\left[1 + \frac{6(J + K)}{F\phi}\right]$$

$$V_1 = V_4 = P$$

When $y_1 < m$

$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) - (M - M_2)\frac{y_1}{h}$$

When $y_1 = m$

$$M_{T1} = M_1\frac{n}{h} - (M - M_2)\frac{m}{h} \qquad \text{and} \qquad M_{T2} = (M + M_1)\frac{n}{h} + M_2\frac{m}{h}$$

When $y_1 > m$

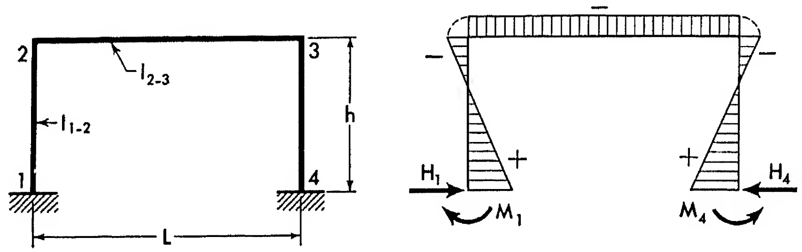
$$M_{y_1} = (M + M_1)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

$$M_x = M_2$$

Moments at corresponding sections in the right half of the frame are identical to those in the left half.

For Notations and Constants, see Arts. 3-1 and 3-2

3-20. Effect of Temperature Rise. Range t° for entire frame.



$$K = \frac{6L\varepsilon t^\circ}{h^2(1+2\phi)} EI_{1-2}$$

$$M_1 = M_4 = \frac{K}{2}(1+\phi)$$

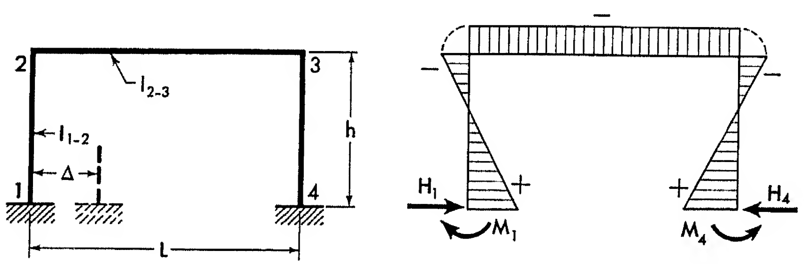
$$M_2 = M_3 = -\frac{K}{2}$$

$$H_1 = H_4 = \frac{K}{h}\left(1+\frac{\phi}{2}\right) \qquad V_1 = V_4 = 0$$

Apply Eqs. (3-1) through (3-3) to obtain the moment at any section of the frame members.

Note: For temperature drop, introduce the value of t° with a negative sign.

3-21. Horizontal Displacement of One Support



$$K = \frac{6\Delta}{h^2(1+2\phi)} EI_{1-2}$$

$$M_1 = M_4 = \frac{K}{2}(1+\phi)$$

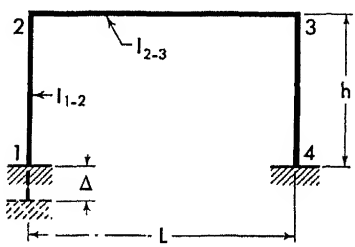
$$M_2 = M_3 = -\frac{K}{2}$$

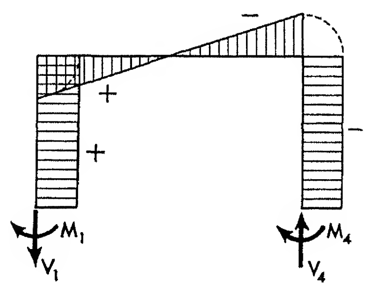
$$H_1 = H_4 = \frac{K}{h}\left(1 + \frac{\phi}{2}\right) \qquad V_1 = V_4 = 0$$

Apply Eqs. (3-1) through (3-3) to obtain the moment at any section of the frame members.

Note: If the direction of the frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.

3-22. Vertical Settlement of One Support





$$M_1 = M_2 = \frac{12\Delta}{DLh\phi} EI_{1\text{-}2}$$

$$M_3 = M_4 = -M_1 \qquad M_{2.5} = 0$$

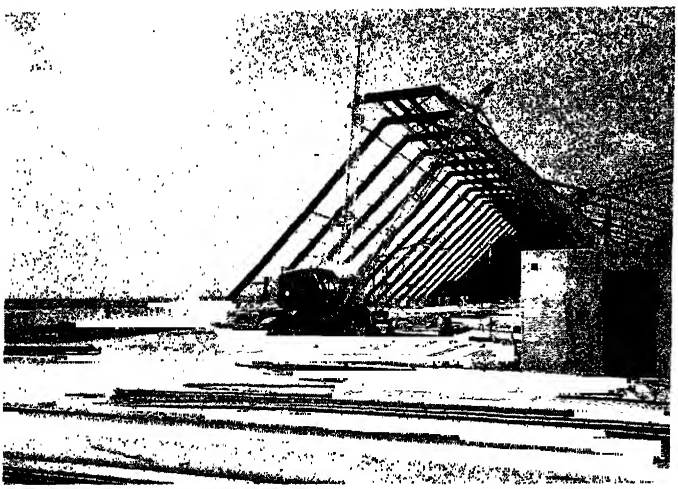
$$H_1 = H_4 = 0$$

$$V_1 = -\frac{24\Delta}{DL^2h\phi} EI_{1\text{-}2} \qquad V_4 = -V_1$$

$$M_{y_1} = M_1 \qquad M_x = M_1 + V_1x$$

$$M_{y_4} = M_4$$

Note: If the direction of frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.

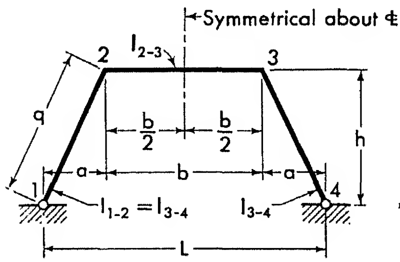


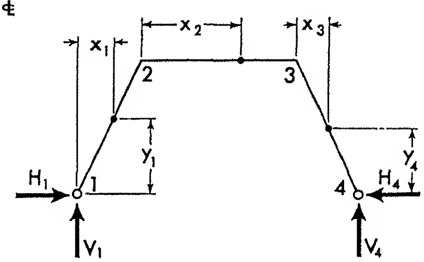
An erection view of the phosphate storage building of the U.S. Phosphoric Products Co., at East Tampa, Florida. This photograph illustrates the judicious use of trapezoidal frames for the framework of a bulk storage building. Slanted columns divert wind flow from the structure and as a result function primarily in tension or compression, thereby lending features of efficiency and economy to the frame. (Courtesy of the Ingalls Iron Works Co., of Birmingham, Ala., steel fabricator.)

SECTION 4

SYMMETRICAL TRAPEZOIDAL FRAMES WITH HINGED SUPPORTS

4-1. Notations, Coordinates, and Frame Constants





The sketch appearing on the left, above, explains notations for a representative trapezoidal frame with members of constant cross section.

The sketch appearing on the right, above, shows positive directions of the vertical and horizontal components of the frame reactions. It also defines the coordinates for any section of the frame. Coordinates are to be considered only in the positive sense.

Frame Constants: $\phi = \frac{I_{1-2}}{I_{2-3}} \cdot \frac{b}{q}$

$$A = 4\left(3 + \frac{2}{\phi}\right)$$

4-2. Equations of Frame Reactions and Moments. The equations for the vertical and the redundant horizontal components of frame reactions are

given on the following pages for a number of loading conditions. Corresponding expressions for the moment and forces at any section of a member with applied load are also provided.

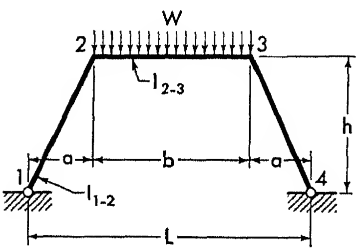
The equations for the moments of load-free members are listed below for reference.

$$M_{x_1} = M_2 \frac{x_1}{a} \tag{4-1}$$

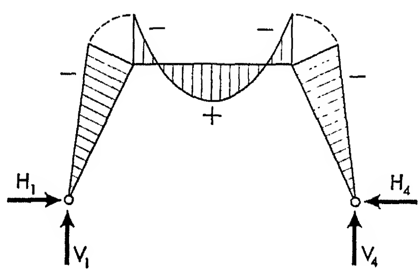
$$M_{x_2} = M_2 \left(1 - \frac{x_2}{b}\right) + M_3 \frac{x_2}{b} \tag{4-2}$$

$$M_{x_3} = M_3 \left(1 - \frac{x_3}{a}\right) \tag{4-3}$$

4-3. Vertical Uniform Load on Girder



W = total load



$$M_2 = M_3 = -\frac{Wb}{A}$$

$$H_1 = H_4 = \frac{W}{h}\left(\frac{a}{2} + \frac{b}{A}\right)$$

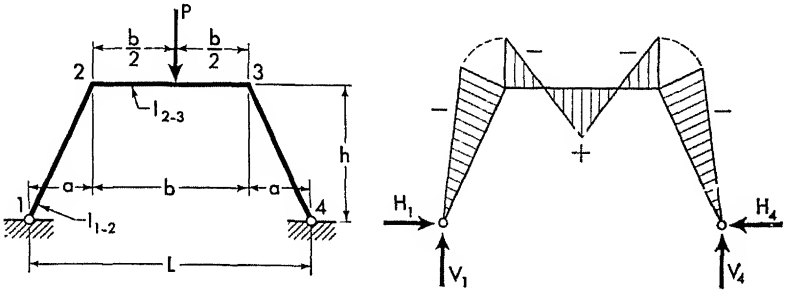
$$V_1 = V_4 = \frac{W}{2}$$

$$M_{x_1} = M_2 \frac{x_1}{a}$$

$$M_{x_2} = \frac{Wx_2}{2}\left(1 - \frac{x_2}{b}\right) + M_2$$

Moments at corresponding sections in the right half of the frame are identical to those in the left half.

4-4. Vertical Concentrated Load at Center of Girder



$$M_2 = M_3 = -\frac{3Pb}{2A}$$

$$H_1 = H_4 = \frac{P}{2h}\left(a + \frac{3b}{A}\right)$$

$$V_1 = V_4 = \frac{P}{2}$$

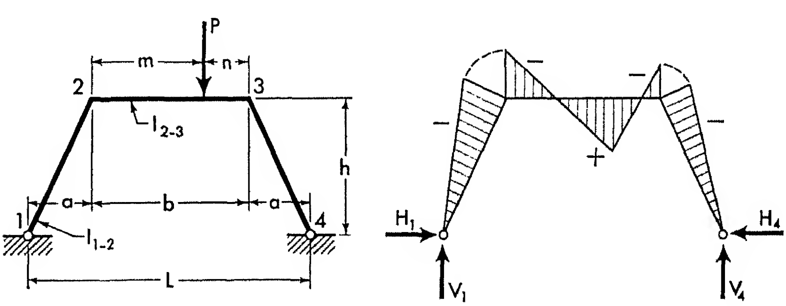
$$M_{x_1} = M_2 \frac{x_1}{a}$$

When $x_2 \leqq \frac{b}{2}$

$$M_{x_2} = \frac{Px_2}{2} + M_2$$

Moments at corresponding sections in the right half of the frame are identical to those in the left half.

4-5. Vertical Concentrated Load at Any Point of Girder



$$K = \frac{6mn}{Ab}$$

For Notations and Constants, see Arts. 4-1 and 4-2

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -P\left[K \mp \frac{a(b-2m)}{2L}\right]$$

$$H_1 = H_4 = \frac{P}{2h}(a + 2K) \qquad \left.\begin{matrix} V_1 \\ V_4 \end{matrix}\right\rangle = \frac{P}{2}\left(1 \pm \frac{b-2m}{L}\right)$$

When $x_2 \leqq m$

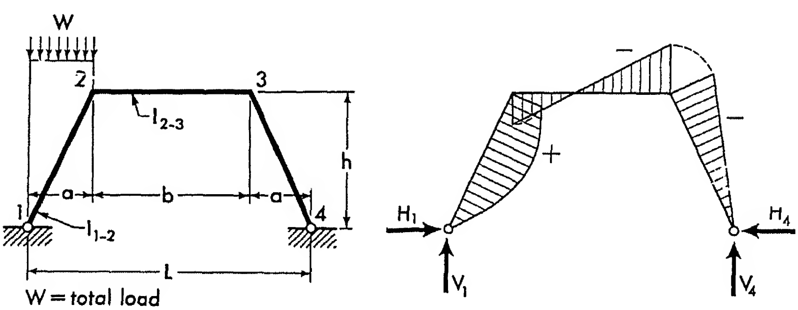
$$M_{x_2} = (Pn + M_3)\frac{x_2}{b} + M_2\left(1 - \frac{x_2}{b}\right)$$

When $x_2 > m$

$$M_{x_2} = (Pm + M_2)\left(1 - \frac{x_2}{b}\right) + M_3\frac{x_2}{b}$$

Apply Eqs. (4-1) and (4-3) to obtain the moment at any section of the frame columns.

4-6. Vertical Uniform Load on Inclined Column



$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -\frac{Wa}{2}\left(\frac{1}{A\phi} \mp \frac{b}{2L}\right)$$

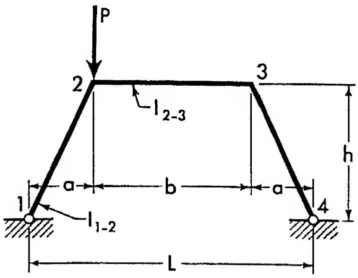
$$H_1 = H_4 = \frac{Wa}{2h}\left(\frac{1}{2} + \frac{1}{A\phi}\right)$$

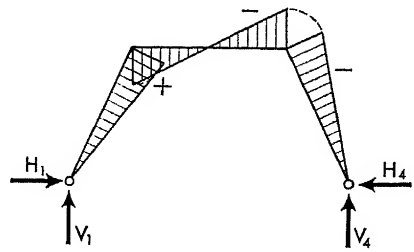
$$V_4 = \frac{Wa}{2L} \qquad V_1 = W - V_4$$

$$M_{x_1} = \frac{Wx_1}{2}\left(1 - \frac{x_1}{a}\right) + M_2\frac{x_1}{a}$$

Apply Eqs. (4-2) and (4-3) to obtain the moment at any section of frame members 2-3 and 3-4.

4-7. Vertical Concentrated Load at Joint 2



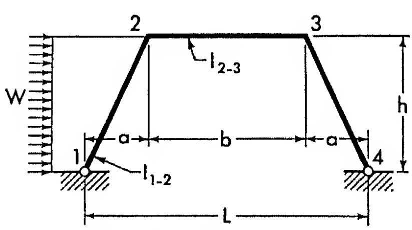


$$M_2 = \frac{Pab}{2L} \qquad M_3 = -\frac{Pab}{2L} \qquad H_1 = H_4 = \frac{Pa}{2h}$$

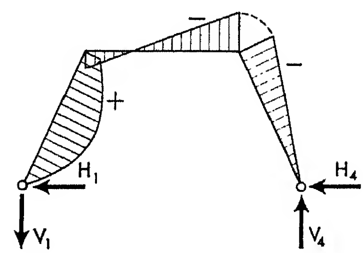
$$V_1 = \frac{P}{2}\left(1 + \frac{b}{L}\right) \qquad V_4 = \frac{P}{2}\left(1 - \frac{b}{L}\right)$$

Apply Eqs. (4-1) through (4-3) to obtain the moment at any section of the frame members.

4-8. Horizontal Uniform Load on Inclined Column



W = total load



$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -\frac{Wh}{2}\left(\frac{1}{A\phi} \mp \frac{b}{2L}\right)$$

$$H_4 = \frac{W}{2}\left(\frac{1}{2} + \frac{1}{A\phi}\right) \qquad H_1 = -(W - H_4)$$

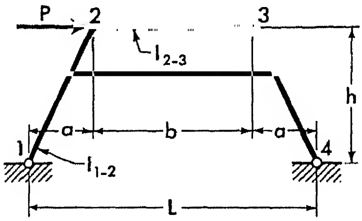
$$V_4 = \frac{Wh}{2L} \qquad V_1 = -V_4$$

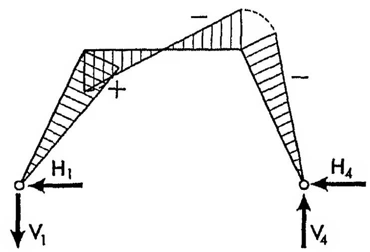
$$M_{y_1} = \frac{Wy_1}{2}\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (4-2) and (4-3) to obtain the moment at any section of frame members 2-3 and 3-4.

For Notations and Constants, see Arts. 4-1 and 4-2

4-9. Horizontal Concentrated Load at Joint 2





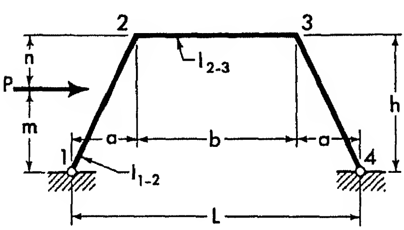
$$M_2 = \frac{Pbh}{2L} \qquad M_3 = -\frac{Pbh}{2L}$$

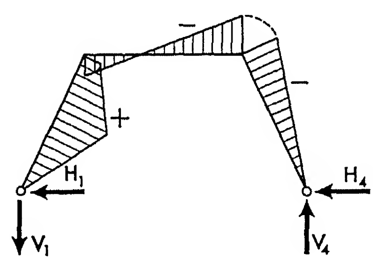
$$H_1 = -\frac{P}{2} \qquad H_4 = \frac{P}{2}$$

$$V_4 = \frac{Ph}{L} \qquad V_1 = -V_4$$

Apply Eqs. (4-1) through (4-3) to obtain the moment at any section of the frame members.

4-10. Horizontal Concentrated Load at Any Point of Inclined Column





$$g = \frac{m}{h} \qquad K = \frac{4(1-g^2)}{A\phi}$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -\frac{Pm}{2}\left(K \mp \frac{b}{L}\right)$$

$$H_4 = \frac{Pm}{2h}(1+K) \qquad H_1 = -(P - H_4)$$

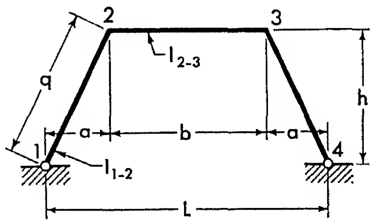
$$V_4 = \frac{Pm}{L} \qquad V_1 = -V_4$$

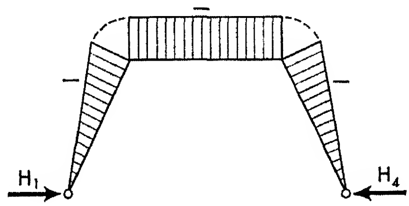
When $y_1 \leqq m$, $\qquad M_{y_1} = (Pn + M_2)\frac{y_1}{h}$

When $y_1 > m$, $\quad M_{y_1} = Pm\left(1 - \frac{y_1}{h}\right) + M_2 \frac{y_1}{h}$

Apply Eqs. (4-2) and (4-3) to obtain the moment at any section of frame members 2-3 and 3-4.

4-11. Effect of Temperature Rise. Range $t°$ for entire frame.





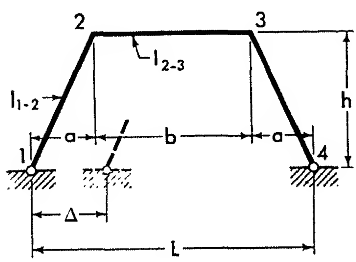
$$H_1 = H_4 = \frac{3L\varepsilon t°}{h^2 q(3\phi + 2)} EI_{1-2}$$

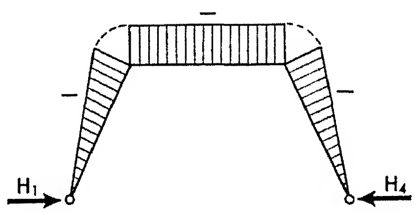
$M_2 = M_3 = -H_4 h \qquad V_1 = V_4 = 0$

Apply Eqs. (4-1) through (4-3) to obtain the moment at any section of frame members.

Note: For temperature drop, use the value of $t°$ with a negative sign.

4-12. Horizontal Displacement of One Support





$$H_1 = H_4 = \frac{3\Delta}{h^2 q(3\phi + 2)} EI_{1-2}$$

$M_2 = M_3 = -H_4 h \qquad V_1 = V_4 = 0$

Apply Eqs. (4-1) through (4-3) to obtain the moment at any section of the frame members.

Note: If the direction of the frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.

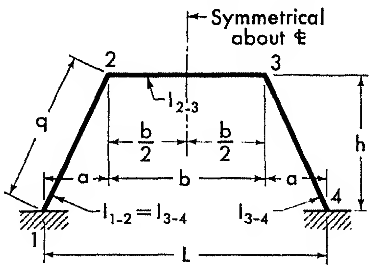
For Notations and Constants, see Arts. 4-1 and 4-2

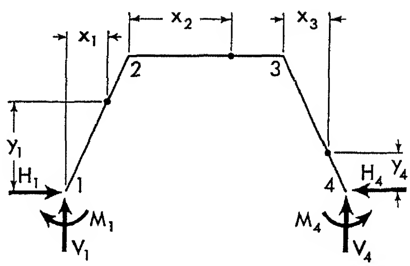
A night view of the gigantic head shaft of the Germania Colliery in Dortmund, Germany. This outstanding structure consists of two trapezoidal frames, braced together for lateral stability. It towers 230 feet in the air, spans a 300-foot distance, and was designed to lift 70 tons of payload from a shaft 3,300 feet deep. Despite its size and the heavy applied load, the structure is characterized by comparatively small members, thus demonstrating the extreme utility of trapezoidal frames in such application. As a matter of record the column is a built-up I member having a web plate only 6 feet 6 inches deep and flanges 24 inches wide. Furthermore, although representing a structure of heavy industrial class, it demonstrates that architectural treatment may be effectively incorporated, imparting neatness and expressiveness to the structure. Designed and erected by Dortmunder Union Bruckenbau-Aktiengesellschaft of Dortmund, Germany. (Courtesy of Dortmunder Union Bruckenbau-Akt.)

SECTION 5

SYMMETRICAL TRAPEZOIDAL FRAMES WITH FIXED SUPPORTS

5-1. Notations, Coordinates, and Frame Constants





The sketch appearing on the left, above, explains notations for a representative trapezoidal frame with members of constant cross section.

The sketch appearing on the right, above, shows positive directions of the moment and the vertical and horizontal components of the frame reactions. It also defines the coordinates for any section of the frame. Coordinates are to be considered only in the positive sense.

Frame Constants: $\phi = \frac{I_{1-2}}{I_{2-3}} \cdot \frac{b}{q}$

$$\mu = \frac{L}{b} \qquad C = 1 + \frac{1}{2\phi}$$

$$D = \phi + 2 + 2\mu + 2\mu^2$$

$$F = \frac{\phi + 2 + \mu}{D}$$

5-2. Equations of Frame Reactions and Moments. The equations for the redundant moments and the vertical and horizontal components of frame reactions are given on the following pages for a number of loading conditions. Corresponding expressions for the moment and forces at any section of a member with applied load are also provided.

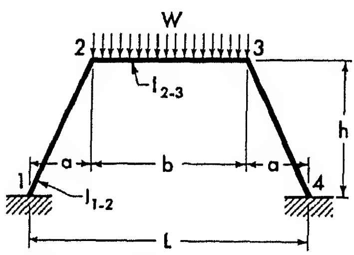
The equations for the moments of load-free members are listed below for reference.

$$M_{x_1} = M_1\left(1 - \frac{x_1}{a}\right) + M_2\frac{x_1}{a} \tag{5-1}$$

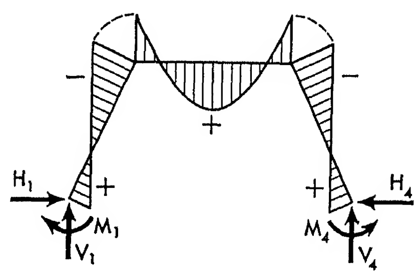
$$M_{x_2} = M_2\left(1 - \frac{x_2}{b}\right) + M_3\frac{x_2}{b} \tag{5-2}$$

$$M_{x_3} = M_3\left(1 - \frac{x_3}{a}\right) + M_4\frac{x_3}{a} \tag{5-3}$$

5-3. Vertical Uniform Load on Girder



W = Total load



$$M_1 = M_4 = \frac{Wb}{24C}$$

$$M_2 = M_3 = -\frac{Wb}{12C}$$

$$H_1 = H_4 = \frac{W}{2h}\left(a + \frac{b}{4C}\right)$$

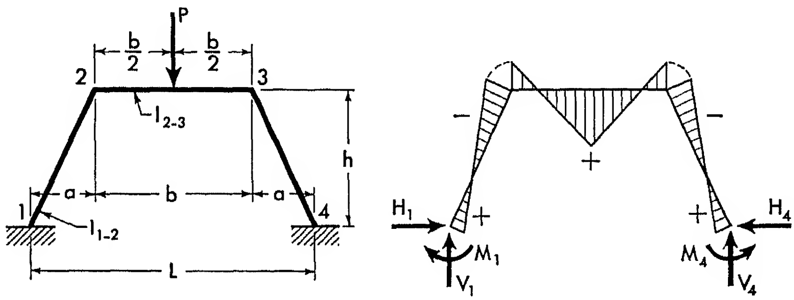
$$V_1 = V_4 = \frac{W}{2}$$

$$M_{x_2} = \frac{Wx_2}{2}\left(1 - \frac{x_2}{b}\right) + M_2$$

Apply Eq. (5-1) to obtain the moment at any section of the left column. Moments at corresponding sections in the right half of the frame are identical to those in the left half.

Members of Constant Section

5-4. Vertical Concentrated Load at Mid-point of Girder



$$M_1 = M_4 = \frac{Pb}{16C} \qquad M_2 = M_3 = -\frac{Pb}{8C}$$

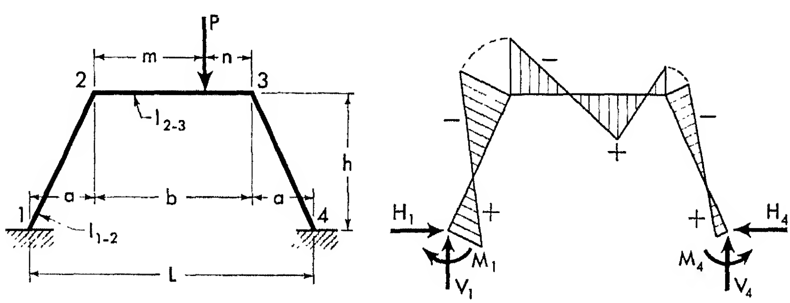
$$H_1 = H_4 = \frac{P}{2h}\left(a + \frac{3b}{8C}\right)$$

$$V_1 = V_4 = \frac{P}{2}$$

When $x_2 \leqq \frac{b}{2}$, $\qquad M_{x_2} = \frac{Px_2}{2} + M_2$

Apply Eq. (5-1) to obtain the moment at any section of frame member 1-2. Moments at corresponding sections in the right half of the frame are identical to those in the left half.

5-5. Vertical Concentrated Load at Any Point of Girder



$$g = \frac{m}{b} \qquad B = \frac{g(1-g)}{2C}$$

$$G = 2g(1-g)(1-2g)$$

For Notations and Constants, see Arts. 5-1 and 5-2

$$J = \frac{Fa(1-2g)}{b} + \frac{G\mu\phi}{2D}$$

$$K = \frac{a(1-F)(1-2g)}{L} - \frac{G\phi}{2D}$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = \frac{Pb}{2}(B \mp J)$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -\frac{Pb}{2}(2B \mp K)$$

$$H_1 = H_4 = \frac{P}{2h}(a + 3Bb)$$

$$\left.\begin{matrix} V_1 \\ V_4 \end{matrix}\right\rangle = \frac{P}{2} \pm \frac{P}{2}\left[\frac{(1-2g)(2Fa+b)}{L} + \frac{G\phi}{D}\right]$$

When $x_2 \leqq m$

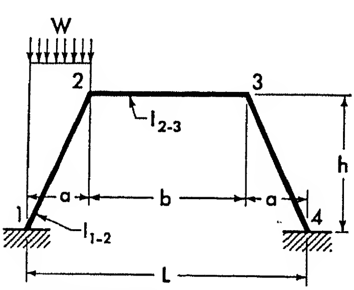
$$M_{x_2} = (Pn + M_3)\frac{x_2}{b} + M_2\left(1 - \frac{x_2}{b}\right)$$

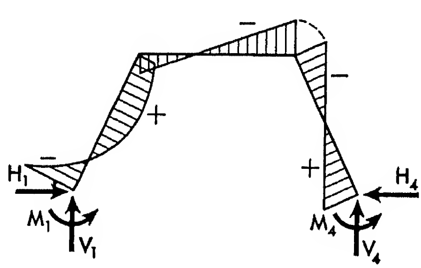
When $x_2 > m$

$$M_{x_2} = (Pm + M_2)\left(1 - \frac{x_2}{b}\right) + M_3\frac{x_2}{b}$$

Apply Eqs. (5-1) and (5-3) to obtain the moment at any section of the frame columns.

5-6. Vertical Uniform Load on Inclined Column





$$G = \frac{1}{48C\phi} \qquad J = F + \frac{\mu(\mu+1)}{2D}$$

Members of Constant Section

$$K = \frac{1-F}{\mu} - \frac{\mu+1}{2D}$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = -\frac{Wa}{4}\left[4G(1+3\phi) \pm J\right]$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -\frac{Wa}{4}(4G \mp K)$$

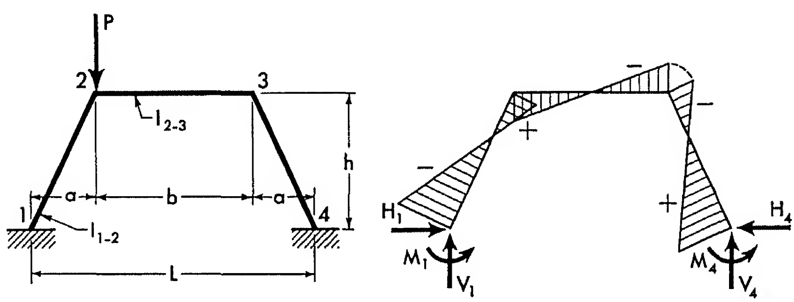
$$H_1 = H_4 = \frac{Wa}{4h}\left(1 - \frac{1}{4C}\right)$$

$$V_4 = \frac{Wa}{2L}(1-J) \qquad V_1 = W - V_4$$

$$M_{x_1} = \left(\frac{Wx_1}{2} + M_1\right)\left(1 - \frac{x_1}{a}\right) + M_2\frac{x_1}{a}$$

Apply Eqs. (5-2) and (5-3) to obtain the moment at any section of frame members 2-3 and 3-4.

5-7. Vertical Concentrated Load at Joint 2



$$J = \frac{Fa}{b} \qquad K = \frac{a(1-F)}{L}$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = \mp\frac{PJb}{2}$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = \pm\frac{PKb}{2}$$

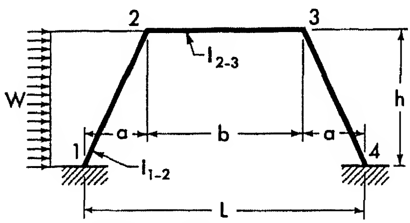
$$H_1 = H_4 = \frac{Pa}{2h}$$

For Notations and Constants, see Arts. 5-1 and 5-2

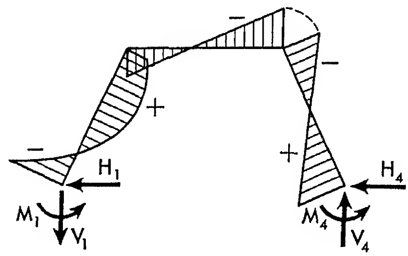
$$\left.\begin{matrix} V_1 \\ V_4 \end{matrix}\right\} = \frac{P}{2}\left(1 \pm \frac{2Fa + b}{L}\right)$$

Apply Eqs. (5-1) through (5-3) to obtain the moment at any section of the frame members.

5-8. Horizontal Uniform Load on Inclined Column



W = total load



$$G = \frac{\mu + 1}{2D} \qquad J = F + G\mu$$

$$K = \frac{1 - F}{\mu} - G$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\} = -\frac{Wh}{4}\left(\frac{1 + 3\phi}{12C\phi} \pm J\right)$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\} = -\frac{Wh}{4}\left(\frac{1}{12C\phi} \mp K\right)$$

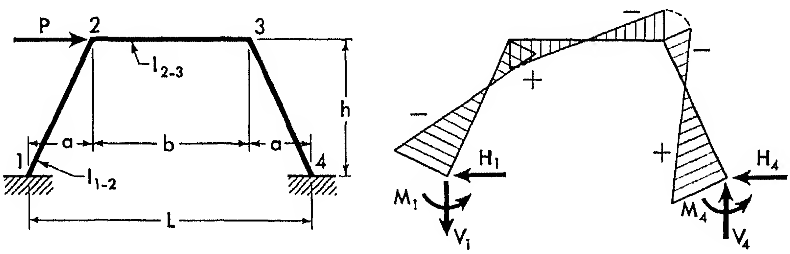
$$H_4 = \frac{W}{4}\left(1 - \frac{1}{4C}\right) \qquad H_1 = -(W - H_4)$$

$$V_4 = \frac{Wh}{2}\left(\frac{1 - F}{L} - \frac{G}{b}\right) \qquad V_1 = -V_4$$

$$M_{y_1} = \left(\frac{Wy_1}{2} + M_1\right)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (5-2) and (5-3) to obtain the moment at any section of frame members 2-3 and 3-4.

5-9. Horizontal Concentrated Load at Joint 2



$$\left.\begin{matrix}M_1\\ M_4\end{matrix}\right\rangle = \mp \frac{PFh}{2}$$

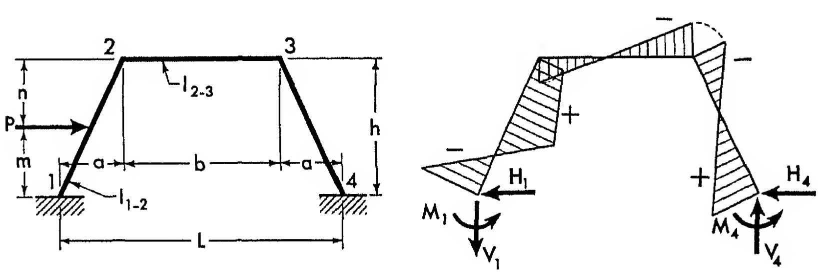
$$\left.\begin{matrix}M_2\\ M_3\end{matrix}\right\rangle = \pm \frac{Ph(1-F)}{2\mu}$$

$$H_1 = -\frac{P}{2} \qquad H_4 = \frac{P}{2}$$

$$V_4 = \frac{Ph(1-F)}{L} \qquad V_1 = -V_4$$

Apply Eqs. (5-1) through (5-3) to obtain the moment at any section of the frame members.

5-10. Horizontal Concentrated Load at Any Point of Inclined Column



$$A = \frac{n[3Lh - 2a(h+m)]}{2Dbh^2} \qquad B = \frac{mn}{2Ch^2\phi}$$

$$G = F + 2A\mu \qquad J = \frac{B[n + \phi(h+n)]}{m}$$

$$K = \frac{1-F}{\mu} - 2A$$

For Notations and Constants, see Arts. 5-1 and 5-2

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = -\frac{Pm}{2}(J \pm G) \qquad \left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -\frac{Pm}{2}(B \mp K)$$

$$H_4 = \frac{Pm}{2h}(1 + B - J) \qquad H_1 = -(P - H_4)$$

$$V_4 = \frac{Pm}{L}(1 - G) \qquad V_1 = -V_4$$

When $y_1 \leqq m$

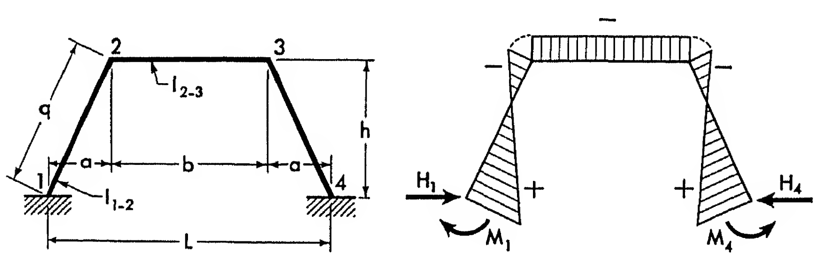
$$M_{y_1} = (Pn + M_2)\frac{y_1}{h} + M_1\left(1 - \frac{y_1}{h}\right)$$

When $y_1 > m$

$$M_{y_1} = (Pm + M_1)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (5-2) and (5-3) to obtain the moment at any section of frame members 2-3 and 3-4.

5-11. Effect of Temperature Rise. Range t° for entire frame.



$$K = \frac{3L\varepsilon t^\circ}{hq(1 + 2\phi)} EI_{1-2}$$

$$M_1 = M_4 = K(1 + \phi)$$

$$M_2 = M_3 = -K$$

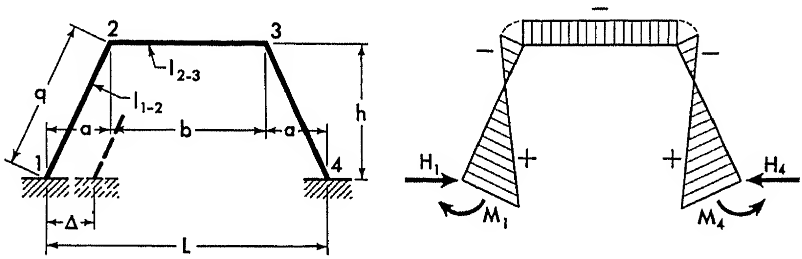
$$H_1 = H_4 = \frac{K}{h}(2 + \phi) \qquad V_1 = V_4 = 0$$

Apply Eqs. (5-1) through (5-3) to obtain the moment at any section of the frame members.

Note: For temperature drop, introduce the value of t° with a negative sign.

Members of Constant Section

5-12. Horizontal Displacement of One Support



$$K = \frac{3\Delta}{hq(1+2\phi)} EI_{1\text{-}2}$$

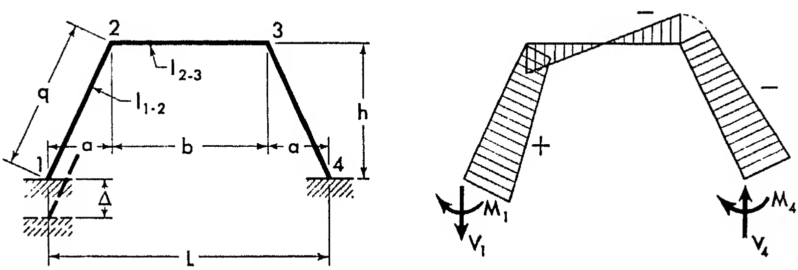
$$M_1 = M_4 = K(1+\phi)$$

$$M_2 = M_3 = -K \qquad H_1 = H_4 = \frac{K}{h}(2+\phi) \qquad V_1 = V_4 = 0$$

Apply Eqs. (5-1) through (5-3) to obtain the moment at any section of the frame members.

Note: If the direction of the displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.

5-13. Vertical Settlement of One Support



$$K = \frac{6\Delta}{Db^2q} EI_{1\text{-}2}$$

$$M_1 = -M_4 = KL$$

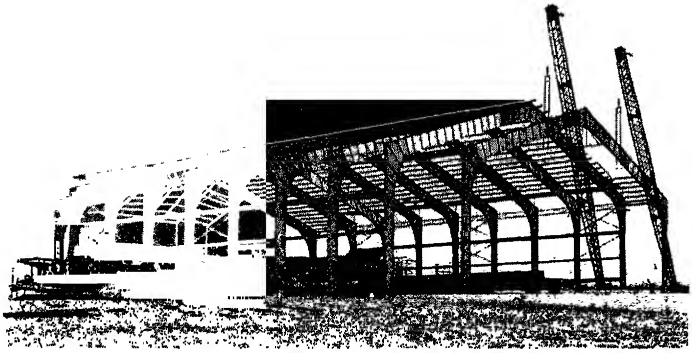
$$M_2 = -M_3 = Kb$$

$$V_1 = -2K \qquad V_4 = 2K \qquad H_1 = H_4 = 0$$

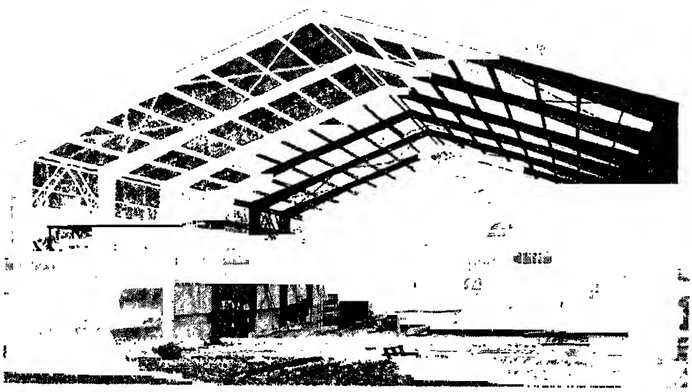
Apply Eqs. (5-1) through (5-3) to obtain the moment at any section of the frame members.

Note: If the direction of the frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.

For Notations and Constants, see Arts. 5-1 and 5-2



The structural framework of the Allen County War Memorial Coliseum, Fort Wayne, Indiana. These welded gable frames, which are 88 feet high and which span 227 feet, are the largest of their type in the United States. Each frame weighs 67 tons and has eight splices: six made on the ground and two made in place during assembly. Another interesting feature of this design is the introduction of cables between the columns' bases to restrain their movement. Designed by A. M. Strauss, Inc., of Fort Wayne, Indiana. (Courtesy of the Lincoln Electric Co., Cleveland, Ohio.)

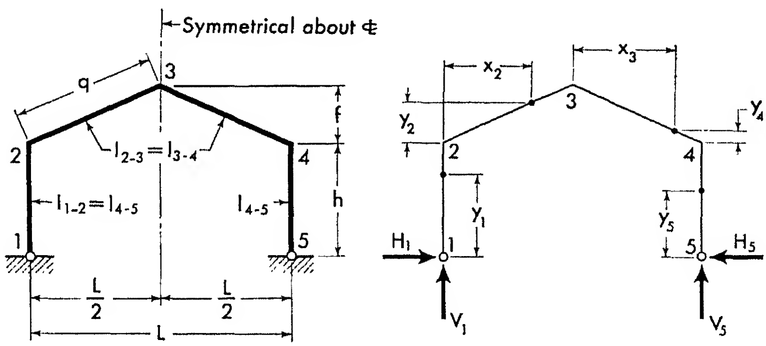


The problem of designing structural frameworks that will conform to provisions of building codes and provide optimum working space in the building has been economically solved by the use of gable frames. The solution is so attractive economically that several steel fabricators carry a complete stock of gable frames of various spans and rises. The illustration above shows a typical gable frame with a span of 80 feet, manufactured by the Butler Company of Kansas City, Missouri. Note that the girders are slightly haunched toward the columns, thereby providing additional strength and giving the frames a neat appearance. (Courtesy of the Lincoln Electric Co., Cleveland, Ohio.)

SECTION 6

SYMMETRICAL GABLE FRAMES WITH HINGED SUPPORTS

6-1. Notations, Coordinates, and Frame Constants



The sketch appearing on the left, above, explains notations for a representative gable frame with members of constant cross section.

The sketch appearing on the right, above, shows positive directions of the vertical and horizontal components of the frame reactions. It also defines the coordinates for any section of the frame. Coordinates are to be considered only in the positive sense.

General Frame Constants: $\phi = \dfrac{I_{1-2}}{I_{2-3}} \cdot \dfrac{q}{h}$ $\qquad \psi = \dfrac{f}{h}$

$$A = 4\left(3 + 3\psi + \psi^2 + \frac{1}{\phi}\right) \qquad B = 2(3 + 2\psi)$$

Constant C. To be used only in cases of horizontal load on the frame.

$$C = 2\left(3 + \psi + \frac{2}{\phi}\right)$$

6-2. Equations of Frame Reactions and Moments. The equations for the vertical and the redundant horizontal components of frame reactions are given on following pages for a number of loading conditions. Corresponding expressions for the moment and forces at any section of a member with applied load are also provided.

The equations for the moments of load-free members are listed below for reference.

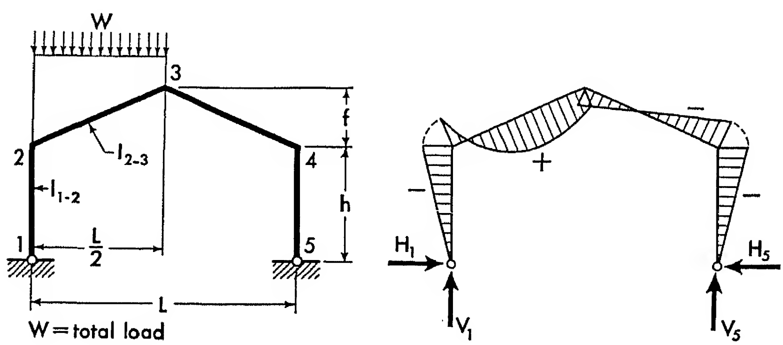
$$M_{y_1} = M_2 \frac{y_1}{h} \tag{6-1}$$

$$M_{x_2} = M_2 \left(1 - \frac{2x_2}{L}\right) + M_3 \frac{2x_2}{L} \tag{6-2}$$

$$M_{x_3} = M_3 \left(1 - \frac{2x_3}{L}\right) + M_4 \frac{2x_3}{L} \tag{6-3}$$

$$M_{y_5} = M_4 \frac{y_5}{h} \tag{6-4}$$

6-3. Vertical Uniform Load on Left Inclined Member



$K = 2 + \psi \qquad H_1 = H_5 = \dfrac{WL}{8Ah}(B + K)$

$M_2 = M_4 = -H_5 h \qquad M_3 = \dfrac{WL}{8} - H_5 h(1 + \psi)$

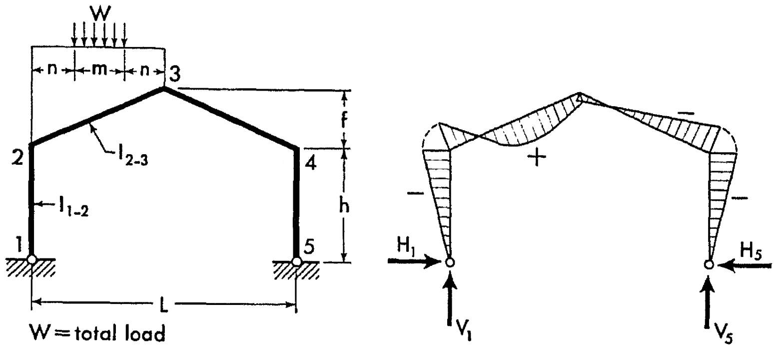
$V_1 = \dfrac{3W}{4} \qquad V_5 = \dfrac{W}{4}$

$$M_{x_2} = \left(\frac{Wx_2}{2} + M_2\right)\left(1 - \frac{2x_2}{L}\right) + M_3 \frac{2x_2}{L}$$

Apply Eqs. (6-1), (6-3), and (6-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

Members of Constant Section

6-4. Vertical Uniform Load over Center Part of Left Inclined Member



$$J = \frac{3}{4} - \left(\frac{m}{L}\right)^2 \qquad K = J(2 + \psi)$$

$$H_1 = H_5 = \frac{WL}{8Ah}(B + 2K)$$

$$M_2 = M_4 = -H_5 h \qquad M_3 = \frac{WL}{8} - H_5 h(1 + \psi)$$

$$V_1 = \frac{3W}{4} \qquad V_5 = \frac{W}{4}$$

When $x_2 \leqq n$

$$M_{x_2} = \frac{Wx_2}{2} + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

When $x_2 > n$, but $\leqq n + m$

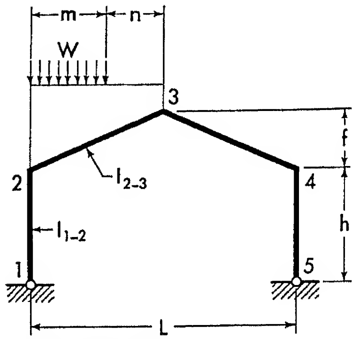
$$M_{x_2} = \frac{W}{2}\left[x_2 - \frac{(x_2 - n)^2}{m}\right] + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

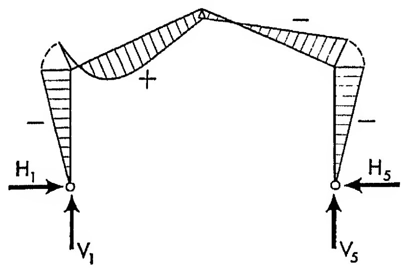
When $x_2 > n + m$

$$M_{x_2} = \left(M_2 + \frac{WL}{4}\right)\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (6-1), (6-3), and (6-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

6-5. Vertical Uniform Load over Part of Left Inclined Member





W = total load

$$g = \frac{2m}{L} \qquad G = \frac{2 - g^2}{(2 - g)^2}$$

$$J = \frac{g}{2}(2 - g)^2 \qquad K = J(1 + G + G\psi)$$

$$H_1 = H_5 = \frac{W}{4Ah}(Bm + KL) \qquad M_2 = M_4 = -H_5h$$

$$M_3 = \frac{Wm}{4} - H_5h(1 + \psi) \qquad V_5 = \frac{Wm}{2L}$$

$$V_1 = W - V_5$$

When $x_2 \leqq m$

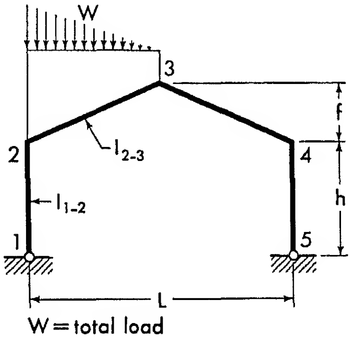
$$M_{x_2} = \frac{Wx_2}{2}\left(\frac{2n}{L} + \frac{m - x_2}{m}\right) + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

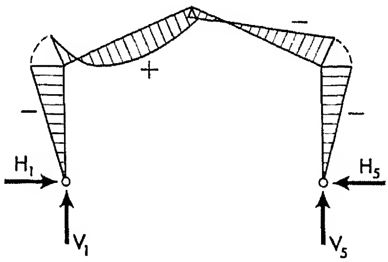
When $x_2 > m$

$$M_{x_2} = \left(\frac{Wm}{2} + M_2\right)\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (6-1), (6-3), and (6-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

6-6. Vertical Triangular Load on Left Inclined Member





$$K = 3 + \frac{7\psi}{5}$$

$$H_1 = H_5 = \frac{WL}{12Ah}(B + K)$$

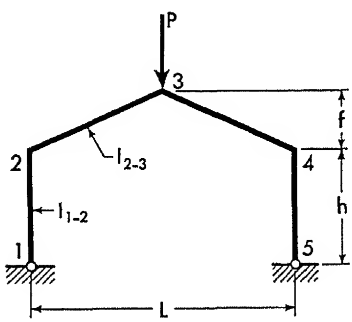
$$M_2 = M_4 = -H_5h$$

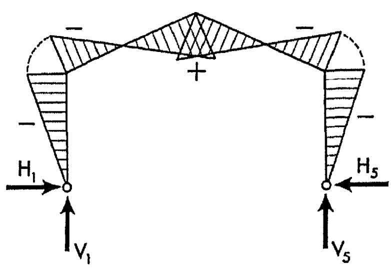
$$M_3 = \frac{WL}{12} - H_5h(1 + \psi) \qquad V_1 = \frac{5W}{6} \qquad V_5 = \frac{W}{6}$$

$$M_{x_2} = \left[M_2 + \frac{2Wx_2}{3}\left(1 - \frac{x_2}{L}\right)\right]\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (6-1), (6-3), and (6-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

6-7. Vertical Concentrated Load at Joint 3





$$H_1 = H_5 = \frac{PLB}{4Ah}$$

For Notations and Constants, see Arts. 6-1 and 6-2

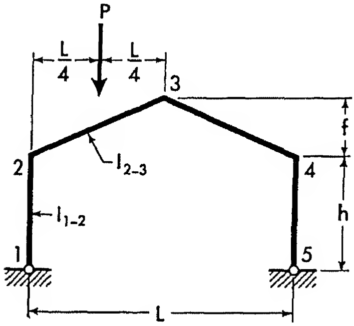
$$M_2 = M_4 = -H_5h$$

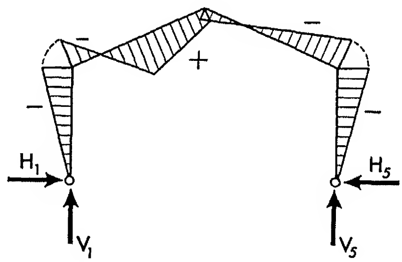
$$M_3 = \frac{PL}{4} - H_5h(1 + \psi)$$

$$V_1 = V_5 = \frac{P}{2}$$

Apply Eqs. (6-1) through (6-4) to obtain the moment at any section of the frame members.

6-8. Vertical Concentrated Load at Mid-point of Left Inclined Member





$$K = 3(2 + \psi)$$

$$H_1 = H_5 = \frac{PL(2B + K)}{16Ah}$$

$$M_2 = M_4 = -H_5h$$

$$M_3 = \frac{PL}{8} - H_5h(1 + \psi) \qquad V_1 = \frac{3P}{4} \qquad V_5 = \frac{P}{4}$$

When $x_2 \leqq \frac{L}{4}$

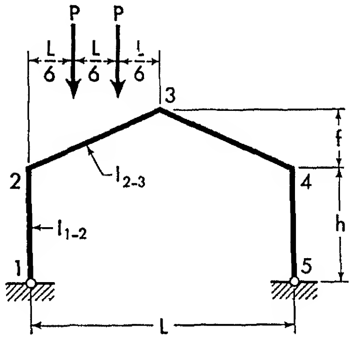
$$M_{x_2} = \frac{Px_2}{2} + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

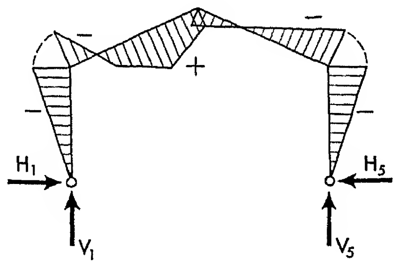
When $x_2 > \frac{L}{4}$

$$M_{x_2} = \frac{P}{4}(L - 2x_2) + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (6-1), (6-3), and (6-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

6-9. Two Equal Vertical Concentrated Loads on Left Inclined Member





$$K = \frac{4}{3}(2 + \psi)$$

$$H_1 = H_5 = \frac{PL}{4Ah}(B + K) \qquad M_2 = M_4 = -H_5h$$

$$M_3 = \frac{PL}{4} - H_5h(1 + \psi) \qquad V_1 = \frac{3P}{2} \qquad V_5 = \frac{P}{2}$$

When $x_2 \leqq \frac{L}{6}$

$$M_{x_2} = Px_2 + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

When $x_2 > \frac{L}{6}$, but $\leqq \frac{L}{3}$

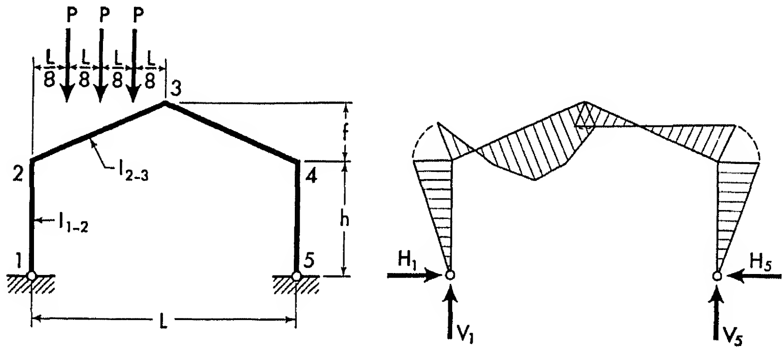
$$M_{x_2} = \frac{PL}{6} + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

When $x_2 > \frac{L}{3}$

$$M_{x_2} = P\left(\frac{L}{2} - x_2\right) + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (6-1), (6-3), and (6-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

6-10. Three Equal Vertical Concentrated Loads on Left Inclined Member



$$K = \frac{5}{4}(2 + \psi)$$

$$H_1 = H_5 = \frac{3PL}{8Ah}(B + K) \qquad M_2 = M_4 = -H_5h$$

$$M_3 = \frac{3PL}{8} - H_5h(1 + \psi) \qquad V_1 = \frac{9P}{4} \qquad V_5 = \frac{3P}{4}$$

When $x_2 \leqq \frac{L}{8}$

$$M_{x_2} = \frac{3Px_2}{2} + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

When $x_2 > \frac{L}{8}$, but $\leqq \frac{L}{4}$

$$M_{x_2} = \frac{P}{8}(L + 4x_2) + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

When $x > \frac{L}{4}$, but $\leqq \frac{3}{8}L$

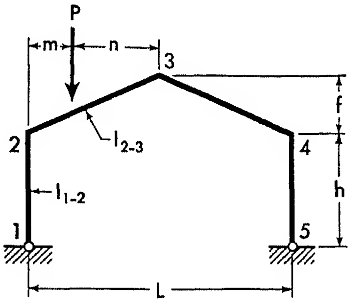
$$M_{x_2} = \frac{P}{8}(3L - 4x_2) + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

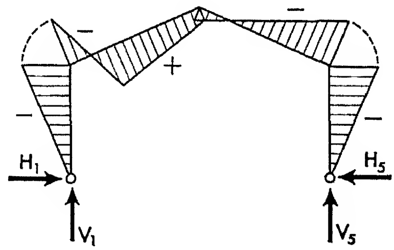
When $x_2 > \frac{3}{8}L$, but $\leqq \frac{L}{2}$

$$M_{x_2} = \frac{3P}{4}(L - 2x_2) + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (6-1), (6-3), and (6-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

6-11. Vertical Concentrated Load at Any Point of Left Inclined Member





$$G = \frac{L + 2m}{L + 2n} \qquad J = \frac{8mn(L + 2n)}{L^3}$$

$$K = J(1 + G + G\psi) \qquad H_1 = H_5 = \frac{P}{4Ah}(2Bm + KL)$$

$$M_2 = M_4 = -H_5 h \qquad M_3 = \frac{Pm}{2} - H_5 h(1 + \psi)$$

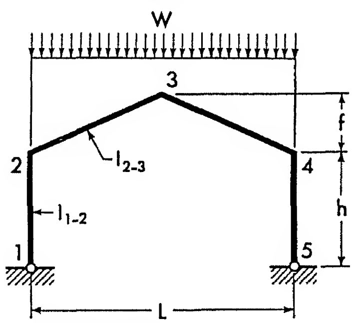
$$V_1 = P\left(1 - \frac{m}{L}\right) \qquad V_5 = \frac{Pm}{L}$$

When $x_2 \leqq m$, $\quad M_{x_2} = (Pn + M_3)\frac{2x_2}{L} + M_2\left(1 - \frac{2x_2}{L}\right)$

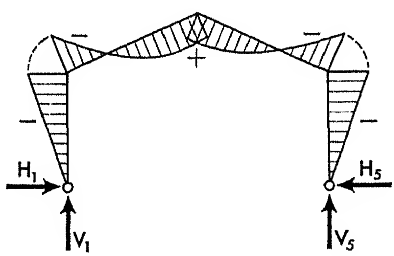
When $x_2 > m$, $\quad M_{x_2} = (Pm + M_2)\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$

Apply Eqs. (6-1), (6-3), and (6-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

6-12. Vertical Uniform Load over Entire Girder



W = total load



$$H_1 = H_5 = \frac{WL}{8Ah}(2 + B + \psi)$$

For Notations and Constants, see Arts. 6-1 and 6-2

$$M_2 = M_4 = -H_5h$$

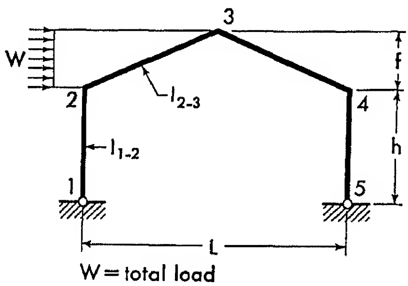
$$M_3 = \frac{WL}{8} - H_5h(1+\psi) \qquad V_1 = V_5 = \frac{W}{2}$$

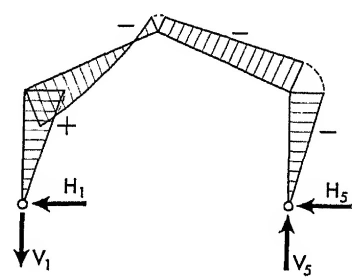
For member 2-3:

$$M_{x_2} = \left(M_2 + \frac{Wx_2}{4}\right)\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eq. (6-1) to obtain the moment at any section of the left column. Moments and forces at corresponding sections in the right half of the frame are identical to those in the left half.

6-13. Horizontal Uniform Load on Left Inclined Member





$$G = 1 + \frac{\psi}{2} \qquad K = A + B + C + 2G\psi$$

$$H_5 = \frac{WK}{4A} \qquad H_1 = -(W - H_5)$$

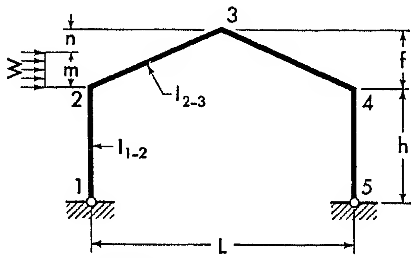
$$M_2 = h(W - H_5) \qquad M_3 = \frac{Wh}{4}(2+\psi) - H_5h(1+\psi)$$

$$M_4 = -H_5h \qquad V_5 = \frac{Wh}{2L}(2+\psi) \qquad V_1 = -V_5$$

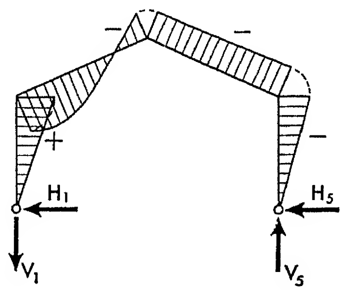
$$M_{y_2} = \left(M_2 + \frac{Wy_2}{2}\right)\left(1 - \frac{y_2}{f}\right) + M_3\frac{y_2}{f}$$

Apply Eqs. (6-1), (6-3), and (6-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

6-14. Horizontal Uniform Load over Part of Left Inclined Member



W = total load



$$g = \frac{m}{f} \qquad G = \frac{2 - g^2}{(2 - g)^2}$$

$$J = \frac{g}{2}(2 - g)^2 \qquad K = J\psi(1 + G + G\psi) + \frac{Bm\psi}{2f}$$

$$H_5 = \frac{W}{2A}(B + C + K) \qquad H_1 = -(W - H_5)$$

$$M_2 = h(W - H_5) \qquad M_3 = \frac{V_5 L}{2} - H_5 h(1 + \psi)$$

$$M_4 = -H_5 h \qquad V_5 = \frac{Wh}{L}\left(1 + \frac{m\psi}{2f}\right)$$

$$V_1 = -V_5$$

When $y_2 \leqq m$

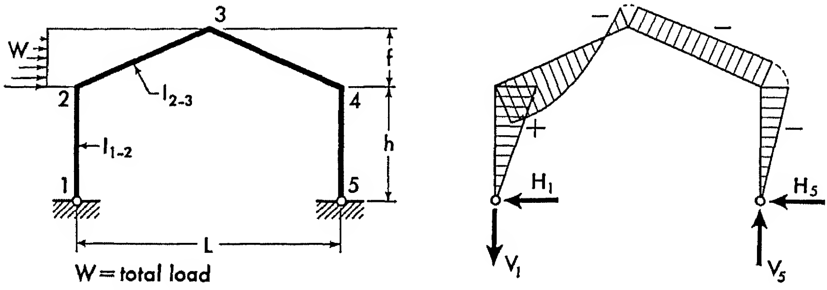
$$M_{y_2} = \frac{Wy_2}{2}\left(1 + \frac{n}{f} - \frac{y_2}{m}\right) + M_2\left(1 - \frac{y_2}{f}\right) + M_3\frac{y_2}{f}$$

When $y_2 > m$

$$M_{y_2} = \left(\frac{Wm}{2} + M_2\right)\left(1 - \frac{y_2}{f}\right) + M_3\frac{y_2}{f}$$

Apply Eqs. (6-1), (6-3), and (6-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

6-15. Horizontal Triangular Load on Left Inclined Member



$$K = B\left(1 + \frac{\psi}{3}\right) + \psi\left(1 + \frac{7\psi}{15}\right)$$

$$H_5 = \frac{W}{2A}(C + K) \qquad H_1 = -(W - H_5)$$

$$M_2 = h(W - H_5) \qquad M_3 = \frac{V_5 L}{2} - H_5 h(1 + \psi)$$

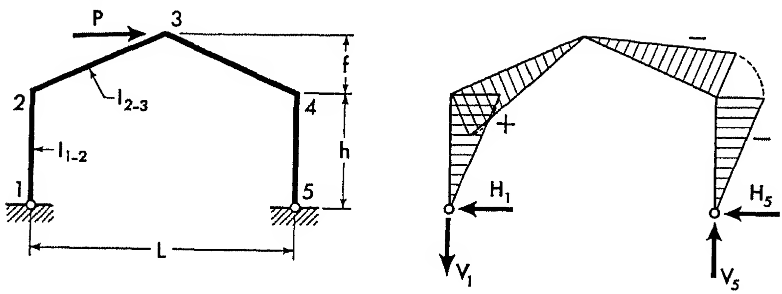
$$M_4 = -H_5 h \qquad V_5 = \frac{Wh}{L}\left(1 + \frac{\psi}{3}\right)$$

$$V_1 = -V_5$$

$$M_{y_2} = \left[M_2 + \frac{Wy_2}{3}\left(2 - \frac{y_2}{f}\right)\right]\left(1 - \frac{y_2}{f}\right) + M_3\frac{y_2}{f}$$

Apply Eqs. (6-1), (6-3), and (6-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

6-16. Horizontal Concentrated Load at Joint 3



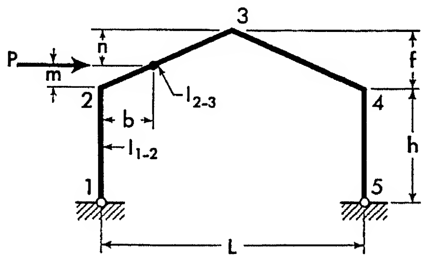
$$H_1 = -\frac{P}{2} \qquad H_5 = \frac{P}{2}$$

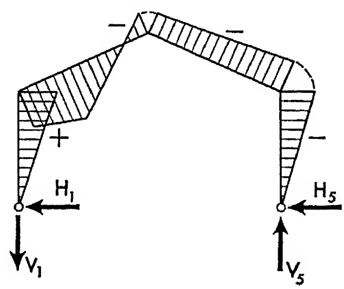
$$M_2 = \frac{Ph}{2} \qquad M_3 = 0 \qquad M_4 = -\frac{Ph}{2}$$

$$V_5 = \frac{Ph}{L}(1 + \psi) \qquad V_1 = -V_5$$

Apply Eqs. (6-1) through (6-4) to obtain the moment at any section of the frame members.

6-17. Horizontal Concentrated Load at Any Point of Left Inclined Member





$$G = \frac{f+m}{f+n} \qquad J = \frac{2mn(f+n)}{f^3}$$

$$K = J\psi(1 + G + G\psi) + \frac{Bm\psi}{f}$$

$$H_5 = \frac{P}{2A}(B + C + K) \qquad H_1 = -(P - H_5)$$

$$M_2 = h(P - H_5)$$

$$M_3 = \frac{V_5L}{2} - H_5h(1 + \psi) \qquad M_4 = -H_5h$$

$$V_5 = \frac{Ph}{L}\left(1 + \frac{m\psi}{f}\right) \qquad V_1 = -V_5$$

When $x_2 \leqq b$

$$M_{x_2} = (M_3 + Pn)\frac{2x_2}{L} + M_2\left(1 - \frac{2x_2}{L}\right)$$

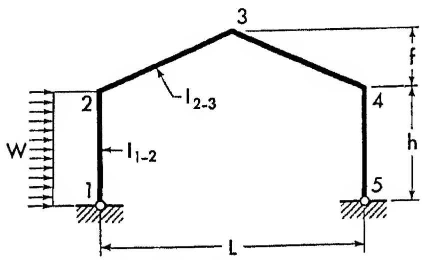
When $x_2 > b$

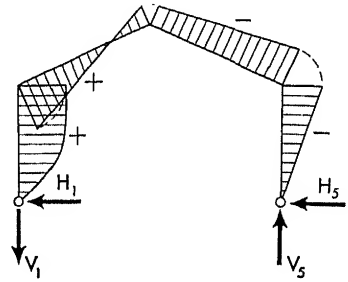
$$M_{x_2} = (M_2 + Pm)\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (6-1), (6-3), and (6-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

For Notations and Constants, see Arts. 6-1 and 6-2

6-18. Horizontal Uniform Load on Column





W = total load

$$H_5 = \frac{W}{4A\phi}\left[1 + \phi(B + C)\right]$$

$$H_1 = -(W - H_5)$$

$$M_2 = \frac{Wh}{2} - H_5 h$$

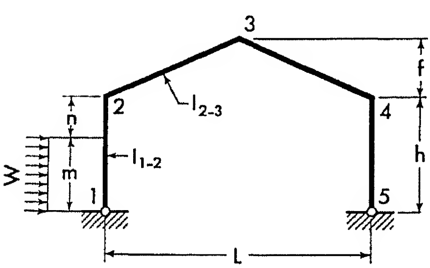
$$M_3 = \frac{Wh}{4} - H_5 h(1 + \psi) \qquad M_4 = -H_5 h$$

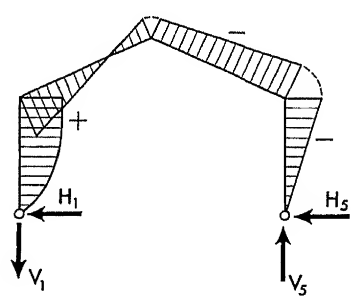
$$V_5 = \frac{Wh}{2L} \qquad V_1 = -V_5$$

$$M_{y_1} = \frac{Wy_1}{2}\left(1 - \frac{y_1}{h}\right) + M_2 \frac{y_1}{h}$$

Apply Eqs. (6-2) through (6-4) to obtain the moment at any section of frame members 2-3, 3-4, and 4-5.

6-19. Horizontal Uniform Load over Part of Column





W = total load

$$g = \frac{m}{h} \qquad J = \frac{g(2 - g^2)}{2\phi}$$

$$K = 2Jh + m(B + C)$$

$$H_5 = \frac{WK}{4Ah} \qquad H_1 = -(W - H_5)$$

$$M_2 = \frac{Wm}{2} - H_5h \qquad M_3 = \frac{Wm}{4} - H_5h(1 + \psi)$$

$$M_4 = -H_5h \qquad V_5 = \frac{Wm}{2L} \qquad V_1 = -V_5$$

When $y_1 \leqq m$

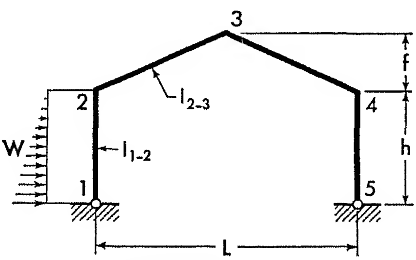
$$M_{y_1} = \frac{Wy_1}{2}\left(\frac{n}{h} + \frac{m - y_1}{m}\right) + M_2\frac{y_1}{h}$$

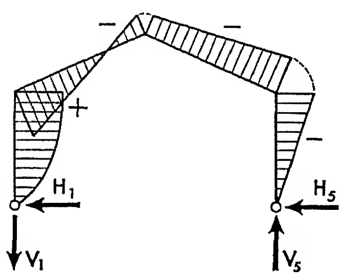
When $y_1 > m$

$$M_{y_1} = \frac{Wm}{2}\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (6-2) through (6-4) to obtain the moment at any section of frame members 2-3, 3-4, and 4-5.

6-20. Horizontal Triangular Load over Entire Column





$$K = \frac{B + C}{3} + \frac{7}{15\phi}$$

$$H_5 = \frac{WK}{2A} \qquad H_1 = -(W - H_5)$$

$$M_2 = h\left(\frac{W}{3} - H_5\right)$$

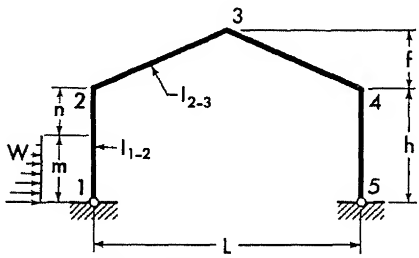
$$M_3 = \frac{Wh}{6} - H_5h(1 + \psi) \qquad M_4 = -H_5h$$

$$V_5 = \frac{Wh}{3L} \qquad V_1 = -V_5$$

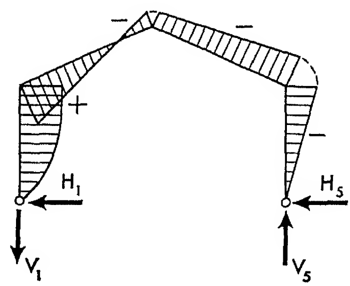
$$M_{y_1} = \frac{Wy_1}{3}\left(1 - \frac{y_1}{h}\right)\left(2 - \frac{y_1}{h}\right) + M_2 \frac{y_1}{h}$$

Apply Eqs. (6-2) through (6-4) to obtain the moment at any section of frame members 2-3, 3-4, and 4-5.

6-21. Horizontal Triangular Load over Part of Column



W = total load



$$g = \frac{m}{h} \qquad J = \frac{g}{15}(10 - 3g^2)$$

$$K = \frac{g}{3}(B + C) + \frac{J}{\phi}$$

$$H_5 = \frac{WK}{2A} \qquad H_1 = -(W - H_5)$$

$$M_2 = \frac{Wm}{3} - H_5 h \qquad M_3 = \frac{Wm}{6} - H_5 h(1 + \psi)$$

$$M_4 = -H_5 h$$

$$V_5 = \frac{Wm}{3L} \qquad V_1 = -V_5$$

When $y_1 \leqq m$

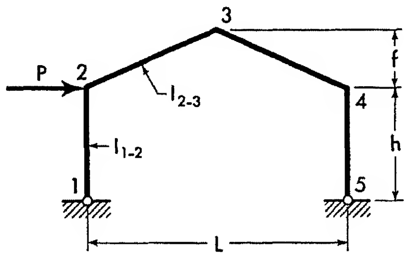
$$M_{y_1} = \frac{Wy_1}{3}\left[\frac{n}{h} + \frac{(m - y_1)(2m - y_1)}{m^2}\right] + M_2 \frac{y_1}{h}$$

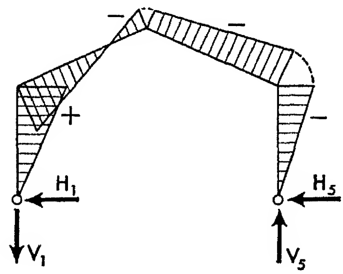
When $y_1 > m$

$$M_{y_1} = \frac{Wm}{3}\left(1 - \frac{y_1}{h}\right) + M_2 \frac{y_1}{h}$$

Apply Eqs. (6-2) through (6-4) to obtain the moment at any section of frame members 2-3, 3-4, and 4-5.

6-22. Horizontal Concentrated Load at Joint 2





$$H_5 = \frac{P(B + C)}{2A} \qquad H_1 = -(P - H_5)$$

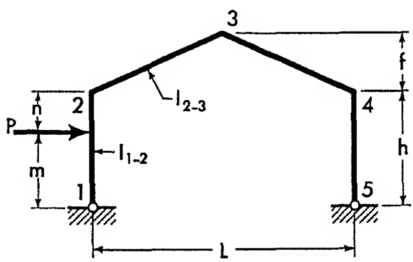
$$M_2 = h(P - H_5)$$

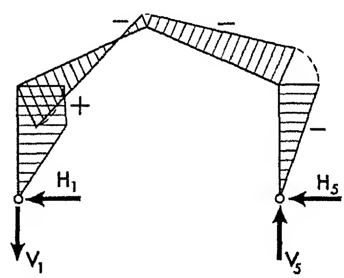
$$M_3 = \frac{Ph}{2} - H_5h(1 + \psi) \qquad M_4 = -H_5h$$

$$V_1 = -\frac{Ph}{L} \qquad V_5 = \frac{Ph}{L}$$

Apply Eqs. (6-1) through (6-4) to obtain the moment at any section of the frame members.

6-23. Horizontal Concentrated Load at Any Point of Column





$$g = \frac{m}{h} \qquad J = 2g(1 - g^2)$$

$$K = g(B + C) + \frac{J}{\phi} \qquad H_5 = \frac{PK}{2A}$$

$$H_1 = -(P - H_5) \qquad M_2 = Pm - H_5h$$

$$M_3 = \frac{Pm}{2} - H_5h(1 + \psi) \qquad M_4 = -H_5h$$

For Notations and Constants, see Arts. 6-1 and 6-2

$$V_5 = \frac{Pm}{L} \qquad V_1 = -V_5$$

When $y_1 \leqq m$

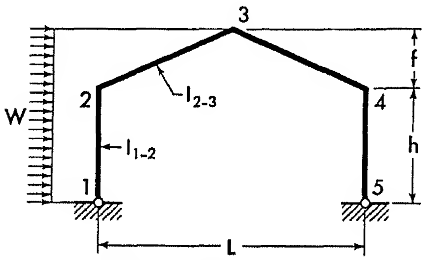
$$M_{y_1} = (Pn + M_2)\frac{y_1}{h}$$

When $y_1 > m$

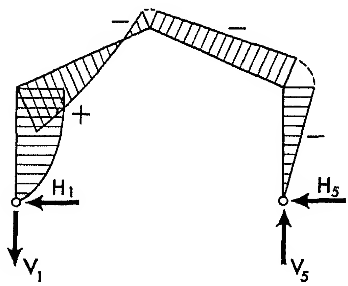
$$M_{y_1} = M_2\frac{y_1}{h} + Pm\left(1 - \frac{y_1}{h}\right)$$

Apply Eqs. (6-2) through (6-4) to obtain the moment at any section of frame members 2-3, 3-4, and 4-5.

6-24. Horizontal Uniform Load over Left Half of Frame



W = total load



$$m = \frac{h}{h+f} \qquad n = 1 - m$$

$$G = 1 + \frac{\psi}{2} \qquad K = A + B + C + 2G\psi$$

$$H_5 = \frac{W}{4A}\left[Kn + m\left(B + C + \frac{1}{\phi}\right)\right]$$

$$H_1 = -(W - H_5) \qquad M_2 = \frac{Wh}{2}(1 + n) - H_5h$$

$$M_3 = \frac{Wh}{4}(1 + n + n\psi) - H_5h(1 + \psi)$$

$$M_4 = -H_5h$$

$$V_5 = \frac{Wh}{2L}(1 + n + n\psi)$$

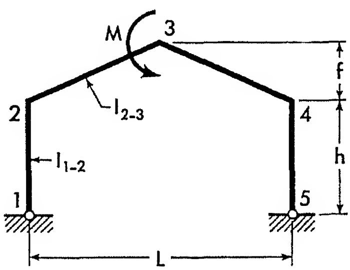
$$V_1 = -V_5$$

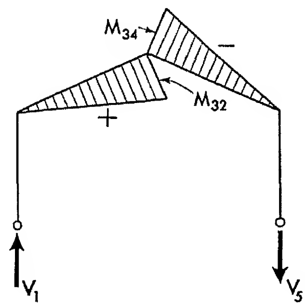
$$M_{y_1} = \frac{Wmy_1}{2}\left(1 - \frac{y_1}{h}\right) + M_2 \frac{y_1}{h}$$

$$M_{y_2} = \left(M_2 + \frac{Wny_2}{2}\right)\left(1 - \frac{y_2}{f}\right) + M_3 \frac{y_2}{f}$$

Apply Eqs. (6-3) and (6-4) to obtain the moment at any section of frame members 3-4 and 4-5.

6-25. Moment Applied at Joint 3





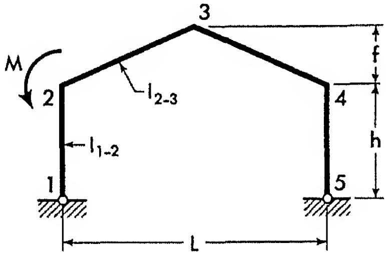
$$H_1 = H_5 = 0$$

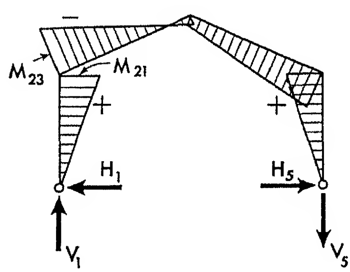
$$M_2 = M_4 = 0 \qquad M_{32} = \frac{M}{2} \qquad M_{34} = -\frac{M}{2}$$

$$V_1 = \frac{M}{L} \qquad V_5 = -\frac{M}{L}$$

$$M_{x_2} = M_{32}\frac{2x_2}{L} \qquad M_{x_3} = M_{34}\left(1 - \frac{2x_3}{L}\right)$$

6-26. Moment Applied at Joint 2





$$H_1 = H_5 = -\frac{M}{2Ah}(6 + B + 2\psi)$$

For Notations and Constants, see Arts. 6-1 and 6-2

$$M_{21} = M_4 = -H_5h \qquad M_{23} = -(M - M_{21})$$

$$M_3 = -\frac{M}{2} - H_5h(1 + \psi)$$

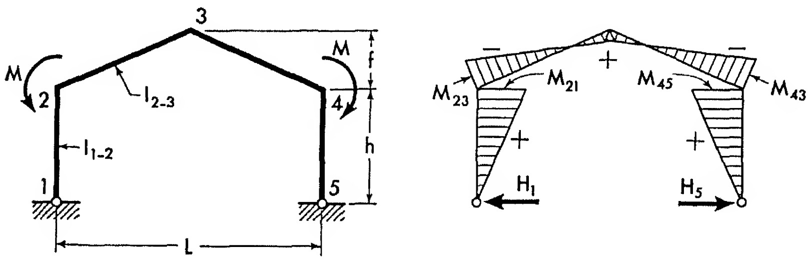
$$V_1 = \frac{M}{L} \qquad V_5 = -\frac{M}{L}$$

$$M_{y_1} = M_{21}\frac{y_1}{h}$$

$$M_{x_2} = M_{23}\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (6-3) and (6-4) to obtain the moment at any section of frame members 3-4 and 4-5.

6-27. Two Equal Moments Applied at Joints 2 and 4



$$H_1 = H_5 = -\frac{M}{Ah}(6 + B + 2\psi)$$

$$M_{21} = M_{45} = -H_5h$$

$$M_{23} = M_{43} = -(M - M_{21})$$

$$M_3 = -M - H_5h(1 + \psi)$$

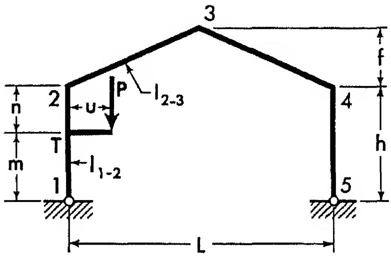
$$V_1 = V_5 = 0$$

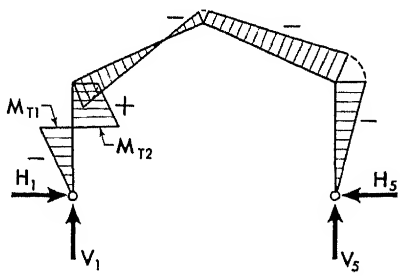
$$M_{y_1} = M_{21}\frac{y_1}{h}$$

$$M_{x_2} = M_{23}\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Moments at corresponding sections in the right half of the frame are identical to those in the left half.

6-28. Vertical Concentrated Load Applied at Bracket





Bracket acts as a simple cantilever and its maximum moment is Pu at point T. The moment diagram of the cantilever is intentionally not shown so that the frame's bending moment diagram may be more clearly illustrated.

$$M = Pu$$

$$J = \frac{2(h^2 - 3m^2)}{h^2\phi} \qquad K = B + C + J$$

$$H_1 = H_5 = \frac{MK}{2Ah} \qquad M_2 = M - H_1 h$$

$$M_3 = \frac{M}{2} - H_5 h(1 + \psi) \qquad M_4 = -H_5 h$$

$$V_1 = P - \frac{M}{L}$$

$$V_5 = \frac{M}{L}$$

When $y_1 < m$

$$M_{y_1} = -(M - M_2)\frac{y_1}{h}$$

When $y_1 = m$

$$M_{T1} = -(M - M_2)\frac{m}{h} \quad \text{and} \quad M_{T2} = M\frac{n}{h} + M_2\frac{m}{h}$$

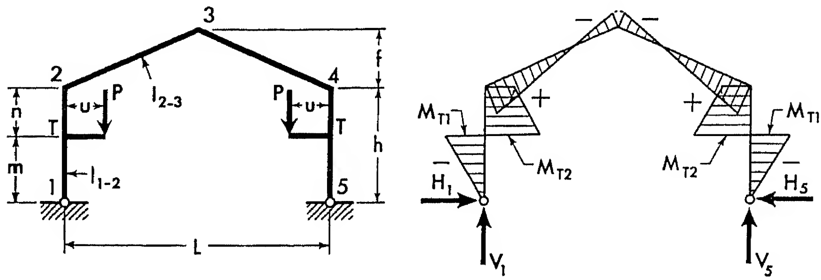
When $y_1 > m$

$$M_{y_1} = M\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (6-2) through (6-4) to obtain the moment at any section of frame members 2-3, 3-4, and 4-5.

For Notations and Constants, see Arts. 6-1 and 6-2

6-29. Two Equal Vertical Concentrated Loads Symmetrically Applied at Brackets



Brackets act as simple cantilevers with the maximum moments of Pu at points T. The moment diagrams of these cantilevers are intentionally not shown so that the frame's bending moment diagram may be more clearly illustrated.

$$M = Pu$$

$$J = \frac{2(h^2 - 3m^2)}{h^2\phi} \qquad K = B + C + J$$

$$H_1 = H_5 = \frac{MK}{Ah} \qquad M_2 = M_4 = M - H_1h$$

$$M_3 = M - H_5h(1 + \psi) \qquad V_1 = V_5 = P$$

When $y_1 < m$

$$M_{y_1} = -(M - M_2)\frac{y_1}{h}$$

When $y_1 = m$

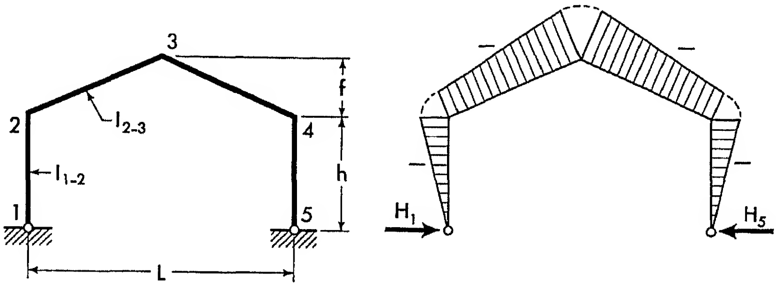
$$M_{T1} = -(M - M_2)\frac{m}{h} \quad \text{and} \quad M_{T2} = M\frac{n}{h} + M_2\frac{m}{h}$$

When $y_1 > m$

$$M_{y_1} = M\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eq. (6-2) to obtain the moment at any section of frame member 2-3. Moments at corresponding sections in the right half of the frame are identical to those in the left half.

6-30. Effect of Temperature Rise. Range t° for entire frame.



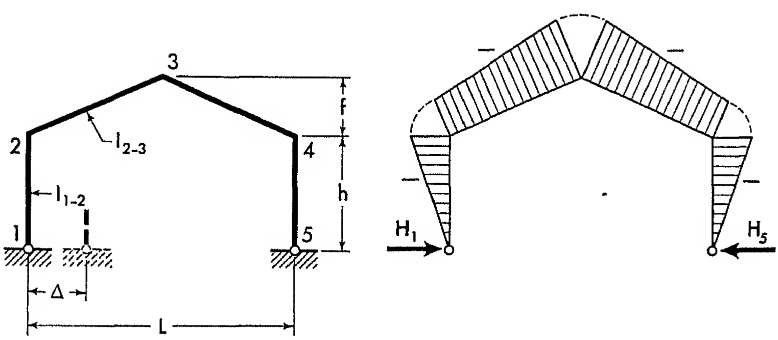
$$H_1 = H_5 = \frac{6L\varepsilon t^\circ}{Ah^3\phi} EI_{1\text{-}2} \qquad M_2 = M_4 = -H_5 h$$

$$M_3 = -H_5 h(1+\psi) \qquad V_1 = V_5 = 0$$

Apply Eqs. (6-1) through (6-4) to obtain the moment at any section of the frame members.

Note: For temperature drop, introduce the value of t° with a negative sign.

6-31. Horizontal Displacement of One Support

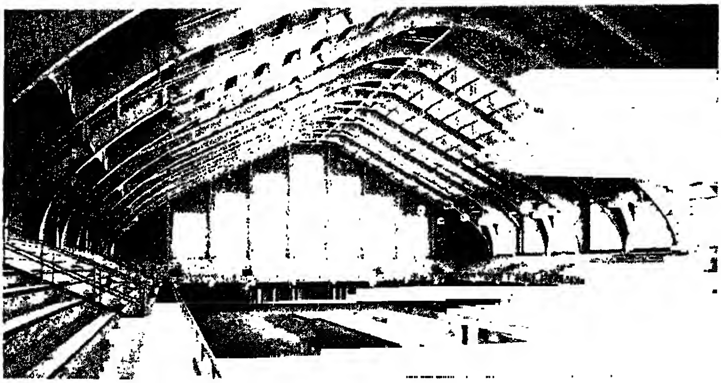


$$H_1 = H_5 = \frac{6\Delta}{Ah^3\phi} EI_{1\text{-}2} \qquad M_2 = M_4 = -H_5 h$$

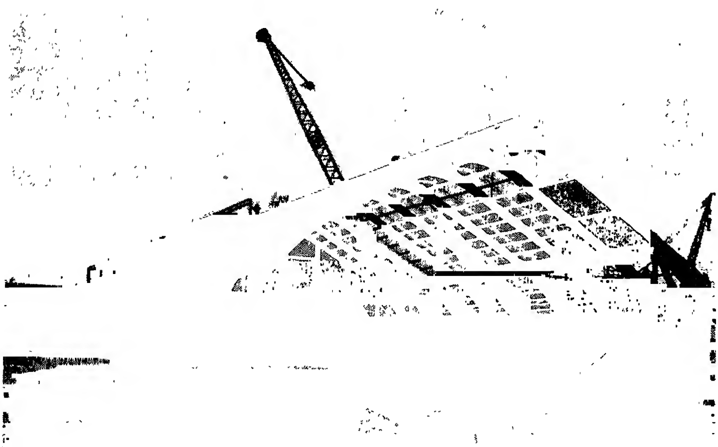
$$M_3 = -H_5 h(1+\psi) \qquad V_1 = V_5 = 0$$

Apply Eqs. (6-1) through (6-4) to obtain the moment at any section of the frame members.

Note: If the direction of the frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.



The Purdue University Field House at the University Campus in Lafayette, Indiana. This is an outstanding illustration of the employment of gable frames for a spacious and handsome field house used for sports activities. The frames are raised high above the floor to permit an unobstructed view from any seat in the bleachers. The frames spanning the building, which is 170 feet wide, have smooth, flaring flanges and provide a neat and attractive appearance to the interior of the field house. Designed by Walter Scholer and Associates, architects of Lafayette, Indiana, in collaboration with United States Steel Corporation. (Courtesy of the American Bridge Co.)

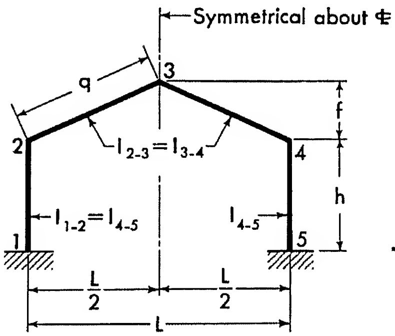


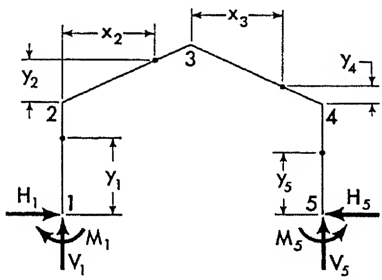
Construction view of the Purdue University Field House illustrates welded gable frames cleverly designed by employing rolled steel sections. At the knees and at the crown, the interior flanges of connecting members were split and rewelded, after insertion of filler plates, thus forming functionally efficient rigid frames. Each frame was delivered to the site in four pieces and welded in place at the crown and at the knees. (Courtesy of United States Steel Corporation.)

SECTION 7

SYMMETRICAL GABLE FRAMES WITH FIXED SUPPORTS

7-1. Notations, Coordinates, and Frame Constants





The sketch appearing on the left, above, explains notations for a representative gable frame with members of constant cross section.

The sketch appearing on the right, above, shows positive directions of the moments and the vertical and horizontal components of the frame reactions. It also defines the coordinates for any section of the frame. Coordinates are to be considered only in the positive sense.

Frame Constants: $\phi = \frac{I_{1\text{-}2}}{I_{2\text{-}3}} \cdot \frac{q}{h} \qquad \psi = \frac{f}{h}$

$$A = \frac{3(1 - \phi\psi)}{2(1 + \phi\psi^2)} \qquad B = \frac{6(1 + \phi)}{1 + \phi\psi^2}$$

$$D = 16(3 + \phi) \qquad F = 12[2 + 2\phi - A(1 - \phi\psi)]$$

7-2. Equations of Frame Reactions and Moments. The equations for the redundant moments and the vertical and horizontal components of frame reactions are given on the following pages for a number of loading conditions. Corresponding expressions for the moment and forces at any section of a member with applied load are also provided.

The equations for the moments of load-free members are listed below for reference.

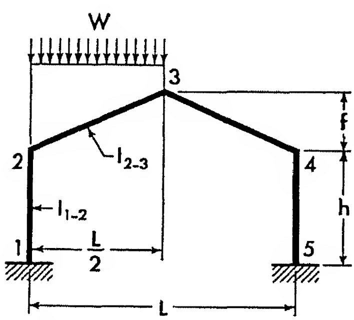
$$M_{y_1} = M_2 \frac{y_1}{h} + M_1 \left(1 - \frac{y_1}{h}\right) \tag{7-1}$$

$$M_{x_2} = M_2 \left(1 - \frac{2x_2}{L}\right) + M_3 \frac{2x_2}{L} \tag{7-2}$$

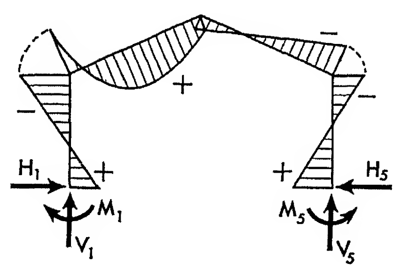
$$M_{x_3} = M_3 \left(1 - \frac{2x_3}{L}\right) + M_4 \frac{2x_3}{L} \tag{7-3}$$

$$M_{y_5} = M_5 \left(1 - \frac{y_5}{h}\right) + M_4 \frac{y_5}{h} \tag{7-4}$$

7-3. Vertical Uniform Load on Left Inclined Member



W = total load



$$G = 2 + \frac{5A\psi}{4} \qquad J = 2A + \frac{5B\psi}{8}$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = -WL\phi\left(\frac{G}{F} \pm \frac{1}{2D}\right)$$

$$H_1 = H_5 = \frac{WLJ\phi}{Fh} \qquad M_1 = M_2 + H_1 h$$

$$M_3 = -\frac{WLG\phi}{F} + \frac{WL}{8} - H_5 f \qquad M_5 = M_4 + H_5 h$$

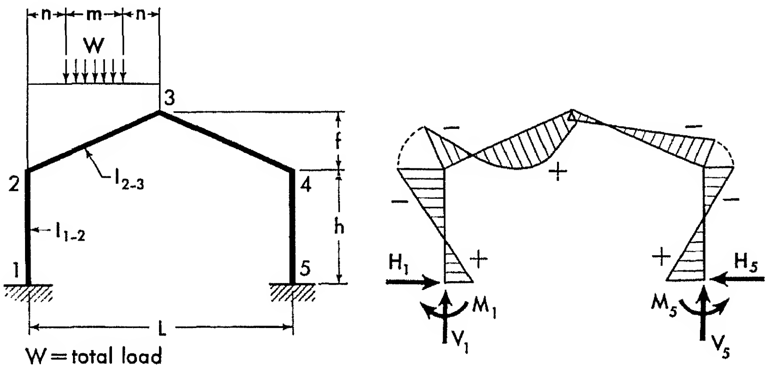
Members of Constant Section

$$V_5 = \frac{W}{4D}(D - 4\phi) \qquad V_1 = W - V_5$$

$$M_{x_2} = \left(\frac{Wx_2}{2} + M_2\right)\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (7-1), (7-3), and (7-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

7-4. Vertical Uniform Load over Center Part of Inclined Member



W = total load

$$G = \frac{3L^2 - 4m^2}{4L^2} \qquad J = \frac{(2 + G)(4A + B\psi)}{2} - A$$

$$K = (2 + G)(2 + A\psi) - 1$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = -WL\phi\left(\frac{K}{2F} \pm \frac{G}{D}\right)$$

$$H_1 = H_5 = \frac{WLJ\phi}{2Fh} \qquad M_1 = M_2 + H_1h$$

$$M_3 = \frac{M_2 + M_4}{2} + \frac{WL}{8} - H_5f$$

$$M_5 = M_4 + H_5h$$

$$V_5 = \frac{W}{4}\left(1 - \frac{8G\phi}{D}\right) \qquad V_1 = W - V_5$$

When $x_2 \leqq n$

$$M_{x_2} = \frac{Wx_2}{2} + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

For Notations and Constants, see Arts. 7-1 and 7-2

When $x_2 > n$, but $\leqq n + m$

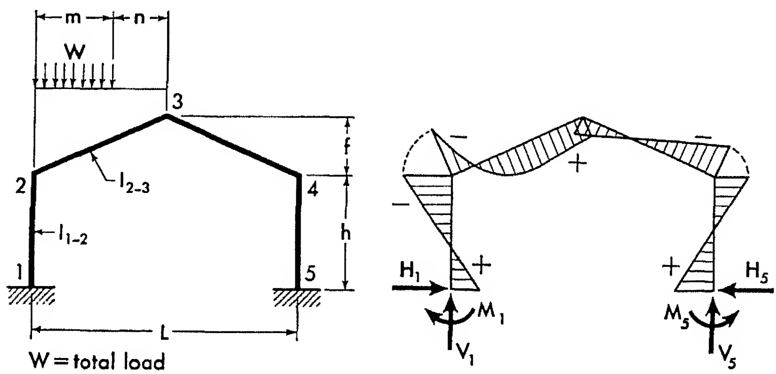
$$M_{x_2} = \frac{W}{2}\left[x_2 - \frac{(x-n)^2}{m}\right] + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

When $x_2 > n + m$, but $\leqq \frac{L}{2}$

$$M_{x_2} = \left(M_2 + \frac{WL}{4}\right)\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (7-1), (7-3), and (7-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

7-5. Vertical Uniform Load over Part of Left Inclined Member



W = total load

$$g = \frac{2m}{L} \qquad G = \frac{2 - g^2}{(2-g)^2} \qquad J = \frac{g}{2}(2-g)^2$$

$$K = JL(1 + G + AG\psi) + 2m(3 + 2A\psi)$$

$$N = JL(2A + 2AG + BG\psi) + 4m(3A + B\psi)$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = -\frac{WK\phi}{2F} \mp \frac{WLJ\phi}{D}$$

$$H_1 = H_5 = \frac{WN\phi}{4Fh} \qquad M_1 = M_2 + H_1 h$$

$$M_3 = -\frac{WK\phi}{2F} + \frac{Wm}{4} - H_5 f \qquad M_5 = M_4 + H_5 h$$

$$V_5 = W\left(\frac{m}{2L} - \frac{2J\phi}{D}\right) \qquad V_1 = W - V_5$$

When $x_2 \leqq m$

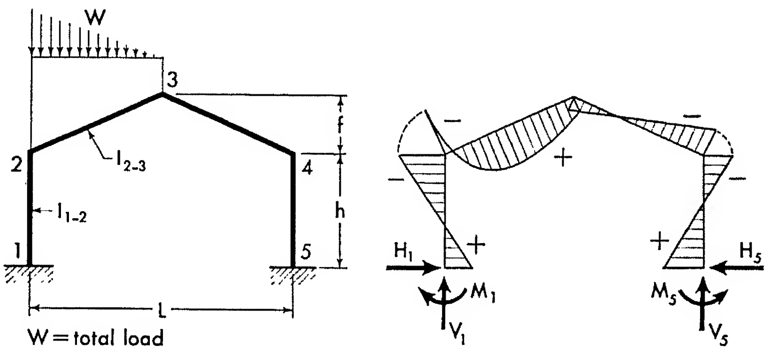
$$M_{x_2} = \frac{Wx_2}{2}\left(\frac{2n}{L} + \frac{m - x_2}{m}\right) + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

When $x_2 > m$

$$M_{x_2} = \left(\frac{Wm}{2} + M_2\right)\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (7-1), (7-3), and (7-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

7-6. Vertical Triangular Load on Left Inclined Member



W = total load

$$G = \frac{3}{5}(5 + 3A\psi) \qquad J = \frac{3}{10}(10A + 3B\psi)$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = -WL\phi\left(\frac{G}{2F} \pm \frac{8}{15D}\right)$$

$$H_1 = H_5 = \frac{WLJ\phi}{2Fh} \qquad M_1 = M_2 + H_1h$$

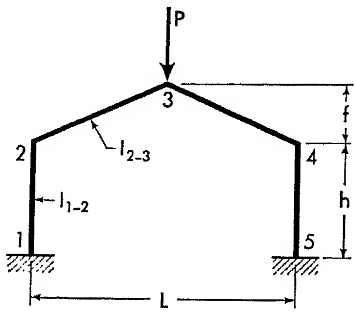
$$M_3 = \frac{M_2 + M_4}{2} + \frac{WL}{12} - H_5f \qquad M_5 = M_4 + H_5h$$

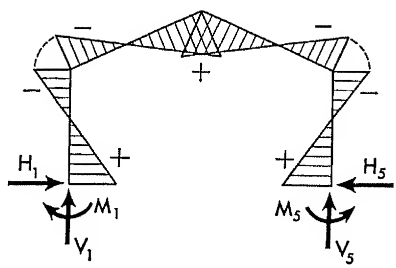
$$V_5 = \frac{W}{6}\left(1 - \frac{32\phi}{5D}\right) \qquad V_1 = W - V_5$$

$$M_{x_2} = \left[M_2 + \frac{2Wx_2}{3}\left(1 - \frac{x_2}{L}\right)\right]\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (7-1), (7-3), and (7-4) to obtain the moment at any section of frame members 1-2, 3-5, and 4-5.

7-7. Vertical Concentrated Load at Joint 3





$$M_2 = M_4 = -\frac{PL\phi}{F}(3 + 2A\psi)$$

$$H_1 = H_5 = \frac{PL\phi}{Fh}(3A + B\psi)$$

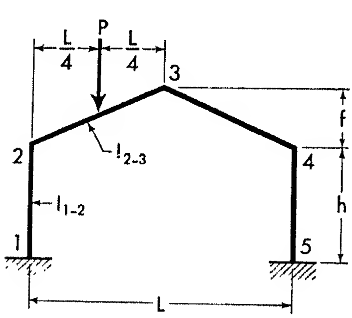
$$M_1 = M_5 = M_2 + H_1h$$

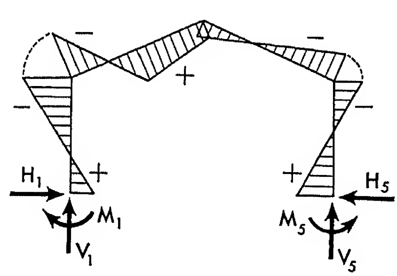
$$M_3 = M_2 + \frac{PL}{4} - H_5f$$

$$V_1 = V_5 = \frac{P}{2}$$

Apply Eqs (7-1) through (7-4) to obtain the moment at any section of the frame members.

7-8. Vertical Concentrated Load at Mid-point of Left Inclined Member





$$G = \frac{9}{2} + \frac{11A\psi}{4} \qquad J = \frac{9A}{2} + \frac{11B\psi}{8}$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\} = -\frac{PL\phi}{2}\left(\frac{G}{F} \pm \frac{3}{2D}\right)$$

$$H_1 = H_5 = \frac{PLJ\phi}{2Fh} \qquad M_1 = M_2 + H_1 h$$

$$M_3 = \frac{M_2 + M_4}{2} + \frac{PL}{8} - H_5 f \qquad M_5 = M_4 + H_5 h$$

$$V_5 = \frac{P}{4}\left(1 - \frac{6\phi}{D}\right) \qquad V_1 = P - V_5$$

When $x_2 \leqq \frac{L}{4}$

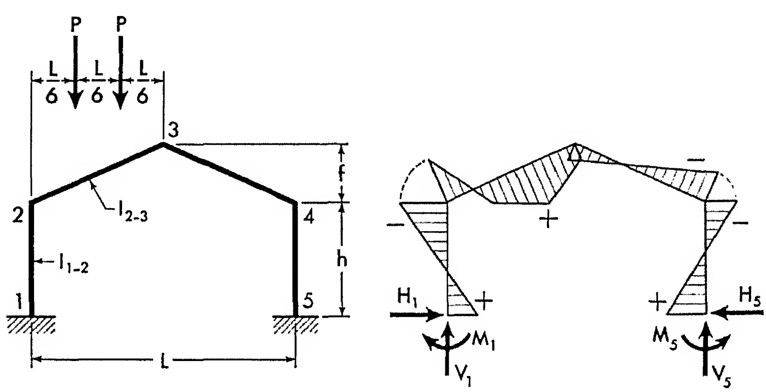
$$M_{x_2} = \frac{Px_2}{2} + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

When $x_2 > \frac{L}{4}$, but $\leqq \frac{L}{2}$

$$M_{x_2} = \frac{P}{4}(L - 2x_2) + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (7-1), (7-3), and (7-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

7-9. Two Equal Vertical Concentrated Loads on Left Inclined Member



$$\left.\begin{matrix} M_2 \\ M_1 \end{matrix}\right\} = -\frac{PL\phi}{3}\left(\frac{13 + 8A\psi}{F} \pm \frac{4}{D}\right)$$

For Notations and Constants, see Arts. 7-1 and 7-2

$$H_1 = H_5 = \frac{PL\phi}{3Fh}(13A + 4B\psi) \qquad M_1 = M_2 + H_1h$$

$$M_3 = \frac{PL}{4} - \frac{PL\phi}{3F}(13 + 8A\psi) - H_5f$$

$$M_5 = M_4 + H_5h \qquad V_5 = \frac{P}{6D}(3D - 16\phi) \qquad V_1 = 2P - V_5$$

When $x_2 \leqq \frac{L}{6}$

$$M_{x_2} = Px_2 + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

When $x_2 > \frac{L}{6}$, but $\leqq \frac{L}{3}$

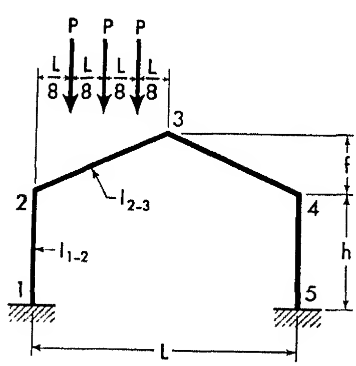
$$M_{x_2} = \frac{PL}{6} + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

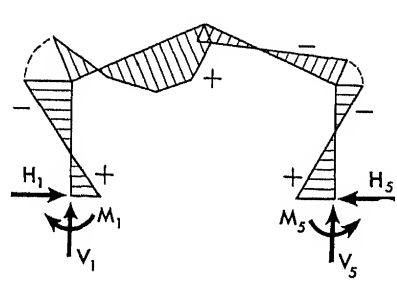
When $x_2 > \frac{L}{3}$, but $\leqq \frac{L}{2}$

$$M_{x_2} = P\left(\frac{L}{2} - x_2\right) + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (7-1), (7-3), and (7-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

7-10. Three Equal Vertical Loads on Left Inclined Member





$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\} = -\frac{3PL\phi}{16}\left(\frac{34 + 21A\psi}{F} \pm \frac{10}{D}\right)$$

$$H_1 = H_5 = \frac{3PL\phi}{32Fh}(68A + 21B\psi) \qquad M_1 = M_2 + H_1h$$

$$M_3 = \frac{3PL}{8}\left[1 - \frac{\phi(34 + 21A\psi)}{2F}\right] - H_5f$$

$$M_5 = M_4 + H_5h \qquad V_5 = \frac{3P}{4D}(D - 5\phi) \qquad V_1 = 3P - V_5$$

When $x_2 \leqq \frac{L}{8}$

$$M_{x_2} = \frac{3Px_2}{2} + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

When $x_2 > \frac{L}{8}$, but $\leqq \frac{L}{4}$

$$M_{x_2} = \frac{P}{8}(L + 4x_2) + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

When $x_2 > \frac{L}{4}$, but $\leqq \frac{3}{8}L$

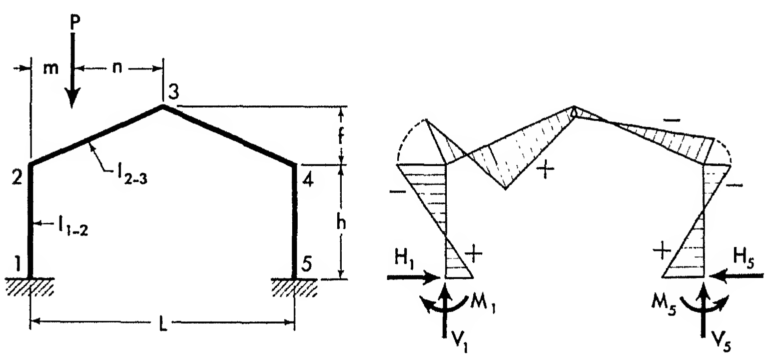
$$M_{x_2} = \frac{P}{8}(3L - 4x_2) + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

When $x_2 > \frac{3}{8}L$, but $\leqq \frac{L}{2}$

$$M_{x_2} = \frac{3P}{4}(L - 2x_2) + M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (7-1), (7-3), and (7-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

7-11. Vertical Concentrated Load at Any Point of Left Inclined Member



$$G = \frac{L + 2m}{L + 2n} \qquad J = \frac{8mn(L + 2n)}{L^3}$$

For Notations and Constants, see Arts. 7-1 and 7-2

$$K = JL(1 + G + AG\psi) + 4m(3 + 2A\psi)$$

$$N = JL[2A(1 + G) + BG\psi] + 8m(3A + B\psi)$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = -P\phi\left(\frac{K}{2F} \pm \frac{JL}{D}\right)$$

$$H_1 = H_5 = \frac{PN\phi}{4Fh} \qquad M_1 = M_2 + H_1h$$

$$M_3 = -\frac{PK\phi}{2F} + \frac{Pm}{2} - H_5f \qquad M_5 = M_4 + H_5h$$

$$V_5 = P\left(\frac{m}{L} - \frac{2J\phi}{D}\right) \qquad V_1 = P - V_5$$

When $x_2 \leqq m$

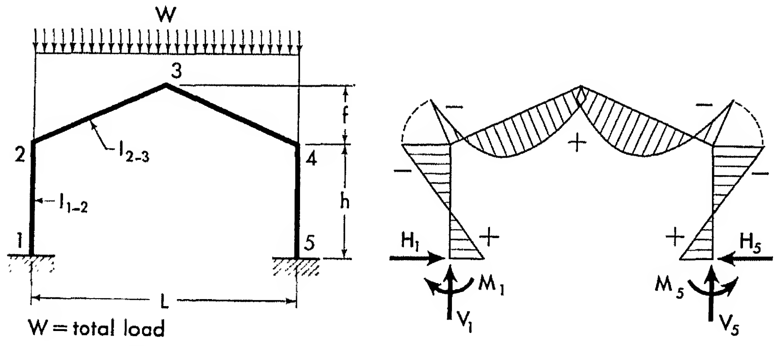
$$M_{x_2} = (Pn + M_3)\frac{2x_2}{L} + M_2\left(1 - \frac{2x_2}{L}\right)$$

When $x_2 > m$

$$M_{x_2} = (Pm + M_2)\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (7-1), (7-3), and (7-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

7-12. Vertical Uniform Load over Entire Girder



$$G = 2 + \frac{5A\psi}{4} \qquad J = 2A + \frac{5B\psi}{8}$$

$$M_2 = M_4 = -\frac{WLG\phi}{F}$$

$$M_1 = M_5 = \frac{WL\phi}{F}(J - G) \qquad H_1 = H_5 = \frac{WLJ\phi}{Fh}$$

$$M_3 = -\frac{WLG\phi}{F} + \frac{WL}{8} - H_5 f$$

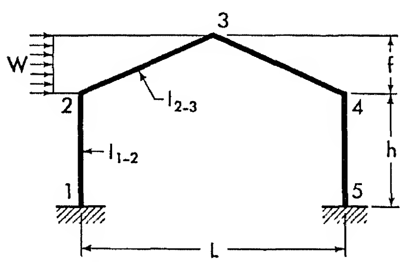
$$V_1 = V_5 = \frac{W}{2}$$

For the left half of girder:

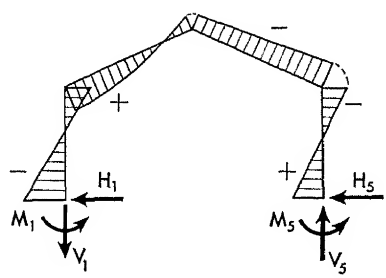
$$M_{x_2} = \left(M_2 + \frac{Wx_2}{4}\right)\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eq. (7-1) to obtain the moment at any section of the left column. Moments and forces at corresponding sections in the right half of the frame are identical to those in the left half.

7-13. Horizontal Uniform Load on Left Inclined Member



W = total load



$$G = 6 - 4A - \phi\psi\left(4 + \frac{5A\psi}{2}\right)$$

$$J = 2B + \phi\psi\left(4A + \frac{5B\psi}{4}\right) - 6A$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = \frac{WGh}{F} \pm \frac{Wh}{D}(12 - \phi\psi)$$

$$H_5 = \frac{WJ}{F} \qquad H_1 = -(W - H_5)$$

$$M_1 = M_2 - h(W - H_5)$$

$$M_3 = \frac{M_2 + M_4}{2} - H_5 f + \frac{Wf}{4} \qquad M_5 = M_4 + H_5 h$$

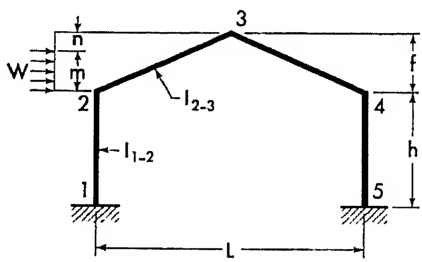
For Notations and Constants, see Arts. 7-1 and 7-2

$$V_5 = \frac{2Wh}{L}\left(\frac{\psi}{4} + \frac{12 - \phi\psi}{D}\right) \qquad V_1 = -V_5$$

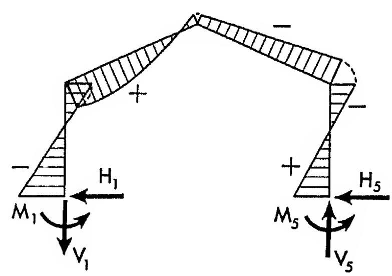
$$M_{y_2} = \left(M_2 + \frac{Wy_2}{2}\right)\left(1 - \frac{y_2}{f}\right) + M_3\frac{y_2}{f}$$

Apply Eqs. (7-1), (7-3), and (7-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

7-14. Horizontal Uniform Load over Part of Left Inclined Member



W = total load



$$g = \frac{m}{f} \qquad G = \frac{2 - g^2}{(2 - g)^2}$$

$$J = \frac{g}{2}(2 - g)^2 \qquad K = Jf\phi(1 + G + AG\psi)$$

$$N = 2h(6 - 4A) - m\phi(6 + 4A\psi)$$

$$S = J\phi\psi[2A(1 + G) + BG\psi]$$

$$T = 4B + g\phi\psi(6A + 2B\psi) - 12A$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = \frac{W}{2F}(N - 2K) \pm \frac{2Wh}{D}(6 - J\phi\psi)$$

$$H_5 = \frac{W}{2F}(S + T) \qquad H_1 = -(W - H_5)$$

$$M_1 = M_2 - h(W - H_5)$$

$$M_3 = \frac{W}{2F}(N - 2K) + \frac{Wm}{4} - H_5 f$$

$$M_5 = M_4 + H_5 h$$

$$V_5 = \frac{W}{L}\left[\frac{m}{2} + \frac{4h}{D}(6 - J\phi\psi)\right]$$

$$V_1 = -V_5$$

When $y_2 \leqq m$

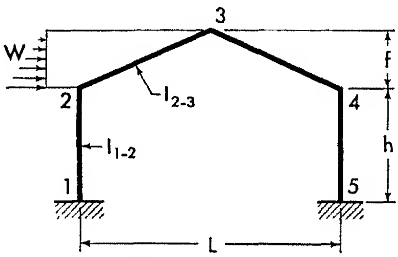
$$M_{y_2} = \frac{Wy_2}{2}\left(1 + \frac{n}{f} - \frac{y_2}{m}\right) + M_2\left(1 - \frac{y_2}{f}\right) + M_3\frac{y_2}{f}$$

When $y_2 > m$

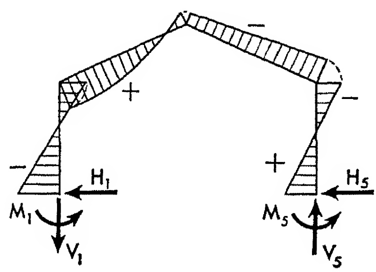
$$M_{y_2} = \left(\frac{Wm}{2} + M_2\right)\left(1 - \frac{y_2}{f}\right) + M_3\frac{y_2}{f}$$

Apply Eqs. (7-1), (7-3), and (7-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

7-15. Horizontal Triangular Load on Left Inclined Member



W = total load



$$G = 6 - 3\phi\psi\left(1 + \frac{3A\psi}{5}\right) - 4A$$

$$J = 2B + \frac{3\phi\psi}{10}(10A + 3B\psi) - 6A$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = \frac{WhG}{F} \pm \frac{4Wh}{15D}(45 - 4\phi\psi)$$

$$H_5 = \frac{WJ}{F} \qquad H_1 = -(W - H_5)$$

$$M_1 = M_2 + H_1h \qquad M_3 = \frac{WhG}{F} + \frac{Wf}{6} - H_5f$$

$$M_5 = M_4 + H_5h$$

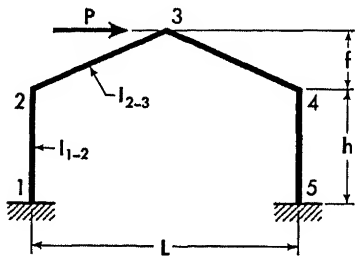
$$V_5 = \frac{W}{3L}\left[f + \frac{8h}{5D}(45 - 4\phi\psi)\right] \qquad V_1 = -V_5$$

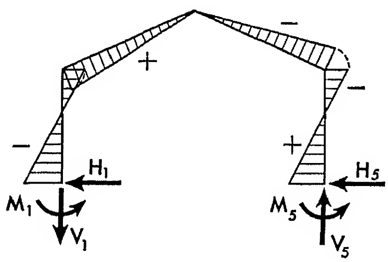
$$M_{y_2} = \left[M_2 + \frac{Wy_2}{3}\left(2 - \frac{y_2}{f}\right)\right]\left(1 - \frac{y_2}{f}\right) + M_3\frac{y_2}{f}$$

Apply Eqs. (7-1), (7-3), and (7-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

For Notations and Constants, see Arts. 7-1 and 7-2

7-16. Horizontal Concentrated Load at Joint 3





$$M_2 = \frac{12Ph}{D} \qquad M_4 = -\frac{12Ph}{D}$$

$$H_1 = -\frac{P}{2} \qquad H_5 = \frac{P}{2}$$

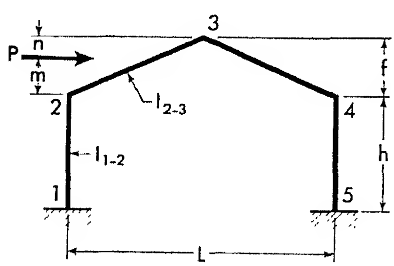
$$M_1 = M_2 + H_1h \qquad M_3 = 0$$

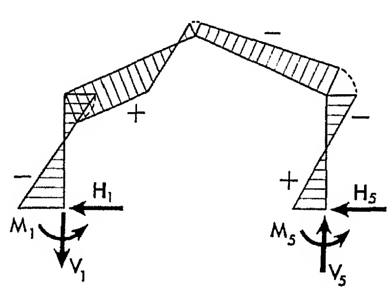
$$M_5 = M_4 + H_5h$$

$$V_5 = \frac{Ph}{DL}(24 + D\psi) \qquad V_1 = -V_5$$

Apply Eqs. (7-1) through (7-4) to obtain the moment at any section of the frame members.

7-17. Horizontal Concentrated Load at Any Point of Left Inclined Member





$$G = \frac{f + m}{f + n} \qquad J = \frac{2mn(f + n)}{f^3}$$

$$K = Jf\phi(1 + G + AG\psi)$$

$$N = h(6 - 4A) - m\phi(6 + 4A\psi)$$

$$S = J\phi\psi[2A(1 + G) + BG\psi]$$

$$T = 2B - 6A + \frac{m\phi\psi}{f}(6A + 2B\psi)$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = \frac{P}{F}(N - K) \pm \frac{2Ph}{D}(6 - J\phi\psi)$$

$$H_5 = \frac{P}{2F}(S + 2T) \qquad H_1 = -(P - H_5)$$

$$M_1 = M_2 + H_1 h \qquad M_3 = \frac{P}{F}(N - K) + \frac{Pm}{2} - H_5 f$$

$$M_5 = M_4 + H_5 h$$

$$V_5 = \frac{P}{DL}[Dm + 4h(6 - J\phi\psi)]$$

$$V_1 = -V_5$$

When $x_2 \leqq \frac{Lm}{2f}$

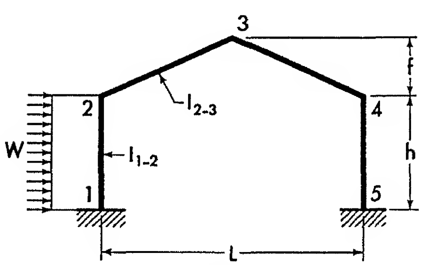
$$M_{x_2} = (M_3 + Pn)\frac{2x_2}{L} + M_2\left(1 - \frac{2x_2}{L}\right)$$

When $x_2 > \frac{Lm}{2f}$

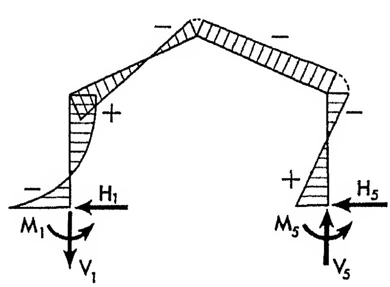
$$M_{x_2} = (M_2 + Pm)\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (7-1), (7-3), and (7-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

7-18. Horizontal Uniform Load on Column



W = total load



$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = \frac{Wh}{2F}(4 - 3A) \pm \frac{4Wh}{D}$$

For Notations and Constants, see Arts. 7-1 and 7-2

$$H_5 = \frac{W}{4F}(3B - 8A) \qquad H_1 = -(W - H_5)$$

$$M_1 = M_2 + H_5h - \frac{Wh}{2}$$

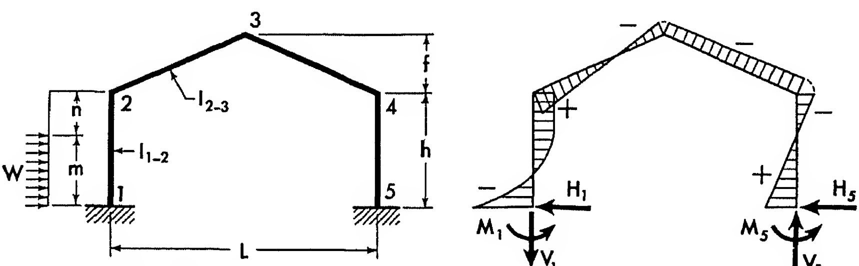
$$M_3 = \frac{Wh}{2F}(4 - 3A) - H_5f \qquad M_5 = M_4 + H_5h$$

$$V_5 = \frac{8Wh}{DL} \qquad V_1 = -V_5$$

$$M_{y_1} = \left(\frac{Wy_1}{2} + M_1\right)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (7-2) through (7-4) to obtain the moment at any section of frame members 2-3, 3-4, and 4-5.

7-19. Horizontal Uniform Load over Part of Column



$$g = \frac{m}{h} \qquad G = \frac{2 - g^2}{(2 - g)^2} \qquad J = \frac{g}{2}(2 - g)^2$$

$$K = m(3 - 2A) - Jh(1 + G - A)$$

$$N = 6m - 2Jh(1 + G)$$

$$S = m(2B - 6A) - Jh(B - 2A - 2AG)$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = W\left(\frac{K}{F} \pm \frac{N}{D}\right) \qquad H_5 = \frac{WS}{2Fh}$$

$$M_1 = M_2 + H_5h - \frac{Wm}{2}$$

$$M_3 = \frac{WK}{F} - H_5f \qquad M_5 = M_4 + H_5h$$

$$H_1 = -(W - H_5)$$

$$V_5 = \frac{4W}{DL}[3m - Jh(1 + G)] \qquad V_1 = -V_5$$

When $y_1 \leqq m$

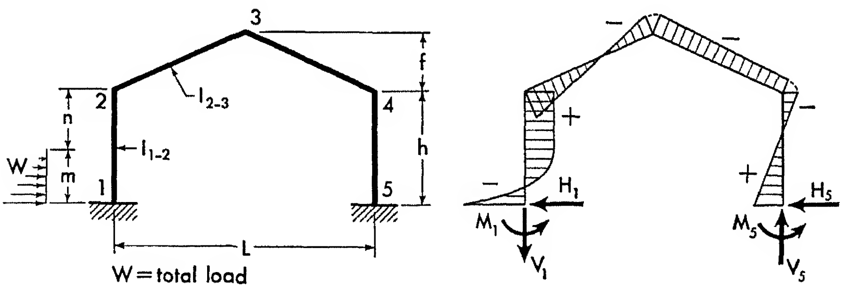
$$M_{y_1} = \frac{Wy_1}{2}\left(\frac{n}{h} + \frac{m - y_1}{m}\right) + M_1\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

When $y_1 > m$

$$M_{y_1} = \left(\frac{Wm}{2} + M_1\right)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (7-2) through (7-4) to obtain the moment at any section of frame members 2-3, 3-4, and 4-5.

7-20. Horizontal Triangular Load over Part of Column



$$g = \frac{m}{h} \qquad G = \frac{10 - 3g^2}{20 - 15g + 3g^2}$$

$$J = \frac{g}{15}(20 - 15g + 3g^2)$$

$$K = \frac{2m}{3}(3 - 2A) - Jh(1 + G - A)$$

$$N = 4g(B - 3A) + 3J(2A + 2AG - B)$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = \frac{WK}{F} \pm \frac{2W}{D}[2m - Jh(1 + G)]$$

$$H_5 = \frac{WN}{6F} \qquad H_1 = -(W - H_5)$$

$$M_1 = M_2 + H_5h - \frac{Wm}{3} \qquad M_3 = \frac{WK}{F} - H_5f$$

$$M_5 = M_4 + H_5h$$

$$V_5 = \frac{4Wh}{DL}[2g - J(1+G)] \qquad V_1 = -V_5$$

When $y_1 \leqq m$

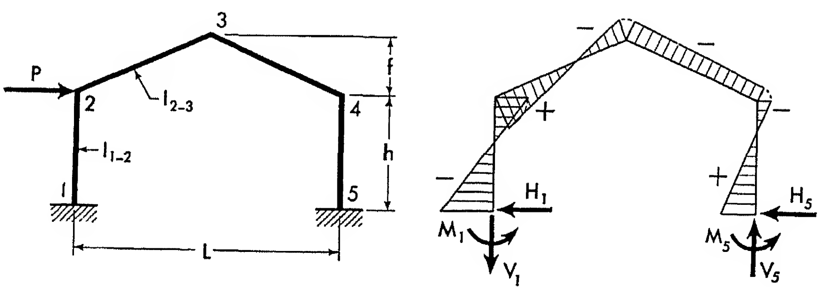
$$M_{y_1} = \frac{Wy_1}{3}\left[\frac{n}{h} + \frac{(m-y_1)(2m-y_1)}{m^2}\right] + M_1\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

When $y_1 > m$

$$M_{y_1} = \left(\frac{Wm}{3} + M_1\right)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (7-2) through (7-4) to obtain the moment at any section of frame members 2-3, 3-4, and 4-5.

7-21. Horizontal Concentrated Load at Joint 2



$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = 4Ph\left(\frac{3-2A}{2F} \pm \frac{3}{D}\right)$$

$$H_5 = \frac{2P}{F}(B - 3A) \qquad H_1 = -(P - H_5)$$

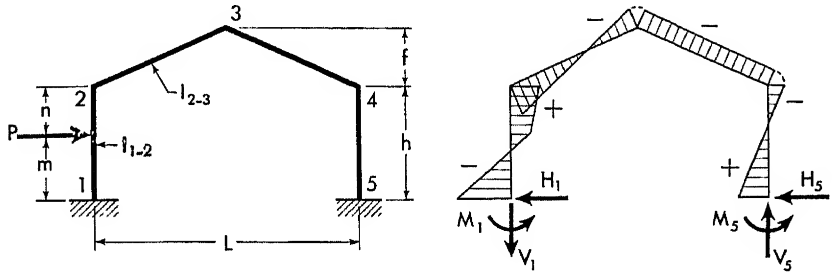
$$M_1 = M_2 - h(P - H_5)$$

$$M_3 = \frac{2Ph}{F}(3 - 2A) - H_5f \qquad M_5 = M_4 + H_5h$$

$$V_5 = \frac{24Ph}{DL} \qquad V_1 = -V_5$$

Apply Eqs. (7-1) through (7-4) to obtain the moment at any section of the frame members.

7-22. Horizontal Concentrated Load at Any Point of Column



$$G = \frac{h+m}{h+n} \qquad J = \frac{2mn(h+n)}{h^3}$$

$$K = m(6 - 4A) - Jh(1 + G - A)$$

$$N = 4m(B - 3A) - Jh[B - 2A(1 + G)]$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = \frac{PK}{F} \pm \frac{2P}{D}\,[6m - Jh(1 + G)]$$

$$H_5 = \frac{PN}{2Fh} \qquad H_1 = -(P - H_5)$$

$$M_1 = M_2 + H_5h - Pm \qquad M_3 = \frac{PK}{F} - H_5f$$

$$M_5 = M_4 + H_5h$$

$$V_5 = \frac{4P}{DL}\,[6m - Jh(1 + G)] \qquad V_1 = -V_5$$

When $y_1 \leqq m$

$$M_{y_1} = (Pn + M_2)\frac{y_1}{h} + M_1\left(1 - \frac{y_1}{h}\right)$$

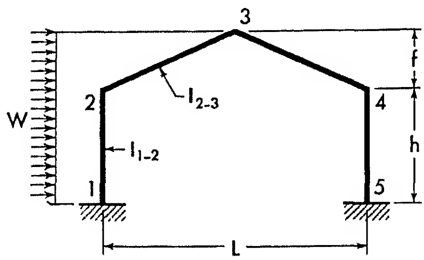
When $y_1 > m$

$$M_{y_1} = (Pm + M_1)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

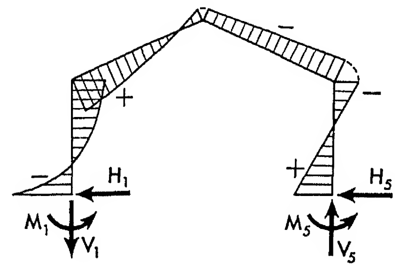
Apply Eqs. (7-2) through (7-4) to obtain the moment at any section of frame members 2-3, 3-4, and 4-5.

For Notations and Constants, see Arts. 7-1 and 7-2

7-23. Horizontal Uniform Load over Left Half of Frame



W = Total load



$$m = \frac{h}{h+f} \qquad n = 1 - m$$

$$G = 6 - 4A - \phi\psi'\left(4 + \frac{5A\psi'}{2}\right)$$

$$J = 2B + \phi\psi'\left(4A + \frac{5B\psi'}{4}\right) - 6A$$

$$K = Gn + m\left(2 - \frac{3}{2}A\right) \qquad N = 4m + n(12 - \phi\psi')$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = Wh\left(\frac{K}{F} \pm \frac{N}{D}\right)$$

$$H_5 = \frac{W}{F}\left(Jn + \frac{3Bm}{4} - 2Am\right) \qquad H_1 = -(W - H_5)$$

$$M_1 = M_2 + H_5h - \frac{Wh}{2}(1 + n)$$

$$M_3 = \frac{M_2 + M_4}{2} - H_5f + \frac{Wfn}{4}$$

$$M_5 = M_4 + H_5h$$

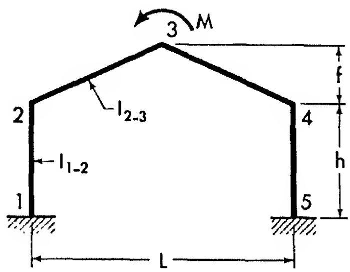
$$V_5 = \frac{Wh}{2DL}(4N + Dn\psi') \qquad V_1 = -V_5$$

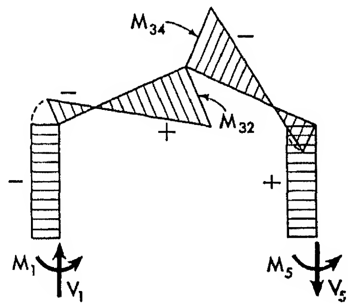
$$M_{y_1} = \left(M_1 + \frac{Wmy_1}{2}\right)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

$$M_{y_2} = \left(M_2 + \frac{Wny_2}{2}\right)\left(1 - \frac{y_2}{f}\right) + M_3\frac{y_2}{f}$$

Apply Eqs. (7-3) and (7-4) to obtain the moment at any section of frame members 3-4 and 4-5.

7-24. Moment Applied at Joint 3





$$M_1 = M_2 = -\frac{4M\phi}{D} \qquad M_{32} = \frac{M}{2}$$

$$M_{34} = -\frac{M}{2} \qquad M_4 = M_5 = \frac{4M\phi}{D}$$

$$V_5 = -\frac{M}{L}\left(1 + \frac{8\phi}{D}\right) \qquad V_1 = -V_5$$

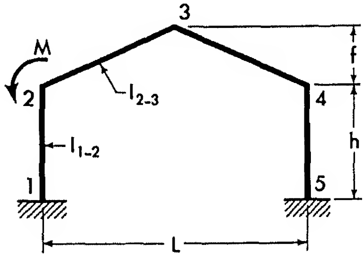
$$H_1 = H_5 = 0$$

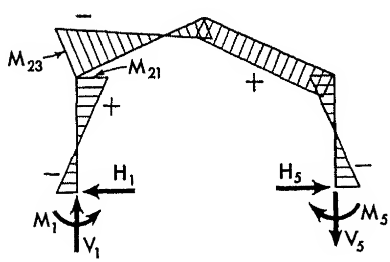
$$M_{x_2} = M_2\left(1 - \frac{2x_2}{L}\right) + M_{32}\frac{2x_2}{L}$$

$$M_{x_3} = M_{34}\left(1 - \frac{2x_3}{L}\right) + M_4\frac{2x_2}{L}$$

Apply Eqs. (7-1) and (7-4) to obtain the moment at any section of frame members 1-2 and 4-5.

7-25. Moment Applied at Joint 2





$$\left.\begin{matrix} M_{21} \\ M_4 \end{matrix}\right\rangle = 2M\phi\left(\frac{6 + 3A\psi}{F} \pm \frac{4}{D}\right)$$

For Notations and Constants, see Arts. 7-1 and 7-2

$$H_1 = H_5 = -\frac{3M\phi}{Fh}(4A + B\psi)$$

$$M_{23} = M_{21} - M \qquad M_1 = M_{21} + H_5 h$$

$$M_3 = 2M\phi\left(\frac{6 + 3A\psi}{F}\right) - \frac{M}{2} - H_5 f$$

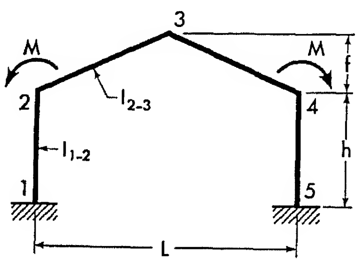
$$M_5 = M_4 + H_5 h \qquad V_5 = -\frac{M}{L}\left(1 - \frac{16\phi}{D}\right) \qquad V_1 = -V_5$$

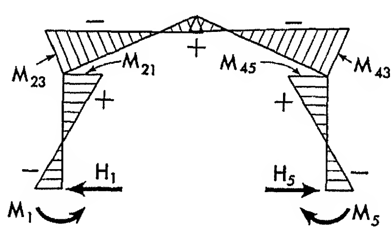
$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) + M_{21}\frac{y_1}{h}$$

$$M_{x_2} = M_{23}\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (7-3) and (7-4) to obtain the moment at any section of frame members 3-4 and 4-5.

7-26. Two Equal Moments Applied at Joints 2 and 4





$$M_{21} = M_{45} = 4M\phi\left(\frac{6 + 3A\psi}{F}\right)$$

$$H_1 = H_5 = -\frac{6M\phi}{Fh}(4A + B\psi)$$

$$M_{23} = M_{43} = M_{21} - M \qquad M_1 = M_5 = M_{21} + H_5 h$$

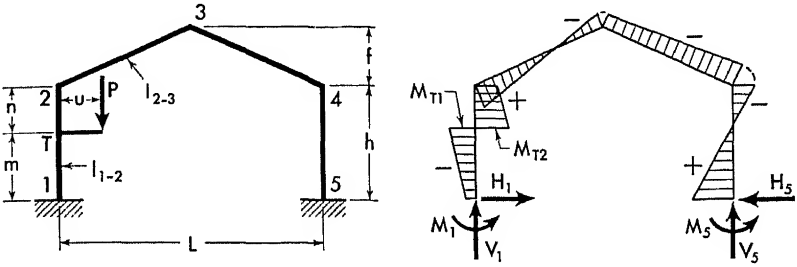
$$M_3 = 4M\phi\left(\frac{6 + 3A\psi}{F}\right) - M - H_5 f \qquad V_1 = V_5 = 0$$

$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) + M_{21}\frac{y_1}{h}$$

$$M_{x_2} = M_{23}\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Moments at corresponding sections in the right half of the frame are identical to those in the left half.

7-27. Vertical Concentrated Load Applied at Bracket



Bracket acts as a simple cantilever and its maximum moment is Pu at point T. The moment diagram of the cantilever is intentionally not shown so that the frame's bending moment diagram may be more clearly illustrated.

$$M = Pu$$

$$G = \frac{h^2 - 3m^2}{h^2 - 3n^2} \qquad J = \frac{2(h^2 - 3n^2)}{h^2}$$

$$K = 6 + J(1 - A - G) - 4A$$

$$N = 4B + J(2AG + B - 2A) - 12A$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = \frac{MK}{F} \pm \frac{2M}{D}[6 + J(1 - G)]$$

$$H_1 = H_5 = \frac{MN}{2Fh} \qquad M_1 = M_2 + H_5 h - M$$

$$M_3 = \frac{MK}{F} - H_5 f \qquad M_5 = M_4 + H_5 h$$

$$V_5 = \frac{4M}{DL}[6 + J(1 - G)]$$

$$V_1 = P - V_5$$

When $y_1 < m$

$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) - (M - M_2)\frac{y_1}{h}$$

When $y_1 = m$

$$M_{T1} = M_1\frac{n}{h} - (M - M_2)\frac{m}{h} \quad \text{and} \quad M_{T2} = (M + M_1)\frac{n}{h} + M_2\frac{m}{h}$$

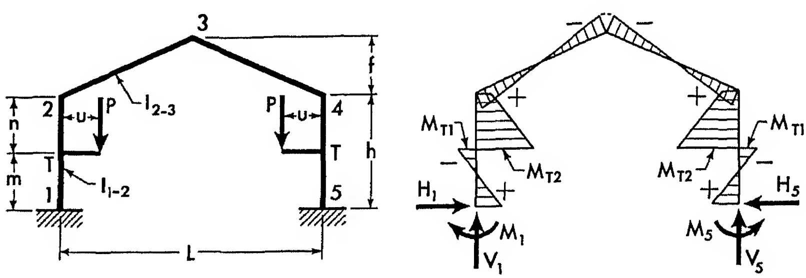
For Notations and Constants, see Arts. 7-1 and 7-2

When $y_1 > m$

$$M_{y_1} = (M + M_1)\left(1 - \frac{y_1}{h}\right) + M_2 \frac{y_1}{h}$$

Apply Eqs. (7-2) through (7-4) to obtain the moment at any section of frame members 2-3, 3-4, and 4-5.

7-28. Two Equal Vertical Concentrated Loads Symmetrically Applied at Brackets



Brackets act as simple cantilevers with the maximum moments of Pu at points T. The moment diagrams of these cantilevers are intentionally not shown so that the frame's bending moment diagram may be clearly illustrated.

$$M = Pu$$

$$G = \frac{h^2 - 3m^2}{h^2 - 3n^2} \qquad J = \frac{2(h^2 - 3n^2)}{h^2}$$

$$K = 6 + J(1 - A - G) - 4A$$

$$N = 4B + J(2AG + B - 2A) - 12A$$

$$M_2 = M_4 = \frac{2MK}{F}$$

$$H_1 = H_5 = \frac{MN}{Fh} \qquad M_1 = M_5 = M_2 + H_5 h - M$$

$$M_3 = \frac{2MK}{F} - H_5 f$$

$$V_1 = V_5 = P$$

When $y_1 < m$

$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) - (M - M_2)\frac{y_1}{h}$$

When $y_1 = m$

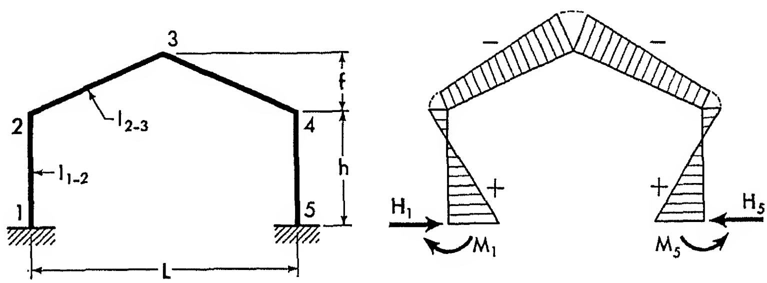
$$M_{T1} = M_1 \frac{n}{h} - (M - M_2)\frac{m}{h} \qquad \text{and} \qquad M_{T2} = (M + M_1)\frac{n}{h} + M_2 \frac{m}{h}$$

When $y_1 > m$

$$M_{y_1} = (M + M_1)\left(1 - \frac{y_1}{h}\right) + M_2 \frac{y_1}{h}$$

Apply Eq. (7-2) to obtain the moment at any section of frame member 2-3. Moments and forces at corresponding sections in the right half of the frame are identical to those in the left half.

7-29. Effect of Temperature Rise. Range t° for entire frame.



$$K = \frac{6L\varepsilon t^\circ}{Fh^2} EI_{1-2}$$

$$M_2 = M_4 = -2AK \qquad M_1 = M_5 = K(B - 2A)$$

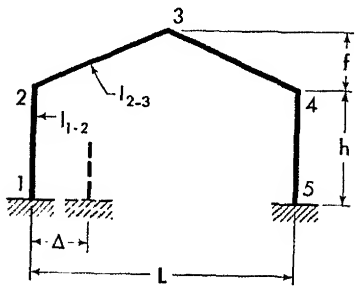
$$H_1 = H_5 = \frac{KB}{h}$$

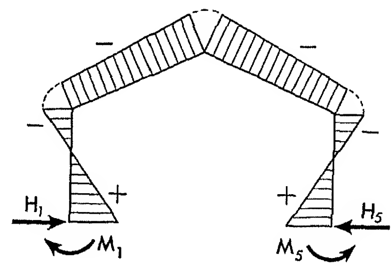
$$M_3 = M_1 - H_1(h + f) \qquad V_1 = V_5 = 0$$

Apply Eqs. (7-1) through (7-4) to obtain the moment at any section of the frame members.

Note: For temperature drop, introduce the value of t° with a negative sign.

7-30. Horizontal Displacement of One Support





$$K = \frac{6\Delta}{Fh^2} EI_{1\text{-}2}$$

$M_2 = M_4 = -2AK$ $\qquad$ $M_1 = M_5 = K(B - 2A)$

$$H_1 = H_5 = \frac{KB}{h}$$

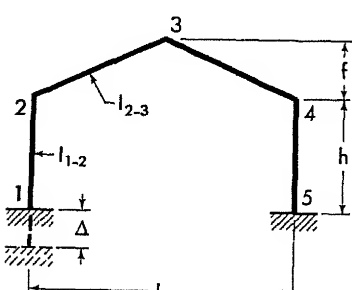
$$M_3 = M_1 - H_1(h + f)$$

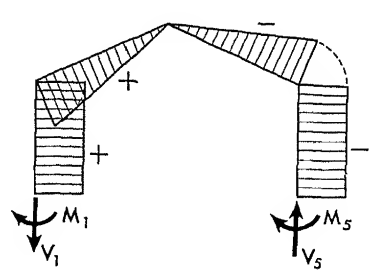
$$V_1 = V_5 = 0$$

Apply Eqs. (7-1) through (7-4) to obtain the moment at any section of the frame members.

Note: If the direction of the frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.

7-31. Vertical Settlement of One Support





$$M_1 = M_2 = \frac{48\Delta}{DLh} EI_{1\text{-}2}$$

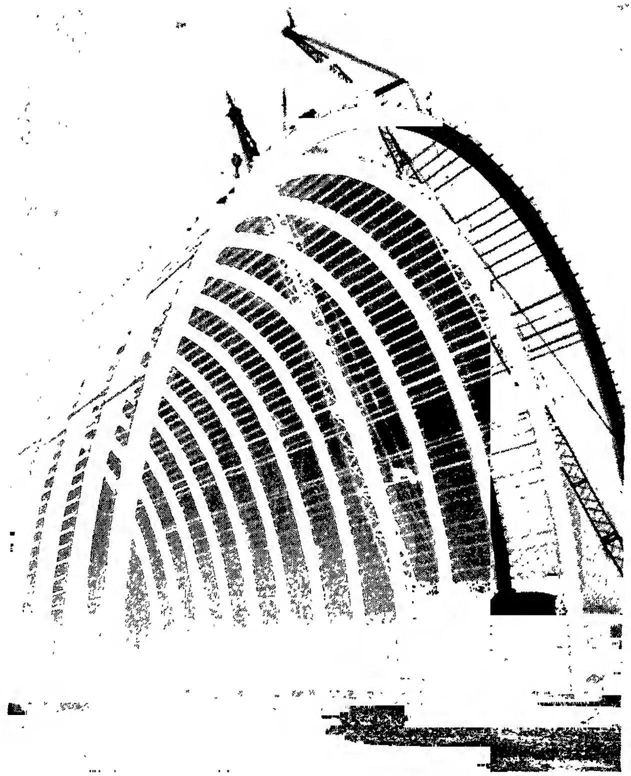
$M_4 = M_5 = -M_1$ $\qquad$ $M_3 = 0$

$$H_1 = H_5 = 0 \qquad V_5 = \frac{2M_1}{L}$$

$$V_1 = -V_5$$

Apply Eqs. (7-1) through (7-4) to obtain the moment at any section of the frame members.

Note: If the direction of the frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.



The skeleton framework of the U.S. Corps of Engineers Radar Laboratory at Wright Field in Dayton, Ohio. The parabolic, glue-laminated, two-hinged arches of 80-foot height and 80-foot span were built to shelter laboratory equipment. These arches were fabricated by gluing 2- by 8-inch planks together. As can be clearly seen in the illustration, the present glue-laminated technique ingeniously solves the problem of fabricating arches of variable curvature and thickness. Hazelet and Erdal are architects for the building. The Timber Structures of Portland, Oregon, are designers and fabricators. (Courtesy of Timber Structures.)

SECTION 8

INTRODUCTION TO ANALYSIS OF FRAMES WITH CURVED MEMBERS

8-1. General. Condensed solutions for the analysis of more commonly used parabolic arches and arched frames under various loads are presented in Sections 9 to 12. While these solutions have been derived, as mentioned above, for members with parabolic axes, it should be noted that other curved members may be also analyzed by the use of these condensed solutions.

Thus, for example, in a segmental member of low rise to span ratio, that is, a flat member with the axis defined by a circular arc, it may be noted that the arc curvature approaches that of a parabola. Therefore, the solutions derived for flat arched members with parabolic axes are applicable to flat arched members with circular arc axes.

Moreover, numerous investigations indicate that elliptical, transformed catenarian and other curves only slightly deviate from the parabolic curve; hence the given solutions provide sufficiently accurate results for members of various curvatures which are in the range of low rise to span ratio.[1]

8-2. Coordinates of Parabolic Axes. Curved members which are considered in the text are symmetrical members with parabolic axes. The coordinates of their axes are defined by the quadratic equation:

[1] For comprehensive discussion of this subject, see T. Merriman, *American Civil Engineers' Handbook*, 5th ed., John Wiley & Sons, Inc., New York, 1930, p. 966. See also J. A. Parcel and G. A. Maney, *An Elementary Treatise on Statically Indeterminate Stresses*, John Wiley & Sons, Inc., New York, 1936, pp. 295 and 327.

$$y = 4f\left(1 - \frac{x}{L}\right)\frac{x}{L} \tag{8-1}$$

where

f = rise of arched member
L = span of arched member
x and y = coordinates of axis with origin at left end of member

For convenience, the coordinates of a parabolic axis, expressed in terms of the rise (f) and the span (L), are listed in Table 9, in the Appendix at intervals of $0.01L$.

8-3. Geometry of Arched Members. The arched members which are used in derivation of the condensed solutions of analysis given in Sections 9 through 12 are characterized by the relation of their cross sections to the angles of inclination of their axes. Expressed in mathematical terms, this means that the sectional moments of inertia of arched members about their neutral axes vary directly with sec φ. Restated in another way, it means that the thickness of these members varies from crown to springing lines as a function of the angle of inclination of the members' axes to the horizontal, conforming to the following equation:

$$d = d_0 \sqrt[3]{\sec \varphi} \tag{8-2}$$

where

d = thickness of arched member at section defined by angle of inclination φ
d_0 = thickness of arched member at crown
$\sec \varphi$ = secant of angle of inclination of member's axis, at the considered section

Examination of Eq. (8-2) reveals that these arched members are only slightly haunched. To illustrate this point, an arch with a ratio of rise to length of 0.15 is considered. For this arch the relative thickness at the springing line is only 5.3 per cent greater than at the crown. For flat arches these variations are even less significant. Because the depth variations of these arched members are minor, they are grouped in this text with the structures comprised of members of constant section.

Considering the effect of haunching of the arched member on the redundant quantities of the structures, analyzed in Sections 9 through 12, it was found that these quantities are not sensitive to minor cross-sectional

variation of the arched member. This fact justifies the use of these condensed solutions for parabolic arches of constant section or for frames containing these arches.

For practical purposes, if the rise to span ratio of an arched member is less than 0.2, the equations of Sections 9 through 12, having been derived for slightly haunched arched members defined by Eq. (8-2), nevertheless provide entirely satisfactory results for structures having arched members of constant section. For greater ratios, the solutions are exact for structures having slightly haunched arched members and approximate for those containing arched members of constant section. The analysis of structures having arched members of constant section and large f/L ratio may be accomplished with high precision by the use of the condensed solutions given in Sections 21 through 24.

8-4. Method of Analysis. As stated previously, the condensed solutions for arches or arched frames are derived by the use of the theory of virtual work, and only the effect of flexural deformation is considered. The effects of shearing deformation and axial deformation (rib shortening) are neglected since their contribution to the total energy of deformation is insignificant. Only in certain cases is a greater refinement justified, and for such cases additional solutions allowing for the effect of axial deformation are given. In this text, the solution of the first type is termed method A, and of the second, method B.

Solutions by method A are given for all loading conditions and for them the bending moment diagrams are shown in the text. Supplementary solutions by method B are given only for cases of vertical loadings on the hingeless arches, the cases more frequently encountered.

It is a generally recognized fact that the effect of axial deformation is of a practical importance only in flat hingeless arches. When the arch rise to length ratio is greater than 0.2, the effect of axial deformation can be neglected, and solution by method A provides satisfactory results. When the ratio is less than 0.2, the refinement in calculations is justified and solution by method B is recommended.

8-5. Illustrative Example. A hingeless parabolic arch of constant section carries a uniform load of 2 kip/ft over the left half of the arch, as shown in Fig. 8-1. The span and rise are 40 ft and 4 ft, respectively, and the thickness of the arch is 1.5 ft. Assuming that the width of the arch is 1 ft, find the redundant quantities of the arch.

The above-described arch has the rise to span ratio of 0.1; therefore,

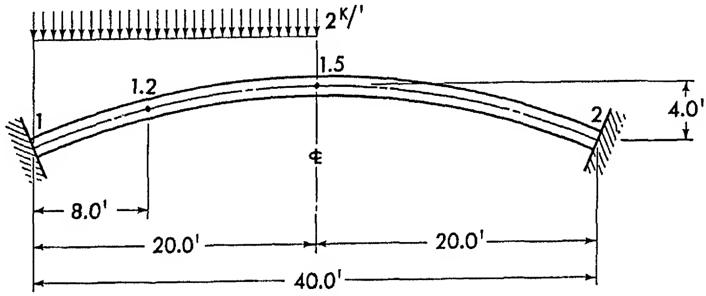


FIG. 8-1. Parabolic arch of constant section with applied vertical load

as stated in Art. 8-3, the condensed solutions of analysis given in the text for slightly haunched arches may be advantageously applied for analysis of the given arch of the constant section.

Analysis by Method A. Applying notations and equations of the condensed solution given in Arts. 10-1 and 10-4, the redundant moments and reactions are readily obtained. Total load $W = 2 \times 20 = 40$ kip. Then,

$$M_1 = -\frac{WL}{32} = -\frac{40 \times 40}{32} = -50 \text{ ft-kip}$$

$$M_2 = \frac{WL}{32} = \frac{40 \times 40}{32} = 50 \text{ ft-kip}$$

$$H_1 = H_2 = \frac{WL}{8f} = \frac{40 \times 40}{8 \times 4} = 50 \text{ kip}$$

$$V_1 = \frac{13}{16} W = \frac{13}{16} \times 40 = 32.5 \text{ kip}$$

$$V_2 = \frac{3}{16} W = \frac{3}{16} \times 40 = 7.5 \text{ kip}$$

For the left half of the arch, the equation of the bending moment at any section is

$$M_x = M_1 + Wx\left(\frac{13}{16} - \frac{x}{L}\right) - \frac{WLy}{8f} \tag{8-3}$$

To apply Eq. (8-3), the coordinates x and y for any section of this half must be known. With the aid of Table 9, in the Appendix, they may be conveniently determined. For example, the coordinates for sections 1.1 and 1.2 of the arch are

$$x_{1.1} = 0.1L = 4.0 \text{ ft} \qquad y_{1.1} = 0.36f = 1.44 \text{ ft}$$

$$x_{1.2} = 0.2L = 8.0 \text{ ft} \qquad y_{1.2} = 0.64f = 2.56 \text{ ft}$$

Inserting the numerical values into Eq. (8-3), the moment $M_{1.1}$ is

$$M_{1.1} = (-50) + 40 \times 4\left(\frac{13}{16} - \frac{4}{40}\right) - \frac{40 \times 40 \times 1.44}{8 \times 4} = -8 \text{ ft-kip}$$

and similarly

$$M_{1.2} = 18 \text{ ft-kip}$$

For the right half of the arch, the bending moments may be determined by the use of the equation

$$M_x = M_2 + \frac{3W}{16}(L - x) - \frac{WLy}{8f}$$

giving, for example, the moment at section 1.6 as

$$M_{1.6} = 50 + \frac{3 \times 40}{16}(40 - 24) - \frac{40 \times 40 \times 3.84}{8 \times 4} = -22 \text{ ft-kip}$$

The complete bending moment diagram of the arch is shown in Fig. 8-2.

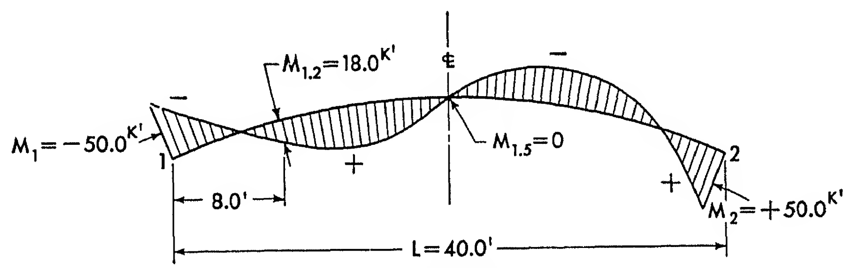


FIG. 8-2. Bending moment diagram of the arch

The equation of the axial force, for any section of the left half of the arch, is given in Art. 10-4 as

$$N_x = H_1 \cos\varphi + \left(V_1 - \frac{2Wx}{L}\right)\sin\varphi \tag{8-4}$$

To apply this equation, the trigonometric functions of the angle of inclination for any section of the arch must be known. Table 10, in the Appendix, provides angles of inclination at different sections for a variety of arches. Thus, for sections 1 and 1.2 the angles of inclination of the arch axis are 21°48′ and 13°30′, respectively; their trigonometric functions are

$\sin 21°48' = 0.3714 \qquad \cos 21°48' = 0.9285$

$\sin 13°30' = 0.2335 \qquad \cos 13°30' = 0.9724$

Introducing numerical values into Eq. (8-4), the following axial forces at sections 1 and 1.2 are found:

$$N_1 = 50 \times 0.9285 + \left(32.5 - \frac{2 \times 40 \times 0}{40}\right) 0.3714 = 58.50 \text{ kip}$$

$$N_{1.2} = 50 \times 0.9724 + \left(32.5 - \frac{2 \times 40 \times 8}{40}\right) 0.2335 = 52.47 \text{ kip}$$

In a similar manner, the shearing force at any section of the arch may be determined by the use of equations given in the text. For example, applying the equation

$$Q_x = -H_1 \sin \varphi + \left(V_1 - \frac{2Wx}{L}\right) \cos \varphi$$

the shearing forces at sections 1 and 1.2 are

$$Q_1 = (-50) \times 0.3714 + \left(32.5 - \frac{2 \times 40 \times 0}{40}\right) 0.9285 = 11.61 \text{ kip}$$

and

$$Q_{1.2} = 4.37 \text{ kip}$$

The complete shearing force diagram is given in Fig. 8-3.

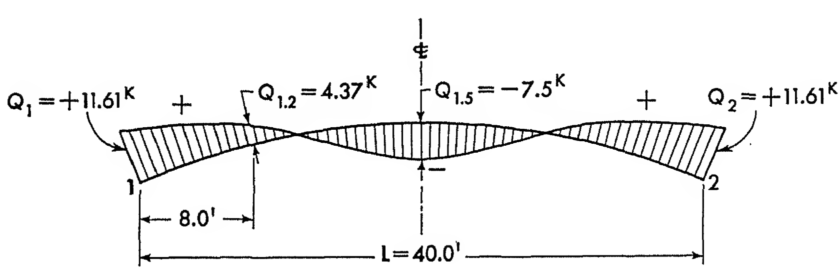


FIG. 8-3. Shearing force diagram of the arch

Static Check. It is advisable to make a check of static equilibrium when all redundant forces have been found. With reference to Fig. 8-4, a check is made by considering the segment of the arch between sections 1.2 and 2 as a free body.

Observing that at section 1.2 the horizontal coordinate is 8 ft and the vertical coordinate is 2.56 ft, the following equation may be written:

$$M_{1.2} = M_2 + V_2(40 - 8) - H_2 \times 2.56 - 2 \times 12 \times 6$$

Inserting numerical values of the redundants,

$$M_{1.2} = 50 + 7.5 \times 32 - 50 \times 2.56 - 2 \times 12 \times 6 = 18 \text{ ft-kip}$$

and good agreement with previous calculations is obtained.

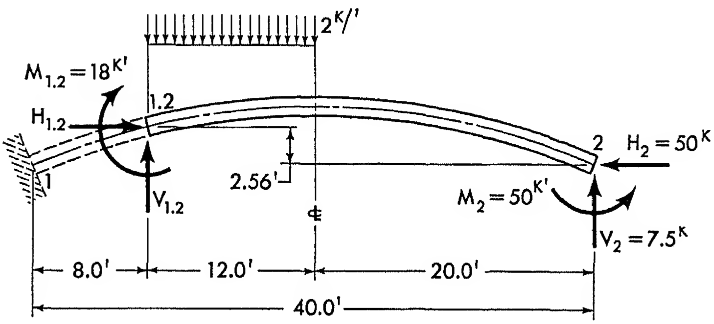


FIG. 8-4. Free body diagram for the segment of the arch

Analysis by Method B. The analysis of the arch by this method provides a refined solution because the effect of axial deformation is taken into consideration. Remembering that the foot-kip dimensional system is used in this example, and applying equations given in Arts. 10-1 and 10-4, the following solution is obtained:

$$G = \tau \frac{d^2}{f^2} = 0.893 \frac{1.5^2}{4^2} = 0.1256$$

$$H_1 = H_2 = \frac{WL}{8f(1+G)} = \frac{40 \times 40}{8 \times 4(1+0.1256)} = 44.42 \text{ kip}$$

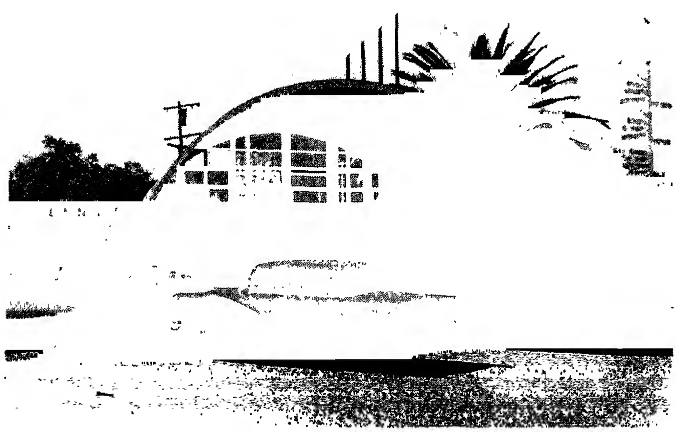
$$\left.\begin{matrix} M_1 \\ \\ M_2 \end{matrix}\right\rangle = -\frac{WL}{12} \cdot \frac{G}{1+G} \mp \frac{WL}{32}$$

$$= -\frac{40 \times 40}{12} \cdot \frac{0.1256}{1+0.1256} \mp \frac{40 \times 40}{32} = \begin{matrix} -64.88 \text{ ft-kip} \\ 35.12 \text{ ft-kip} \end{matrix}$$

$V_1 = 32.5$ kip $\qquad V_2 = 7.5$ kip

Axial and shearing forces may be determined without difficulty by the use of Eqs. (10-6) and (10-7).

A public lobby in the General Assembly Building of the United Nations Headquarters, United Nations, New York. The elegantly shaped steel arches in the background support the large step-ramp leading to the Assembly Halls. This photograph demonstrates the successful use of arches in contributing to the over-all magnificence of the building. (Courtesy of the United Nations, N.Y.)

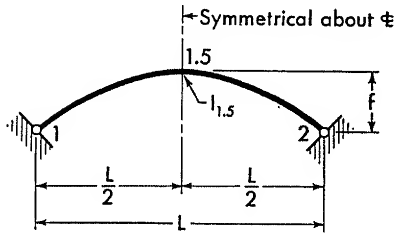


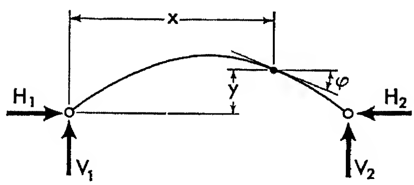
A drive-in ticket office of the United Airlines, in Redwood City, California. This photograph illustrates the application of arcuately shaped rolled steel members to serve as arch ribs of the building. In addition to being highly functional, the structure is characterized by its attractiveness and neatness. (Courtesy of Nick Guins.)

SECTION 9

SYMMETRICAL PARABOLIC TWO-HINGED ARCHES

9-1. Notations and Coordinates





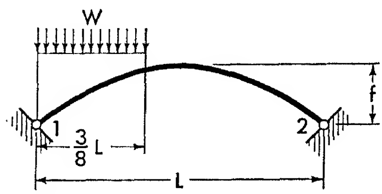
The sketch appearing on the left, above, explains notations for a representative arch with minor depth variations, as defined by Eq. (8-2).[1] The arch axis is a symmetrical parabola conforming to Eq. (8-1).

The sketch appearing on the right, above, shows positive directions of the vertical and horizontal components of the arch reactions. It also defines the angle of inclination and coordinates at any section of an arch. Angles of inclination and coordinates are to be considered only in the positive sense.

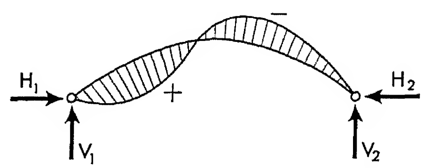
9-2. Equations of Forces and Moments. The equations for the vertical and the redundant horizontal components of the arch reactions are given on the following pages for a number of loading conditions. Corresponding expressions for the moment and the axial and shearing forces at any section of the arch are also provided.

[1] The application of the condensed solutions of analysis given in this section may be extended to arches of constant section, by the reasons given in Art. 8-3.

9-3. Vertical Uniform Load over Three-eighths of Span



W = total load



$$H_1 = H_2 = \frac{423}{4{,}096}\,\frac{WL}{f} \qquad V_1 = \frac{13}{16}W \qquad V_2 = \frac{3}{16}W$$

When $x \leqq \frac{3}{8}L$

$$M_x = Wx\left(\frac{13}{16} - \frac{4x}{3L}\right) - H_1 y$$

When $x > \frac{3}{8}L$

$$M_x = \frac{3W}{16}(L - x) - H_1 y$$

When $x \leqq \frac{3}{8}L$

$$\left.\begin{aligned} N_x &= W\left(\frac{13}{16} - \frac{8x}{3L}\right)\sin\varphi + H_1\cos\varphi \\ Q_x &= W\left(\frac{13}{16} - \frac{8x}{3L}\right)\cos\varphi - H_1\sin\varphi \end{aligned}\right\} \qquad (9\text{-}1)$$

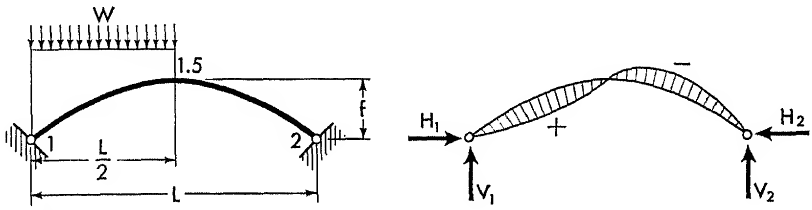
When $x > \frac{3}{8}L$, but $\leqq \frac{L}{2}$

$$\left.\begin{aligned} N_x &= H_1\cos\varphi - \frac{3W}{16}\sin\varphi \\ Q_x &= -H_1\sin\varphi - \frac{3W}{16}\cos\varphi \end{aligned}\right\} \qquad (9\text{-}2)$$

When $x > \frac{L}{2}$

$$\left.\begin{aligned} N_x &= H_1\cos\varphi + \frac{3W}{16}\sin\varphi \\ Q_x &= H_1\sin\varphi - \frac{3W}{16}\cos\varphi \end{aligned}\right\} \qquad (9\text{-}3)$$

9-4. Vertical Uniform Load over Left Half of Span



W = total load

$$H_1 = H_2 = \frac{WL}{8f} \qquad V_1 = \frac{3}{4}W \qquad V_2 = \frac{W}{4}$$

When $x \leqq \frac{L}{2}$

$$M_x = Wx\left(\frac{3}{4} - \frac{x}{L}\right) - H_1 y$$

$$N_x = H_1 \cos\varphi + W\left(\frac{3}{4} - \frac{2x}{L}\right)\sin\varphi \tag{9-4}$$

$$Q_x = -H_1 \sin\varphi + W\left(\frac{3}{4} - \frac{2x}{L}\right)\cos\varphi$$

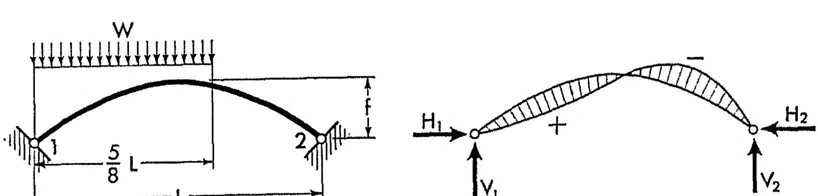
When $x > \frac{L}{2}$

$$M_x = \frac{W}{4}(L - x) - H_1 y$$

$$N_x = H_1 \cos\varphi + \frac{W}{4}\sin\varphi \tag{9-5}$$

$$Q_x = H_1 \sin\varphi - \frac{W}{4}\cos\varphi$$

9-5. Vertical Uniform Load over Five-eighths of Span



W = total load

$$H_1 = H_2 = \frac{283}{2{,}048}\frac{WL}{f} \qquad V_1 = \frac{11}{16}W \qquad V_2 = \frac{5}{16}W$$

For Notations and Constants, see Arts. 9-1 and 9-2

When $x \leqq \frac{5}{8}L$

$$M_x = Wx\left(\frac{11}{16} - \frac{4x}{5L}\right) - H_1 y$$

When $x > \frac{5}{8}L$

$$M_x = \frac{5W}{16}(L - x) - H_1 y$$

When $x \leqq \frac{L}{2}$

$$N_x = W\left(\frac{11}{16} - \frac{8x}{5L}\right)\sin\varphi + H_1\cos\varphi$$

$$Q_x = W\left(\frac{11}{16} - \frac{8x}{5L}\right)\cos\varphi - H_1\sin\varphi \tag{9-6}$$

When $x > \frac{L}{2}$, but $\leqq \frac{5}{8}L$

$$N_x = W\left(\frac{8x}{5L} - \frac{11}{16}\right)\sin\varphi + H_1\cos\varphi$$

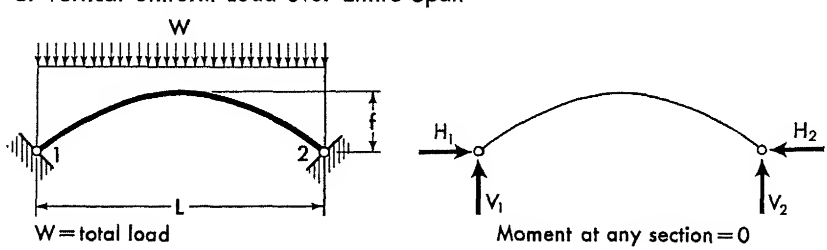
$$Q_x = H_1\sin\varphi - W\left(\frac{8x}{5L} - \frac{11}{16}\right)\cos\varphi \tag{9-7}$$

When $x > \frac{5}{8}L$

$$N_x = H_1\cos\varphi + \frac{5W}{16}\sin\varphi$$

$$Q_x = H_1\sin\varphi - \frac{5W}{16}\cos\varphi \tag{9-8}$$

9-6. Vertical Uniform Load over Entire Span



$$H_1 = H_2 = \frac{WL}{8f} \qquad V_1 = V_2 = \frac{W}{2}$$

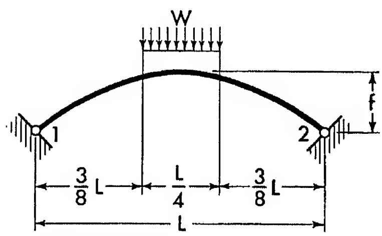
M and Q are zero at any section of the arch.

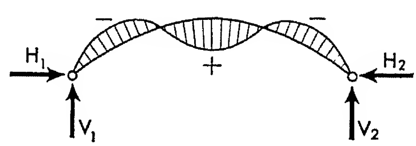
Members of Constant Section

$$\text{When } x \leqq \frac{L}{2} \qquad N_x = H_1 \cos\varphi + W\left(\frac{1}{2} - \frac{x}{L}\right)\sin\varphi$$

$$\text{When } x > \frac{L}{2} \qquad N_x = H_1 \cos\varphi + W\left(\frac{x}{L} - \frac{1}{2}\right)\sin\varphi \qquad (9\text{-}9)$$

9-7. Vertical Uniform Load over Center Quarter of Span





W = total load

$$H_1 = H_2 = \frac{195}{1{,}024}\frac{WL}{f} \qquad V_1 = V_2 = \frac{W}{2}$$

When $x \leqq \frac{3}{8}L$

$$M_x = \frac{Wx}{2} - H_1 y$$

$$N_x = \frac{W}{2}\sin\varphi + H_1\cos\varphi \qquad (9\text{-}10)$$

$$Q_x = \frac{W}{2}\cos\varphi - H_1\sin\varphi$$

When $x > \frac{3}{8}L$, but $\leqq \frac{L}{2}$

$$M_x = \frac{Wx}{2} - \frac{2W}{L}\left(x - \frac{3}{8}L\right)^2 - H_1 y$$

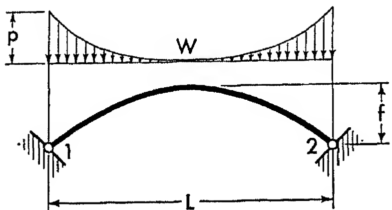
$$N_x = H_1\cos\varphi + 2W\left(1 - \frac{2x}{L}\right)\sin\varphi \qquad (9\text{-}11)$$

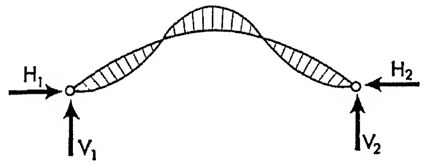
$$Q_x = 2W\left(1 - \frac{2x}{L}\right)\cos\varphi - H_1\sin\varphi$$

Moments and axial forces at corresponding sections in the right half of the arch are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

For Notations and Constants, see Arts. 9-1 and 9-2

9-8. Vertical Complementary Parabolic Load over Entire Arch





Total load $W = \frac{pL}{3}$

$$H_1 = H_2 = \frac{WL}{14f} \qquad V_1 = V_2 = \frac{W}{2}$$

When $x \leqq \frac{L}{2}$

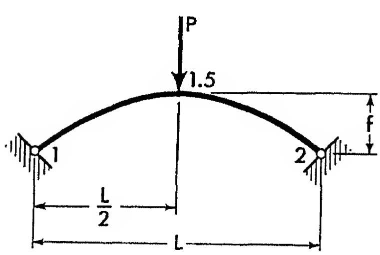
$$M_x = \frac{WL}{16}\left[1 - \left(\frac{L-2x}{L}\right)^4\right] - H_1 y$$

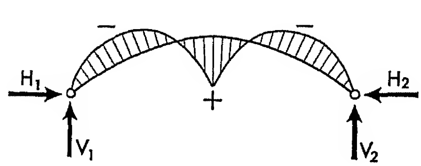
$$N_x = \frac{W}{2}\left(\frac{L-2x}{L}\right)^3 \sin\varphi + H_1 \cos\varphi \tag{9-12}$$

$$Q_x = \frac{W}{2}\left(\frac{L-2x}{L}\right)^3 \cos\varphi - H_1 \sin\varphi$$

Moments and axial forces at corresponding sections in the right half of the arch are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

9-9. Vertical Concentrated Load at Crown





$$H_1 = H_2 = \frac{25}{128}\frac{PL}{f} \qquad V_1 = V_2 = \frac{P}{2}$$

$$N_x = H_1 \cos\varphi + \frac{P}{2} \sin\varphi \tag{9-13}$$

When $x \leqq \frac{L}{2}$

$$M_x = \frac{P}{2}x - H_1 y$$

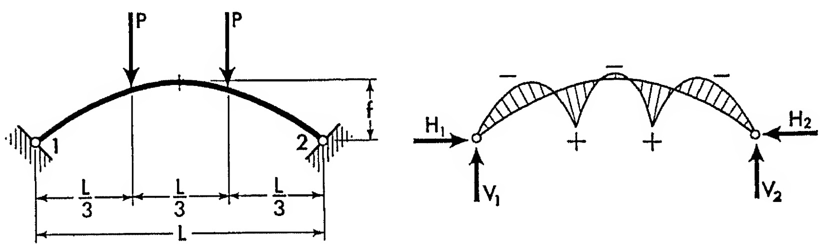
$$Q_x = -H_1 \sin\varphi + \frac{P}{2}\cos\varphi \tag{9-14}$$

When $x > \frac{L}{2}$

$$M_x = \frac{P}{2}(L - x) - H_2 y$$

$$Q_x = H_1 \sin\varphi - \frac{P}{2}\cos\varphi \tag{9-15}$$

9-10. Two Vertical Concentrated Loads on Arch



$$H_1 = H_2 = \frac{55}{162}\frac{PL}{f} \qquad V_1 = V_2 = P$$

When $x \leqq \frac{L}{3}$

$$M_x = Px - H_1 y$$

$$N_x = H_1 \cos\varphi + P\sin\varphi$$

$$Q_x = -H_1 \sin\varphi + P\cos\varphi$$

When $x > \frac{L}{3}$, but $\leqq \frac{L}{2}$

$$M_x = \frac{PL}{3} - H_1 y$$

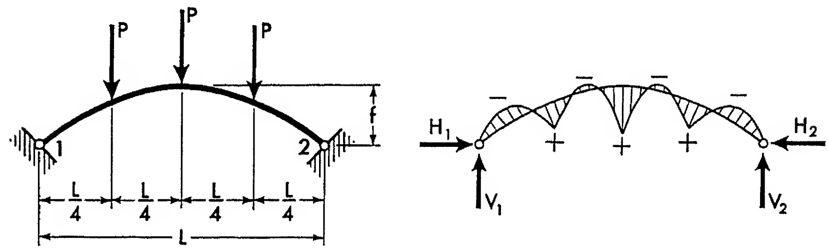
$$N_x = H_1 \cos\varphi$$

$$Q_x = -H_1 \sin\varphi$$

For Notations and Constants, see Arts. 9-1 and 9-2

Moments and axial forces at corresponding sections in the right half of the arch are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

9-11. Three Vertical Concentrated Loads on Arch



$$H_1 = H_2 = \frac{485}{1{,}024}\,\frac{PL}{f} \qquad V_1 = V_2 = \frac{3}{2}P$$

When $x \leqq \frac{L}{4}$

$$M_x = \frac{3Px}{2} - H_1 y$$

$$N_x = H_1 \cos\varphi + \frac{3}{2} P \sin\varphi$$

$$Q_x = -H_1 \sin\varphi + \frac{3}{2} P \cos\varphi$$

When $x > \frac{L}{4}$, but $\leqq \frac{L}{2}$

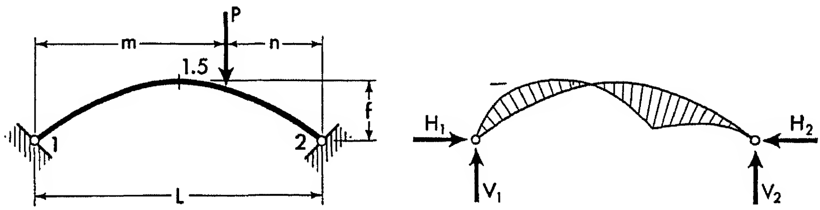
$$M_x = \frac{P(L + 2x)}{4} - H_1 y$$

$$N_x = H_1 \cos\varphi + \frac{P}{2} \sin\varphi$$

$$Q_x = -H_1 \sin\varphi + \frac{P}{2} \cos\varphi$$

Moments and axial forces at corresponding sections in the right half of the arch are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

9-12. Vertical Concentrated Load on Arch



$$H_1 = H_2 = \frac{5PL}{8f}\frac{m}{L}\left[1 - 2\left(\frac{m}{L}\right)^2 + \left(\frac{m}{L}\right)^3\right]$$

$$V_2 = \frac{Pm}{L} \qquad V_1 = P - V_2$$

When $x \leqq m$ $\qquad M_x = \frac{Pnx}{L} - H_1 y$

When $x > m$ $\qquad M_x = Pm\left(1 - \frac{x}{L}\right) - H_1 y$

When $x \leqq m$ and $\frac{L}{2}$

$$N_x = H_1 \cos\varphi + P\frac{n}{L}\sin\varphi$$
$$Q_x = -H_1 \sin\varphi + P\frac{n}{L}\cos\varphi \tag{9-16}$$

When $x \leqq m$, but $\geqq \frac{L}{2}$

$$N_x = H_1 \cos\varphi - P\frac{n}{L}\sin\varphi$$
$$Q_x = H_1 \sin\varphi + P\frac{n}{L}\cos\varphi \tag{9-17}$$

When $x \geqq m$, but $\leqq \frac{L}{2}$

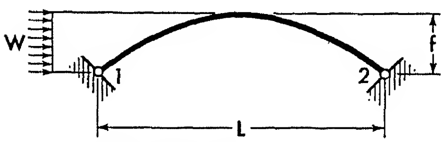
$$N_x = H_1 \cos\varphi - P\frac{m}{L}\sin\varphi$$
$$Q_x = -H_1 \sin\varphi - P\frac{m}{L}\cos\varphi \tag{9-18}$$

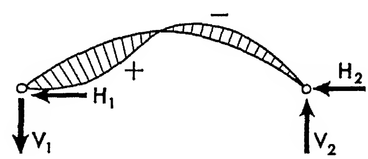
For Notations and Constants, see Arts. 9-1 and 9-2

When $x \geqq m$ and $\frac{L}{2}$

$$N_x = H_1 \cos\varphi + P\frac{m}{L}\sin\varphi$$
$$Q_x = H_1 \sin\varphi - P\frac{m}{L}\cos\varphi \tag{9-19}$$

9-13. Horizontal Uniform Load on Left Half of Arch





$$H_1 = -\frac{5W}{7} \qquad H_2 = \frac{2W}{7}$$

$$V_2 = \frac{Wf}{2L} \qquad V_1 = -V_2$$

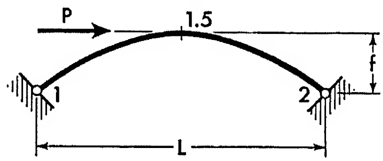
When $x \leqq \frac{L}{2}$

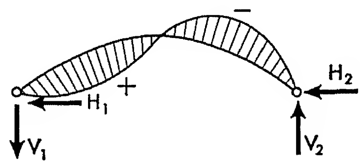
$$M_x = -\frac{Wf}{2}\left(\frac{x}{L} + \frac{y^2}{f^2}\right) - H_1 y$$
$$N_x = \left(\frac{Wy}{f} + H_1\right)\cos\varphi - \frac{Wf}{2L}\sin\varphi \tag{9-20}$$
$$Q_x = -\left(\frac{Wy}{f} + H_1\right)\sin\varphi - \frac{Wf}{2L}\cos\varphi$$

When $x > \frac{L}{2}$

$$M_x = \frac{Wf}{2}\left(1 - \frac{x}{L}\right) - H_2 y$$
$$N_x = (W + H_1)\cos\varphi + \frac{Wf}{2L}\sin\varphi \tag{9-21}$$
$$Q_x = -\frac{Wf}{2L}\cos\varphi + (W + H_1)\sin\varphi$$

9-14. Horizontal Concentrated Load at Crown





$$H_1 = -\frac{P}{2} \qquad H_2 = \frac{P}{2}$$

$$V_1 = -\frac{Pf}{L} \qquad V_2 = \frac{Pf}{L}$$

$$Q_x = \frac{P}{2}\sin\varphi - \frac{Pf}{L}\cos\varphi \tag{9-22}$$

When $x \leqq \frac{L}{2}$

$$M_x = Pf\left(\frac{y}{2f} - \frac{x}{L}\right)$$

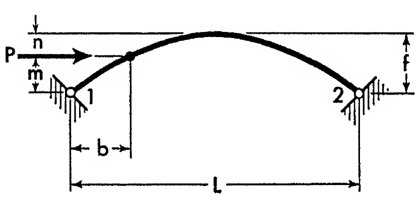
$$N_x = -\frac{P}{2}\cos\varphi - \frac{Pf}{L}\sin\varphi \tag{9-23}$$

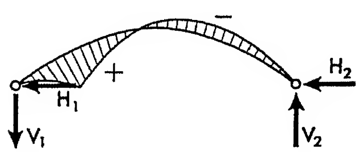
When $x > \frac{L}{2}$

$$M_x = \frac{Pf}{L}(L - x) - \frac{Py}{2}$$

$$N_x = \frac{P}{2}\cos\varphi + \frac{Pf}{L}\sin\varphi \tag{9-24}$$

9-15. Horizontal Concentrated Load at Any Point of Left Half of Arch





Obtain value of K from Table 9-1.

$$H_2 = PK \qquad H_1 = -(P - H_2)$$

For Notations and Constants, see Arts. 9-1 and 9-2

$$V_2 = \frac{Pm}{L} \qquad V_1 = -\frac{Pm}{L}$$

When $x \leqq b$ $\qquad M_x = -\frac{Pmx}{L} - H_1 y$

When $x > b$ $\qquad M_x = Pm\left(1 - \frac{x}{L}\right) - H_2 y$

When $x \leqq b$

$$\left.\begin{aligned} N_x &= H_1 \cos\varphi - \frac{Pm}{L}\sin\varphi \\ Q_x &= -H_1 \sin\varphi - \frac{Pm}{L}\cos\varphi \end{aligned}\right\} \tag{9-25}$$

When $x \geqq b$, but $< \frac{L}{2}$

$$\left.\begin{aligned} N_x &= (P + H_1) \cos\varphi - \frac{Pm}{L}\sin\varphi \\ Q_x &= -(P + H_1) \sin\varphi - \frac{Pm}{L}\cos\varphi \end{aligned}\right\} \tag{9-26}$$

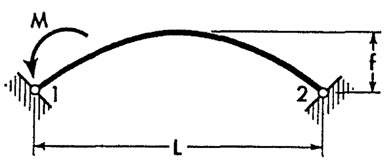
When $x \geqq \frac{L}{2}$

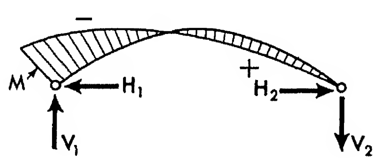
$$\left.\begin{aligned} N_x &= (P + H_1) \cos\varphi + \frac{Pm}{L}\sin\varphi \\ Q_x &= (P + H_1) \sin\varphi - \frac{Pm}{L}\cos\varphi \end{aligned}\right\} \tag{9-27}$$

Table 9-1. Values of K for Various m

m	0	0.2f	0.4f	0.6f	0.8f	1.0f
K	0	0.1243	0.2444	0.3545	0.4463	0.5000

9-16. Moment Applied at Left End of Arch





$$H_1 = H_2 = -\frac{5M}{8f} \qquad V_1 = \frac{M}{L} \qquad V_2 = -V_1$$

Members of Constant Section

$$M_x = -M\left(1 - \frac{x}{L} - \frac{5y}{8f}\right)$$

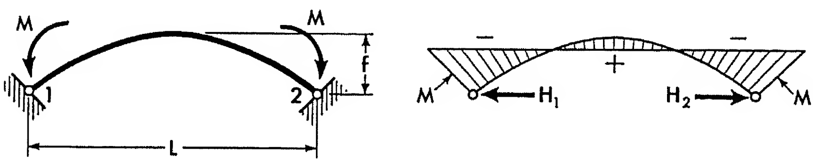
When $x \leqq \frac{L}{2}$

$$N_x = H_1 \cos\varphi + \frac{M}{L}\sin\varphi$$
$$Q_x = \frac{M}{L}\cos\varphi - H_1 \sin\varphi \qquad (9\text{-}28)$$

When $x > \frac{L}{2}$

$$N_x = H_1 \cos\varphi - \frac{M}{L}\sin\varphi$$
$$Q_x = \frac{M}{L}\cos\varphi + H_1 \sin\varphi \qquad (9\text{-}29)$$

9-17. Equal Moments Applied at Both Ends of Arch



$$H_1 = H_2 = -\frac{5M}{4f} \qquad V_1 = V_2 = 0$$

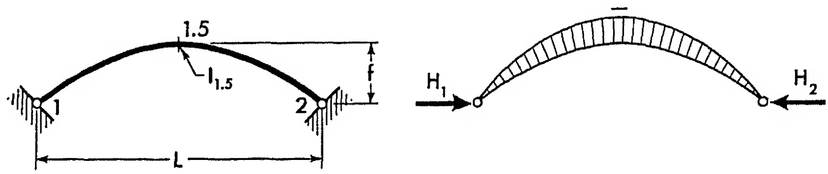
$$M_x = -M\left(1 - \frac{5y}{4f}\right)$$

$$N_x = H_1 \cos\varphi \qquad (9\text{-}30)$$

When $x \leqq \frac{L}{2}$ $\quad Q_x = -H_1 \sin\varphi$

When $x > \frac{L}{2}$ $\quad Q_x = H_1 \sin\varphi \qquad (9\text{-}31)$

9-18. **Effect of Temperature Rise.** Range $t°$ for entire arch.



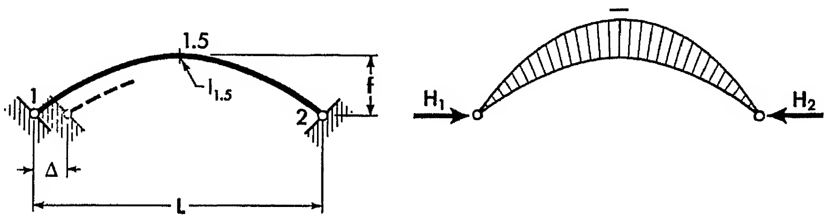
$$H_1 = H_2 = \frac{15\varepsilon t°}{8f^2} EI_{1.5} \qquad V_1 = V_2 = 0$$

$$M_y = -H_1 y \qquad M_{1.5} = -H_1 f$$

Apply Eqs. (9-30) and (9-31) to obtain the axial and shearing forces at any section of the arch.

Note: For temperature drop, introduce the value of $t°$ with a negative sign.

9-19. Horizontal Displacement of One Support

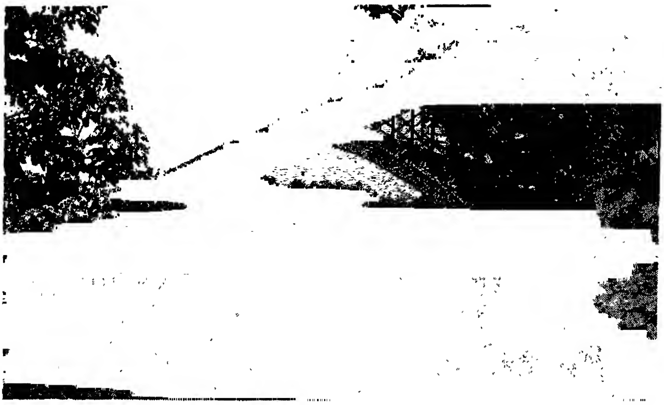


$$H_1 = H_2 = \frac{15\Delta}{8Lf^2} EI_{1.5} \qquad V_1 = V_2 = 0$$

$$M_y = -H_1 y \qquad M_{1.5} = -H_1 f$$

Apply Eqs. (9-30) and (9-31) to obtain the axial and shearing forces at any section of the arch.

Note: If the direction of arch displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.



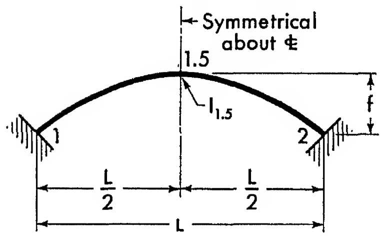
The Henry Hudson Bridge at Spuyten Duyvil in New York City. This magnificent bridge connecting the banks of the Harlem River has an arch span of 800 feet and is one of the largest structures of its type in the United States. A uniform arch of parabolic curvature with a high rise to span ratio lends a highly aesthetic appearance to the bridge. Robinson & Steinman of New York are consulting engineers on design. (Courtesy of the American Institute of Steel Construction.)

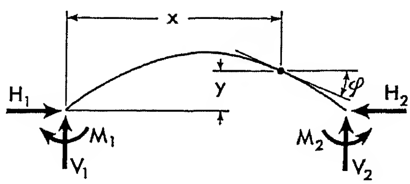
The Orford-Fairlee highway bridge over the Connecticut River on the boundary line of the states of New Hampshire and Vermont. This is an example of an overhead plate girder arch bridge having a span of 425 feet and a rise of 85 feet. In contrast to the preceding illustration, the highway deck of this bridge is suspended from the arches by means of stiff hangers. The parabolic arch curvature of the plate girder with smooth haunching toward the abutments contributes to the graceful appearance of the bridge. (Courtesy of U.S. Steel Corporation, steel fabricator.)

SECTION 10

SYMMETRICAL PARABOLIC HINGELESS ARCHES

10-1. Notations, Coordinates, and Arch Constant





The sketch appearing on the left, above, explains notations for a representative arch with minor depth variations, as defined by Eq. (8-2).[1] The arch axis is a symmetrical parabola conforming to Eq. (8-1).

The sketch appearing on the right, above, shows positive directions of the moments and the vertical and horizontal components of the arch reactions. It also defines the angle of inclination and coordinates at any section of an arch. Angles of inclination and coordinates are to be considered only in the positive sense.

Arch Constant, G, for the Analysis by Method B

$$G = \frac{d^2_{1.5}\tau}{f^2}$$

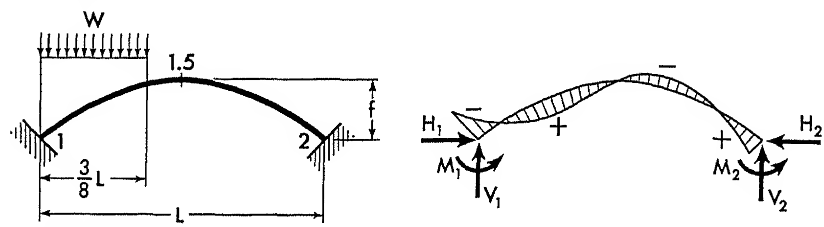
where

$d_{1.5}$ = arch thickness at crown (in proper dimensional units)
τ = numerical constant from Table 14 in the Appendix

[1] The application of the condensed solutions of analysis given in this section may be extended to arches of constant section, by the reasons given in Art. 8-3.

10-2. Equations of Moments and Forces. The equations for the redundant moments and the vertical and horizontal components of the arch reactions are given on the following pages for a number of loading conditions. Corresponding expressions for the moment and the axial and shearing forces at any section of the arch are also provided. In addition, precise solutions of arch analysis, defined in Art. 8-4 as method B, are also included for many loading conditions.

10-3. Vertical Uniform Load over Three-eighths of Span



W = Total load

METHOD A

$$M_1 = -\frac{375}{8{,}192}WL \qquad M_2 = \frac{225}{8{,}192}WL \qquad H_1 = H_2 = \frac{751}{8{,}192}\frac{WL}{f}$$

$$V_1 = \frac{907W}{1{,}024} \qquad V_2 = \frac{117W}{1{,}024} \qquad M_{1.5} = -\frac{58WL}{8{,}192}$$

When $x \leqq \frac{3}{8}L$

$$M_x = M_1 + \frac{907Wx}{1{,}024} - \frac{4Wx^2}{3L} - H_1y \tag{10-1}$$

When $x > \frac{3}{8}L$

$$M_x = M_2 + \frac{117W}{1{,}024}(L - x) - H_1y \tag{10-2}$$

When $x \leqq \frac{3}{8}L$

$$N_x = H_1 \cos\varphi - \left(\frac{8Wx}{3L} - V_1\right)\sin\varphi$$

$$Q_x = -H_1 \sin\varphi - \frac{8Wx}{3L}\left(-V_1\right)\cos\varphi \tag{10-3}$$

When $x > \frac{3}{8}L$, but $\leqq \frac{L}{2}$

$$\begin{aligned} N_x &= H_1 \cos\varphi - (W - V_1)\sin\varphi \\ Q_x &= -H_1 \sin\varphi - (W - V_1)\cos\varphi \end{aligned} \tag{10-4}$$

When $x > \frac{L}{2}$

$$\begin{aligned} N_x &= H_1 \cos\varphi + (W - V_1)\sin\varphi \\ Q_x &= H_1 \sin\varphi - (W - V_1)\cos\varphi \end{aligned} \tag{10-5}$$

METHOD B

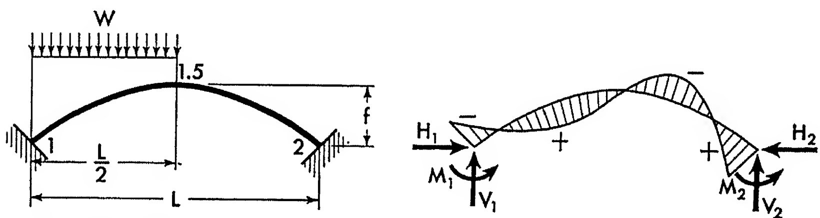
$$H_1 = H_2 = \frac{751}{8{,}192}\,\frac{WL}{f(1+G)}$$

$$V_1 = \frac{907W}{1{,}024} \qquad V_2 = \frac{117W}{1{,}024}$$

$$\left.\begin{matrix} M_1 \\ M_2 \end{matrix}\right\rangle = \frac{1}{12}\left(8H_1f - \frac{27WL}{32}\right) \mp \frac{75WL}{2{,}048}$$

Moment and axial and shearing forces, at any section of the arch, may be determined by the use of Eqs. (10-1) through (10-5).

10-4. Vertical Uniform Load over Left Half of Span



W = Total load

METHOD A

$$M_1 = -\frac{WL}{32} \qquad M_2 = \frac{WL}{32} \qquad H_1 = H_2 = \frac{WL}{8f}$$

$$V_1 = \frac{13}{16}W \qquad V_2 = \frac{3W}{16} \qquad M_{1.5} = 0$$

When $x \leqq \frac{L}{2}$

$$M_x = M_1 + Wx\left(\frac{13}{16} - \frac{x}{L}\right) - H_1 y \tag{10-6}$$

$$\left.\begin{aligned} N_x &= H_1 \cos\varphi + \left(V_1 - \frac{2Wx}{L}\right)\sin\varphi \\ Q_x &= -H_1 \sin\varphi + \left(V_1 - \frac{2Wx}{L}\right)\cos\varphi \end{aligned}\right\} \tag{10-7}$$

When $x > \frac{L}{2}$

$$M_x = M_2 + \frac{3W}{16}(L - x) - H_1 y \tag{10-8}$$

$$\left.\begin{aligned} N_x &= H_1 \cos\varphi + (W - V_1)\sin\varphi \\ Q_x &= H_1 \sin\varphi - (W - V_1)\cos\varphi \end{aligned}\right\} \tag{10-9}$$

METHOD B

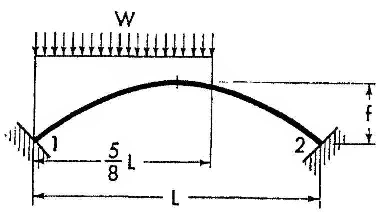
$$H_1 = H_2 = \frac{WL}{8f(1+G)}$$

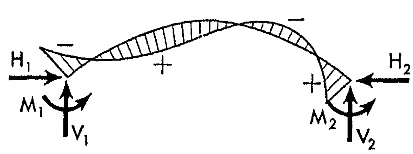
$$V_1 = \frac{13}{16}W \qquad V_2 = \frac{3W}{16}$$

$$\left.\begin{matrix} M_1 \\ M_2 \end{matrix}\right\rangle = -\frac{WL}{12}\left(\frac{G}{1+G}\right) \mp \frac{WL}{32}$$

Moment and axial and shearing forces, at any section of the arch, may be determined by the use of Eqs. (10-6) through (10-9).

10-5. Vertical Uniform Load over Five-eighths of Span





W = Total load

METHOD A

$$M_1 = -\frac{135WL}{8{,}192} \qquad M_2 = \frac{225WL}{8{,}192}$$

Members of Constant Section

$$H_1 = H_2 = \frac{297}{2{,}048}\frac{WL}{f} \qquad V_1 = \frac{749W}{1{,}024}$$

$$V_2 = W - V_1 \qquad M_{1.5} = \frac{35WL}{8{,}192}$$

When $x \leqq \frac{5}{8}L$

$$M_x = M_1 + \frac{749}{1{,}024}Wx - \frac{4Wx^2}{5L} - H_1y \tag{10-10}$$

When $x > \frac{5}{8}L$

$$M_x = M_2 + \frac{275W}{1{,}024}(L - x) - H_1y \tag{10-11}$$

When $x \leqq \frac{L}{2}$

$$\left.\begin{aligned} N_x &= H_1 \cos\varphi - \left(\frac{8Wx}{5L} - V_1\right)\sin\varphi \\ Q_x &= -H_1 \sin\varphi - \left(\frac{8Wx}{5L} - V_1\right)\cos\varphi \end{aligned}\right. \tag{10-12}$$

When $x > \frac{L}{2}$, but $\leqq \frac{5}{8}L$

$$\left.\begin{aligned} N_x &= H_1 \cos\varphi + \left(\frac{8Wx}{5L} - V_1\right)\sin\varphi \\ Q_x &= H_1 \sin\varphi - \left(\frac{8Wx}{5L} - V_1\right)\cos\varphi \end{aligned}\right. \tag{10-13}$$

When $x > \frac{5}{8}L$

$$\left.\begin{aligned} N_x &= H_1 \cos\varphi + (W - V_1)\sin\varphi \\ Q_x &= H_1 \sin\varphi - (W - V_1)\cos\varphi \end{aligned}\right. \tag{10-14}$$

METHOD B

$$H_1 = H_2 = \frac{297}{2{,}048}\frac{WL}{f(1 + G)}$$

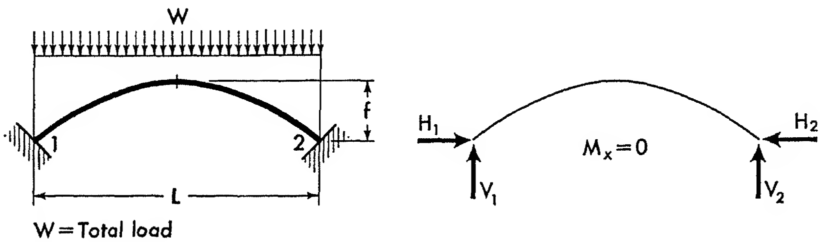
$$V_1 = \frac{749W}{1{,}024} \qquad V_2 = W - V_1$$

For Notations and Constants, see Arts. 10-1 and 10-2

$$\left.\begin{matrix} M_1 \\ M_2 \end{matrix}\right\rangle = \frac{1}{12}\left(8H_1 f - \frac{35WL}{32}\right) \mp \frac{45WL}{2{,}048}$$

Moment and axial and shearing forces, at any section of the arch, may be determined by the use of Eqs. (10-10) through (10-14).

10-6. Vertical Uniform Load over Entire Span



METHOD A

M and Q are zero at any section of the arch.

$$H_1 = H_2 = \frac{WL}{8f} \qquad V_1 = V_2 = \frac{W}{2}$$

$$\text{When } x \leqq \frac{L}{2} \qquad N_x = H_1 \cos\varphi + \frac{W}{2}\left(1 - \frac{2x}{L}\right)\sin\varphi$$

$$\text{When } x > \frac{L}{2} \qquad N_x = H_1 \cos\varphi - \frac{W}{2}\left(1 - \frac{2x}{L}\right)\sin\varphi \qquad (10\text{-}15)$$

METHOD B

$$H_1 = H_2 = \frac{WL}{8f(1+G)} \qquad V_1 = V_2 = \frac{W}{2}$$

$$M_1 = M_2 = -\frac{WL}{12}\left(\frac{G}{1+G}\right) \qquad M_{1.5} = \frac{WL}{24}\left(\frac{G}{1+G}\right)$$

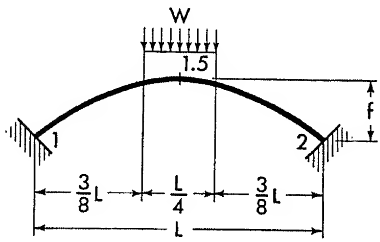
$$M_x = M_1 + \frac{W}{2}\left(1 - \frac{x}{L}\right)x - H_1 y$$

$$\text{When } x \leqq \frac{L}{2} \qquad Q_x = -H_1 \sin\varphi + \frac{W}{2}\left(1 - \frac{2x}{L}\right)\cos\varphi$$

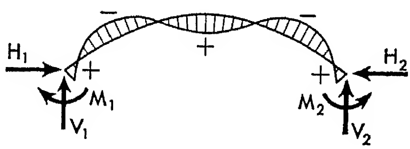
$$\text{When } x > \frac{L}{2} \qquad Q_x = H_1 \sin\varphi + \frac{W}{2}\left(1 - \frac{2x}{L}\right)\cos\varphi \qquad (10\text{-}16)$$

Axial force, at any section of the arch, may be determined by the use of Eq. (10-15).

10-7. Vertical Uniform Load over Center Quarter of Span



W = Total load



METHOD A

$$M_1 = M_2 = \frac{7WL}{256} \qquad H_1 = H_2 = \frac{115}{512}\frac{WL}{f}$$

$$M_{1.5} = \frac{11WL}{512} \qquad V_1 = V_2 = \frac{W}{2}$$

When $x \leqq \frac{3}{8}L$

$$\left.\begin{aligned} M_x &= M_1 + \frac{Wx}{2} - H_1 y \\ N_x &= \frac{W}{2}\sin\varphi + H_1\cos\varphi \\ Q_x &= \frac{W}{2}\cos\varphi - H_1\sin\varphi \end{aligned}\right\} \qquad (10\text{-}17)$$

When $x > \frac{3}{8}L$, but $\leqq \frac{L}{2}$

$$\left.\begin{aligned} M_x &= M_1 + \frac{Wx}{2} - \frac{2W}{L}\left(x - \frac{3L}{8}\right)^2 - H_1 y \\ N_x &= H_1\cos\varphi + 2W\left(1 - \frac{2x}{L}\right)\sin\varphi \\ Q_x &= 2W\left(1 - \frac{2x}{L}\right)\cos\varphi - H_1\sin\varphi \end{aligned}\right\} \qquad (10\text{-}18)$$

Moments and axial forces at corresponding sections in the right half of the arch are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

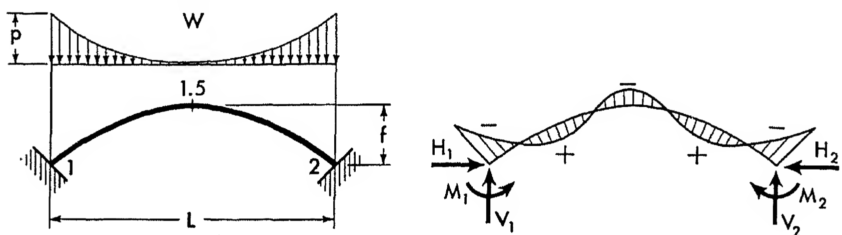
METHOD B

$$H_1 = H_2 = \frac{115}{512}\frac{WL}{f(1+G)} \qquad V_1 = V_2 = \frac{W}{2}$$

$$M_1 = M_2 = \frac{1}{12}\left(8H_1 f - \frac{47WL}{32}\right)$$

Moment and axial and shearing forces at any section of the arch may be determined by the use of Eqs. (10-17) and (10-18) and the application of the symmetry relation described above.

10-8. Vertical Complementary Parabolic Load over Entire Arch



Total load $W = \frac{pL}{3}$

METHOD A

$$M_1 = M_2 = -\frac{WL}{70} \qquad H_1 = H_2 = \frac{3WL}{56f}$$

$$V_1 = V_2 = \frac{W}{2} \qquad M_{1.5} = -\frac{3WL}{560}$$

When $x \leqq \frac{L}{2}$

$$M_x = M_1 + \frac{WL}{16}\left[1 - \left(\frac{L-2x}{L}\right)^4\right] - H_1 y \tag{10-19}$$

$$N_x = \frac{W}{2}\left(\frac{L-2x}{L}\right)^3 \sin\varphi + H_1\cos\varphi$$

$$Q_x = \frac{W}{2}\left(\frac{L-2x}{L}\right)^3 \cos\varphi - H_1\sin\varphi \tag{10-20}$$

Moments and axial forces at corresponding sections in the right half of the arch are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

Members of Constant Section

METHOD B

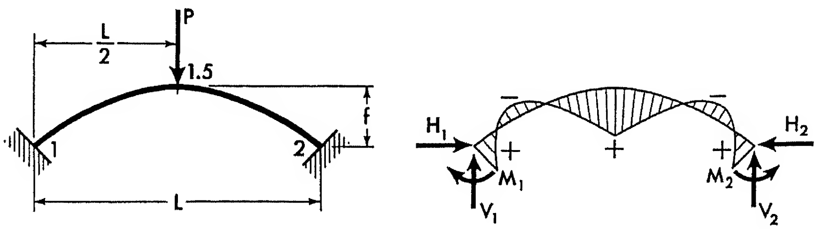
$$H_1 = H_2 = \frac{3}{56}\frac{WL}{f(1+G)}$$

$$M_1 = M_2 = -\frac{WL}{140}\left(\frac{2+7G}{1+G}\right)$$

$$M_{1.5} = -\frac{WL}{560}\left(\frac{3-7G}{1+G}\right) \qquad V_1 = V_2 = \frac{W}{2}$$

Moment and axial and shearing forces, at any section of the arch, may be determined by the use of Eqs. (10-19) and (10-20) and the application of the symmetry relation described above.

10-9. Vertical Concentrated Load at Crown



METHOD A

$$M_1 = M_2 = \frac{PL}{32} \qquad H_1 = H_2 = \frac{15}{64}\frac{PL}{f}$$

$$V_1 = V_2 = \frac{P}{2} \qquad M_{1.5} = \frac{3PL}{64}$$

When $x \leqq \frac{L}{2}$

$$M_x = \frac{P}{32}\left(L + 16x - \frac{15Ly}{2f}\right)$$

$$N_x = H_1 \cos\varphi + \frac{P}{2}\sin\varphi$$

$$Q_x = -H_1 \sin\varphi + \frac{P}{2}\cos\varphi \qquad (10\text{-}21)$$

Moments and axial forces at corresponding sections in the right half of the arch are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

For Notations and Constants, see Arts. 10-1 and 10-2

METHOD B

$$H_1 = H_2 = \frac{15}{64}\frac{PL}{f(1+G)}$$

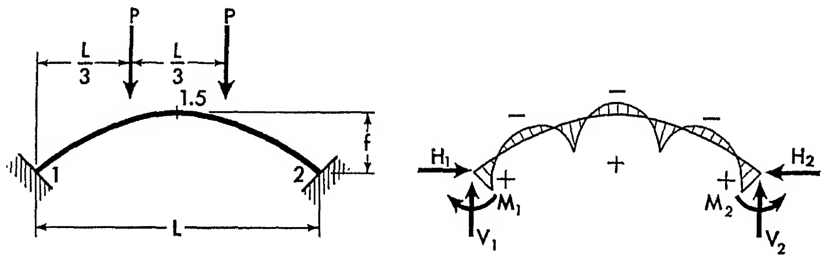
$$M_1 = M_2 = \frac{PL}{32}\left(\frac{1-4G}{1+G}\right) \qquad V_1 = V_2 = \frac{P}{2}$$

When $x \leqq \frac{L}{2}$

$$M_x = M_1 + \frac{Px}{2} - H_1 y$$

Axial and shearing forces, at any section of the arch, may be determined by the use of Eq. (10-21) and the application of the symmetry relation described above.

10-10. Two Vertical Concentrated Loads on Arch



METHOD A

$$M_1 = M_2 = \frac{2PL}{81} \qquad H_1 = H_2 = \frac{10}{27}\frac{PL}{f}$$

$$V_1 = V_2 = P$$

When $x \leqq \frac{L}{3}$

$$\left.\begin{aligned} M_x &= M_1 + Px - H_1 y \\ N_x &= H_1 \cos\varphi + P \sin\varphi \\ Q_x &= -H_1 \sin\varphi + P \cos\varphi \end{aligned}\right\} \qquad (10\text{-}22)$$

When $x > \frac{L}{3}$, but $\leqq \frac{L}{2}$

$$\left.\begin{aligned} M_x &= M_1 + \frac{PL}{3} - H_1 y \\ N_x &= H_1 \cos\varphi \\ Q_x &= -H_1 \sin\varphi \end{aligned}\right\} \qquad (10\text{-}23)$$

Moments and axial forces at corresponding sections in the right half of the arch are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

METHOD B

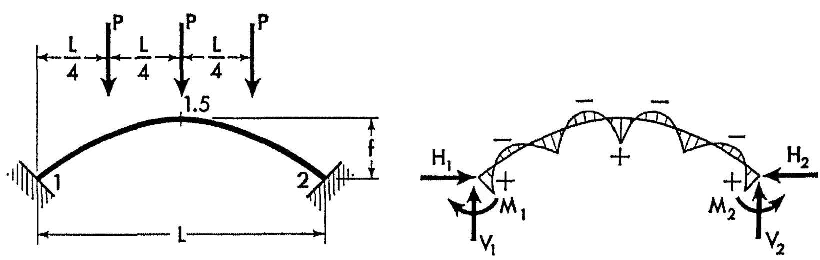
$$M_1 = M_2 = \frac{2PL}{81}\left(\frac{1-9G}{1+G}\right)$$

$$H_1 = H_2 = \frac{10}{27}\frac{PL}{f(1+G)}$$

$$V_1 = V_2 = P$$

Moment and axial and shearing forces, at any section of the arch, may be determined by the use of Eqs. (10-22) and (10-23) and the application of the symmetry relation described above.

10-11. Three Vertical Concentrated Loads on Arch



METHOD A

$$M_1 = M_2 = \frac{5PL}{256} \qquad H_1 = H_2 = \frac{255}{512}\frac{PL}{f}$$

$$V_1 = V_2 = \frac{3P}{2}$$

When $x \leqq \frac{L}{4}$

$$M_x = M_1 + \frac{3Px}{2} - H_1 y \tag{10-24}$$

$$N_x = H_1 \cos \varphi + \frac{3P}{2} \sin \varphi$$

$$Q_x = -H_1 \sin \varphi + \frac{3P}{2} \cos \varphi \tag{10-25}$$

When $x > \frac{L}{4}$, but $\leqq \frac{L}{2}$

$$M_x = M_1 + \frac{P(L + 2x)}{4} - H_1 y \tag{10-26}$$

$$N_x = H_1 \cos \varphi + \frac{P}{2} \sin \varphi$$

$$Q_x = -H_1 \sin \varphi + \frac{P}{2} \cos \varphi \tag{10-27}$$

Moments and axial forces at corresponding sections in the right half of the arch are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

METHOD B

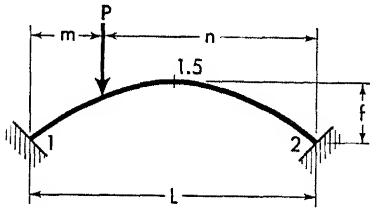
$$M_1 = M_2 = \frac{5PL}{256}\left(\frac{1 - 16G}{1 + G}\right)$$

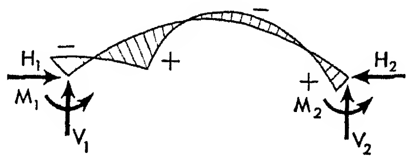
$$H_1 = H_2 = \frac{255}{512}\frac{PL}{f(1 + G)}$$

$$V_1 = V_2 = \frac{3P}{2}$$

Moment and axial and shearing forces, at any section of the arch, may be determined by the use of Eqs. (10-24) through (10-27) and the application of the symmetry relation described above.

10-12. Vertical Concentrated Load on Arch





METHOD A

$$M_1 = Pm\left(\frac{5m}{2L} - 1\right)\left(\frac{n}{L}\right)^2$$

$$M_2 = Pn\left(\frac{5n}{2L} - 1\right)\left(\frac{m}{L}\right)^2$$

$$H_1 = H_2 = \frac{15Pm^2n^2}{4L^3f}$$

$$V_2 = P\left(1 + \frac{2n}{L}\right)\left(\frac{m}{L}\right)^2 \qquad V_1 = P - V_2$$

$$\left.\begin{aligned} &\text{When } x \leqq m && M_x = M_1 + V_1x - H_1y \\ &\text{When } x > m && M_x = M_2 + V_2(L - x) - H_2y \end{aligned}\right\} \qquad (10\text{-}28)$$

When $x \leqq m$ and $\frac{L}{2}$

$$\left.\begin{aligned} N_x &= H_1 \cos\varphi + V_1 \sin\varphi \\ Q_x &= -H_1 \sin\varphi + V_1 \cos\varphi \end{aligned}\right\} \qquad (10\text{-}29)$$

When $x \leqq m$, but $\geqq \frac{L}{2}$

$$\left.\begin{aligned} N_x &= H_1 \cos\varphi - V_1 \sin\varphi \\ Q_x &= H_1 \sin\varphi + V_1 \cos\varphi \end{aligned}\right\} \qquad (10\text{-}30)$$

When $x \geqq m$, but $\leqq \frac{L}{2}$

$$\left.\begin{aligned} N_x &= H_1 \cos\varphi - (P - V_1) \sin\varphi \\ Q_x &= -H_1 \sin\varphi - (P - V_1) \cos\varphi \end{aligned}\right\} \qquad (10\text{-}31)$$

For Notations and Constants, see Arts. 10-1 and 10-2

When $x \geqq m$ and $\frac{L}{2}$

$$N_x = H_1 \cos\varphi + (P - V_1)\sin\varphi$$
$$Q_x = H_1 \sin\varphi - (P - V_1)\cos\varphi \tag{10-32}$$

METHOD B

$$M_1 = Pm\left[\frac{5m}{2L(1+G)} - 1\right]\left(\frac{n}{L}\right)^2$$

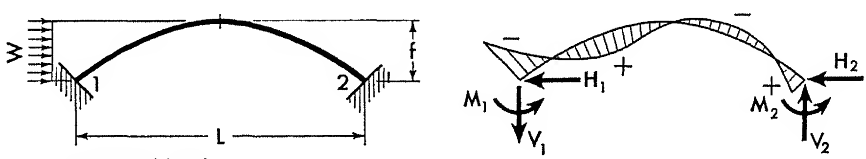
$$M_2 = Pn\left[\frac{5n}{2L(1+G)} - 1\right]\left(\frac{m}{L}\right)^2$$

$$H_1 = H_2 = \frac{15Pm^2n^2}{4L^3f(1+G)}$$

$$V_2 = P\left(1 + \frac{2n}{L}\right)\left(\frac{m}{L}\right)^2 \qquad V_1 = P - V_2$$

Moment and axial and shearing forces, at any section of the arch, may be determined by the use of Eqs. (10-28) through (10-32).

10-13. Horizontal Uniform Load on Left Half of Arch



$$\left.\begin{matrix} M_1 \\ M_2 \end{matrix}\right\} = -\frac{Wf}{8}\left(\frac{117}{256} \pm 1\right)$$

$$H_1 = -\frac{201}{256}W \qquad H_2 = \frac{55W}{256} \qquad V_2 = \frac{Wf}{4L} \qquad V_1 = -V_2$$

When $x \leqq \frac{L}{2}$

$$M_x = M_1 + V_1x - H_1y - \frac{Wy^2}{2f}$$

$$N_x = \left(H_1 + \frac{Wy}{f}\right)\cos\varphi + V_1\sin\varphi$$

$$Q_x = -\left(H_1 + \frac{Wy}{f}\right)\sin\varphi + V_1\cos\varphi \tag{10-33}$$

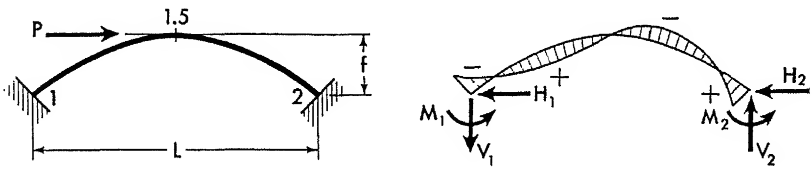
When $x > \frac{L}{2}$

$$M_x = \frac{139Wf}{2{,}048} + V_2(L - x) - H_2 y$$

$$N_x = (W + H_1)\cos\varphi - V_1 \sin\varphi$$

$$Q_x = (W + H_1)\sin\varphi + V_1 \cos\varphi \qquad (10\text{-}34)$$

10-14. Horizontal Concentrated Load at Crown



$$M_1 = -\frac{Pf}{8} \qquad M_2 = \frac{Pf}{8} \qquad H_1 = -\frac{P}{2}$$

$$H_2 = \frac{P}{2} \qquad V_2 = \frac{3Pf}{4L} \qquad V_1 = -V_2$$

$$Q_x = \frac{P}{2}\sin\varphi + V_1\cos\varphi \qquad (10\text{-}35)$$

When $x \leqq \frac{L}{2}$

$$M_x = -\frac{Pf}{8}\left(1 + \frac{6x}{L}\right) + \frac{Py}{2}$$

$$N_x = V_1 \sin\varphi - \frac{P}{2}\cos\varphi \qquad (10\text{-}36)$$

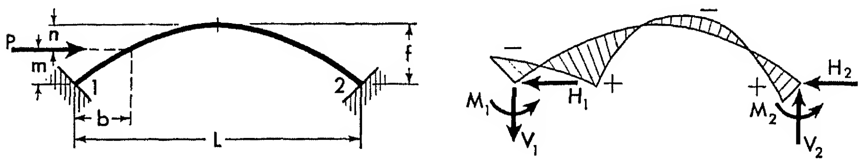
When $x > \frac{L}{2}$

$$M_x = \frac{Pf}{8}\left(7 - \frac{6x}{L}\right) - \frac{Py}{2}$$

$$N_x = -V_1 \sin\varphi + \frac{P}{2}\cos\varphi \qquad (10\text{-}37)$$

For Notations and Constants, see Arts. 10-1 and 10-2

10-15. Horizontal Concentrated Load at Any Point of Left Half of Arch



Obtain values of C, J, and K from Table 10-1.

Table 10-1. Values of C, J, and K for Various m

m =	0	0.2f	0.4f	0.6f	0.8f	1.0f
C	0	7.155	9.325	7.588	3.579	0
J	0	6.988	25.562	51.938	80.419	100.00
K	0	8.500	14.013	16.500	16.000	12.50

$$\left.\begin{matrix} M_1 \\ M_2 \end{matrix}\right\rangle = -\frac{Pf}{100}(C \pm K)$$

$$H_2 = \frac{PJ}{200} \qquad H_1 = -(P - H_2)$$

$$V_2 = \frac{Pf}{L}\left(\frac{m}{f} - \frac{K}{50}\right) \qquad V_1 = -V_2$$

When $x \leqq b$ $\qquad M_x = M_1 + V_1 x - H_1 y$

When $x > b$ $\qquad M_x = M_2 + V_2(L - x) - H_2 y$

When $x \leqq b$

$$N_x = H_1 \cos \varphi + V_1 \sin \varphi$$
$$Q_x = -H_1 \sin \varphi + V_1 \cos \varphi \tag{10-38}$$

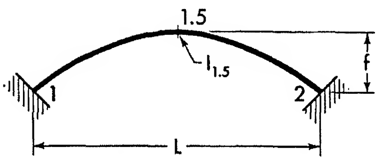
When $x > b$, but $< \frac{L}{2}$

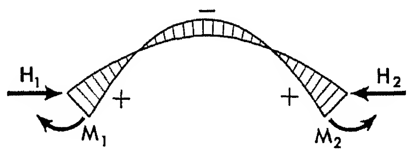
$$N_x = (P + H_1) \cos \varphi + V_1 \sin \varphi$$
$$Q_x = -(P + H_1) \sin \varphi + V_1 \cos \varphi \tag{10-39}$$

When $x \geqq \frac{L}{2}$

$$N_x = (P + H_1)\cos\varphi - V_1 \sin\varphi$$
$$Q_x = (P + H_1)\sin\varphi + V_1 \cos\varphi \qquad (10\text{-}40)$$

10-16. Effect of Temperature Rise. Range t° for entire arch.





$$M_1 = M_2 = \frac{15\varepsilon t^\circ}{2f} EI_{1.5}$$

$$H_1 = H_2 = \frac{3M_1}{2f} \qquad V_1 = V_2 = 0$$

$$M_x = M_1 - H_1 y \qquad M_{1.5} = M_1 - H_1 f$$

$$N_x = H_1 \cos\varphi \qquad (10\text{-}41)$$

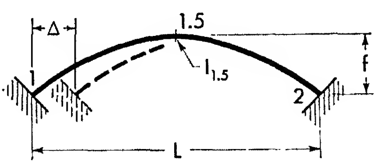
When $x \leqq \frac{L}{2}$ $\qquad Q_x = -H_1 \sin\varphi$

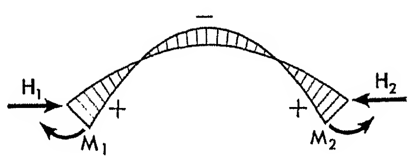
$$(10\text{-}42)$$

When $x > \frac{L}{2}$ $\qquad Q_x = H_1 \sin\varphi$

Note: For temperature drop, introduce the value of t° with a negative sign.

10-17. Horizontal Displacement of One Support





$$M_1 = M_2 = \frac{15\Delta}{2Lf} EI_{1.5}$$

$$H_1 = H_2 = \frac{3M_1}{2f}$$

$$V_1 = V_2 = 0$$

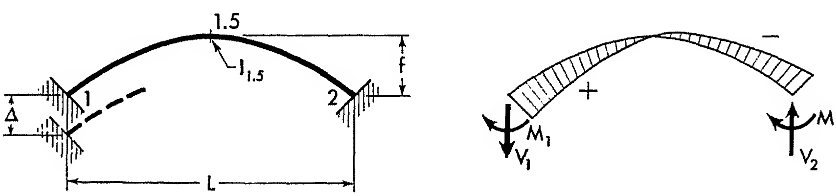
$$M_x = M_1 - H_1 y$$

$$M_{1.5} = M_1 - H_1 f$$

Axial and shearing forces, at any section of the arch, may be determined by the use of Eqs. (10-41) and (10-42).

Note: If the direction of the arch displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.

10-18. Vertical Settlement of One Support



$$M_1 = -M_2 = \frac{6\Delta}{L^2} EI_{1.5}$$

$$H_1 = H_2 = 0$$

$$V_1 = -\frac{2M_1}{L}$$

$$V_2 = -V_1$$

$$M_x = M_1 + V_1 x$$

$$M_{1.5} = 0$$

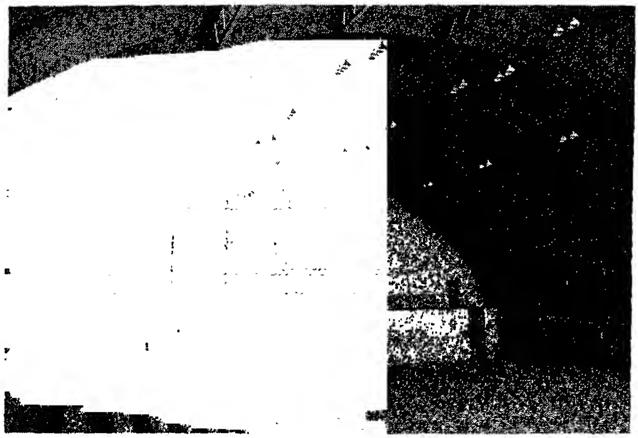
$$Q_x = V_1 \cos \varphi \tag{10-43}$$

When $x \leqq \frac{L}{2}$ $\qquad N_x = V_1 \sin \varphi$

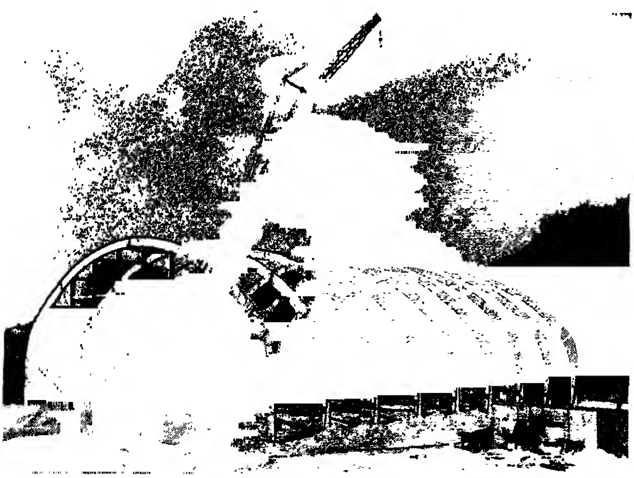
(10-44)

When $x > \frac{L}{2}$ $\qquad N_x = -V_1 \sin \varphi$

Note: If the direction of the arch displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.



An interior view of the David Mead Field House at Allegheny College in Meadville, Pennsylvania. The frames, roof, walls, balcony, and other elements of this field house are constructed entirely of steel. The 11 arcuately shaped frames are covered with specially designed prefabricated Z panels. Notice the simple and neat appearance of all structural members. The structure is a joint design of Lorimer Rich & Associates of Brooklyn, New York, and the Pittsburgh–Des Moines Steel Co., Pittsburgh, Pennsylvania. (Courtesy of Allegheny College and United States Steel Corporation.)

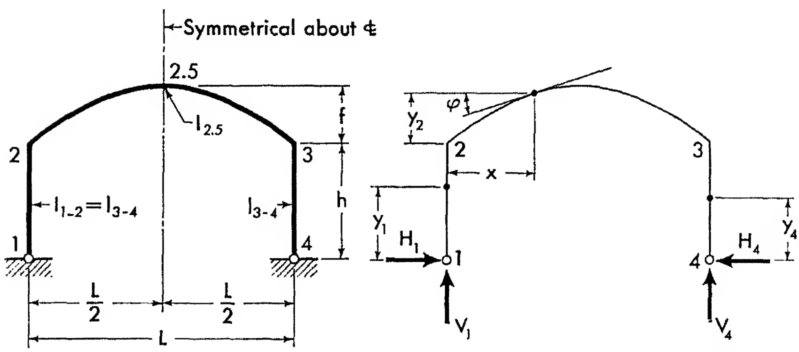


The structural steel framework of the David Mead Field House at Allegheny College during construction. The simplicity and efficiency of installation of this type of structure may be readily realized from the picture. As a matter of fact, each of the depicted rigid frames was fabricated and delivered to the site in three pieces, and was assembled and erected in only two hours' time. (Courtesy of United States Steel Corporation.)

SECTION 11

SYMMETRICAL PARABOLIC FRAMES WITH HINGED SUPPORTS

11-1. Notations, Coordinates, and Frame Constants



The sketch appearing on the left, above, explains notations for a representative arched frame with columns of constant section and a slightly haunched girder, as defined by Eq. (8-2).[1] The axis of the arched girder is a symmetrical parabola conforming to Eq. (8-1).

The sketch appearing on the right, above, shows positive directions of the vertical and horizontal components of the frame reactions. It also defines the angle of inclination and the coordinates at any section of the frame. Angles of inclination and coordinates are to be considered only in the positive sense.

General Frame Constants: $\phi = \frac{I_{1\text{-}2}}{I_{2.5}} \cdot \frac{L}{h}$ $\qquad \psi = \frac{f}{h}$

[1] The application of the condensed solutions of analysis given in this section may be extended to frames with arched members of constant section, by the reasons given in Art. 8-3.

$$A = 8[1 + \phi(1.5 + 2\psi + 0.8\psi^2)]$$

Constant B. For use only in cases of horizontal load on the column.

$$B = \frac{4(1 + 1.5\phi + \phi\psi)}{A}$$

11-2. Equations of Frame Reactions and Moments. The equations for the vertical and the redundant horizontal components of frame reactions are given on the following pages for a number of loading conditions. Corresponding expressions for the moment and forces at any section of a member with applied load are also provided.

The equations for the moments and forces of load-free members are listed below for reference.

1. The bending moment equations are

$$M_{y_1} = M_2 \frac{y_1}{h} \tag{11-1}$$

$$M_x = M_2\left(1 - \frac{x}{L}\right) + M_3 \frac{x}{L} - H_4 y_2 \tag{11-2}$$

$$M_{y_4} = M_3 \frac{y_4}{h} \tag{11-3}$$

2. The equations for axial and shearing forces in the arched member are

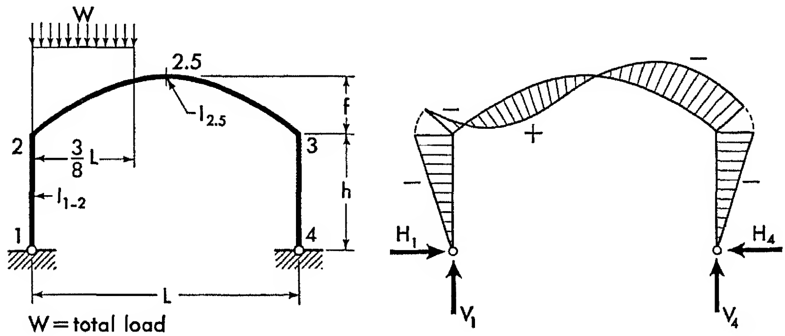
When $x \leqq \frac{L}{2}$

$$\begin{aligned} N_x &= V_1 \sin\varphi + H_4 \cos\varphi \\ Q_x &= V_1 \cos\varphi - H_4 \sin\varphi \end{aligned} \tag{11-4}$$

When $x > \frac{L}{2}$

$$\begin{aligned} N_x &= H_4 \cos\varphi - V_1 \sin\varphi \\ Q_x &= V_1 \cos\varphi + H_4 \sin\varphi \end{aligned} \tag{11-5}$$

11-3. Vertical Uniform Load over Three-eighths of Span



$$H_1 = H_4 = \frac{WL\phi}{Ah}\left(\frac{27}{32} + \frac{2{,}705}{4{,}096}\psi\right)$$

$$M_2 = M_3 = -H_1 h \qquad V_1 = \frac{13}{16}W \qquad V_4 = \frac{3W}{16}$$

When $x \leqq \frac{3}{8}L$

$$M_x = Wx\left(\frac{13}{16} - \frac{4x}{3L}\right) - H_1(h + y_2)$$

When $x > \frac{3}{8}L$

$$M_x = \frac{3W}{16}(L - x) - H_1(h + y_2)$$

When $x \leqq \frac{3}{8}L$

$$N_x = W\left(\frac{13}{16} - \frac{8x}{3L}\right)\sin\varphi + H_1\cos\varphi$$

$$Q_x = W\left(\frac{13}{16} - \frac{8x}{3L}\right)\cos\varphi - H_1\sin\varphi \tag{11-6}$$

When $x > \frac{3}{8}L$, but $\leqq \frac{L}{2}$

$$N_x = H_1\cos\varphi - \frac{3W}{16}\sin\varphi$$

$$Q_x = -H_1\sin\varphi - \frac{3W}{16}\cos\varphi \tag{11-7}$$

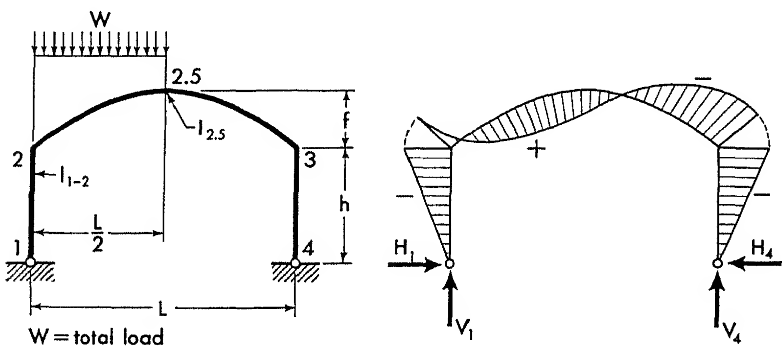
For Notations and Constants, see Arts. 11-1 and 11-2

When $x > \frac{L}{2}$

$$N_x = H_1 \cos\varphi + \frac{3W}{16}\sin\varphi$$
$$Q_x = H_1 \sin\varphi - \frac{3W}{16}\cos\varphi \qquad (11\text{-}8)$$

Apply Eqs. (11-1) and (11-3) to obtain the moment at any section of the frame columns.

11-4. Vertical Uniform Load over Left Half of Span



$$H_1 = H_4 = \frac{WL\phi}{Ah}\left(1 + \frac{4\psi}{5}\right)$$

$$M_2 = M_3 = -H_1 h \qquad V_1 = \frac{3}{4}W \qquad V_4 = \frac{W}{4}$$

When $x \leqq \frac{L}{2}$

$$M_x = Wx\left(\frac{3}{4} - \frac{x}{L}\right) - H_1(h + y_2)$$
$$N_x = H_1 \cos\varphi + W\left(\frac{3}{4} - \frac{2x}{L}\right)\sin\varphi \qquad (11\text{-}9)$$
$$Q_x = -H_1 \sin\varphi + W\left(\frac{3}{4} - \frac{2x}{L}\right)\cos\varphi$$

When $x > \frac{L}{2}$

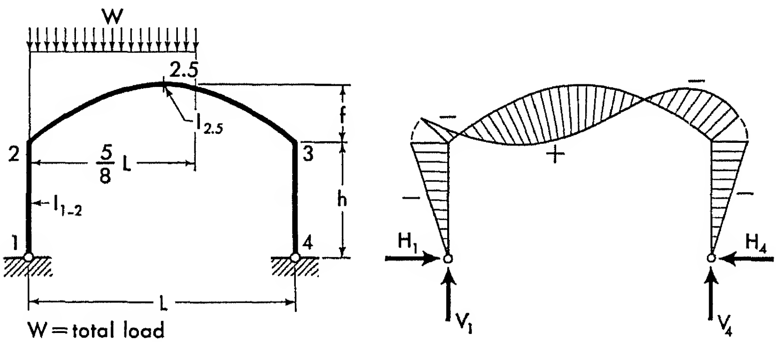
$$M_x = \frac{W}{4}(L - x) - H_1(h + y_2)$$

$$N_x = H_1 \cos\varphi + \frac{W}{4}\sin\varphi \qquad (11\text{-}10)$$

$$Q_x = H_1 \sin\varphi - \frac{W}{4}\cos\varphi$$

Apply Eqs. (11-1) and (11-3) to obtain the moment at any section of the frame columns.

11-5. Vertical Uniform Load over Five-eighths of Span



$$H_1 = H_4 = \frac{WL\phi}{Ah}\left(\frac{35}{32} + \frac{905}{1{,}024}\psi\right)$$

$$M_2 = M_3 = -H_1h \qquad V_1 = \frac{11}{16}W \qquad V_4 = \frac{5W}{16}$$

When $x \leq \frac{5}{8}L$

$$M_x = Wx\left(\frac{11}{16} - \frac{4x}{5L}\right) - H_1(h + y_2)$$

When $x > \frac{5}{8}L$

$$M_x = \frac{5W}{16}(L - x) - H_1(h + y_2)$$

When $x \leqq \frac{L}{2}$

$$N_x = W\left(\frac{11}{16} - \frac{8x}{5L}\right)\sin\varphi + H_1\cos\varphi$$
$$Q_x = W\left(\frac{11}{16} - \frac{8x}{5L}\right)\cos\varphi - H_1\sin\varphi \tag{11-11}$$

When $x > \frac{L}{2}$, but $\leqq \frac{5}{8}L$

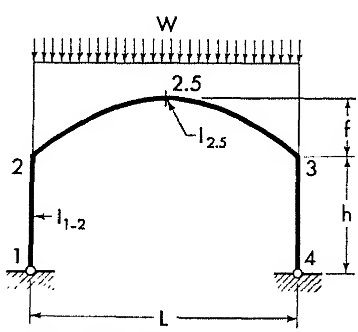
$$N_x = W\left(\frac{8x}{5L} - \frac{11}{16}\right)\sin\varphi + H_1\cos\varphi$$
$$Q_x = H_1\sin\varphi - W\left(\frac{8x}{5L} - \frac{11}{16}\right)\cos\varphi \tag{11-12}$$

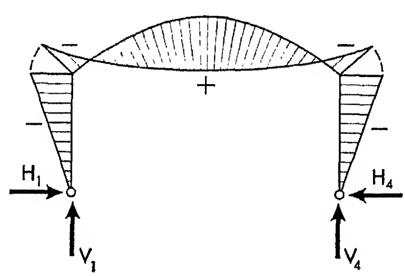
When $x > \frac{5}{8}L$

$$N_x = H_1\cos\varphi + \frac{5W}{16}\sin\varphi$$
$$Q_x = H_1\sin\varphi - \frac{5W}{16}\cos\varphi \tag{11-13}$$

Apply Eqs. (11-1) and (11-3) to obtain the moment at any section of the frame columns.

11-6. Vertical Uniform Load over Entire Span





$$H_1 = H_4 = \frac{WL\phi}{Ah}\left(1 + \frac{4\psi}{5}\right)$$

$$M_2 = M_3 = -H_1 h \qquad V_1 = V_4 = \frac{W}{2}$$

$$M_x = \frac{Wx}{2}\left(1 - \frac{x}{L}\right) - H_1(h + y_2)$$

When $x \leqq \frac{L}{2}$

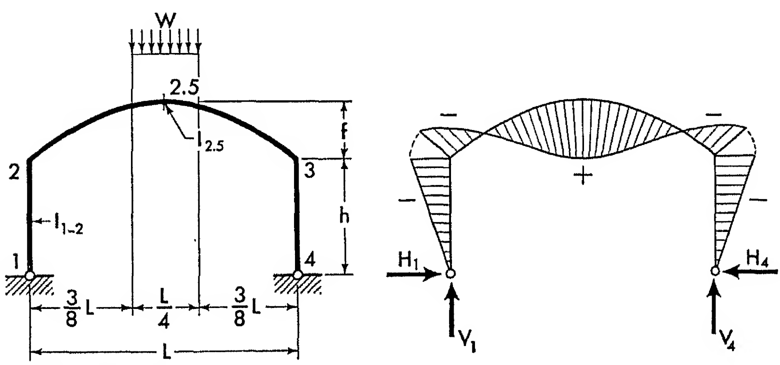
$$\begin{aligned} N_x &= H_1 \cos\varphi + W\left(\frac{1}{2} - \frac{x}{L}\right)\sin\varphi \\ Q_x &= W\left(\frac{1}{2} - \frac{x}{L}\right)\cos\varphi - H_1 \sin\varphi \end{aligned} \qquad (11\text{-}14)$$

When $x > \frac{L}{2}$

$$\begin{aligned} N_x &= H_1 \cos\varphi + W\left(\frac{x}{L} - \frac{1}{2}\right)\sin\varphi \\ Q_x &= H_1 \sin\varphi - W\left(\frac{x}{L} - \frac{1}{2}\right)\cos\varphi \end{aligned} \qquad (11\text{-}15)$$

Apply Eqs. (11-1) and (11-3) to obtain the moment at any section of the frame columns.

11-7. Vertical Uniform Load over Center Quarter of Span



W = total load

$$H_1 = H_4 = \frac{WL\phi}{Ah}\left(\frac{47 + 39\psi}{32}\right)$$

$$M_2 = M_3 = -H_1 h \qquad V_1 = V_4 = \frac{W}{2}$$

For Notations and Constants, see Arts. 11-1 and 11-2

When $x \leqq \frac{3}{8} L$

$$M_x = \frac{Wx}{2} - H_1(h + y_2)$$

$$N_x = \frac{W}{2} \sin \varphi + H_1 \cos \varphi \qquad (11\text{-}16)$$

$$Q_x = \frac{W}{2} \cos \varphi - H_1 \sin \varphi$$

When $x > \frac{3}{8} L$, but $\leqq \frac{L}{2}$

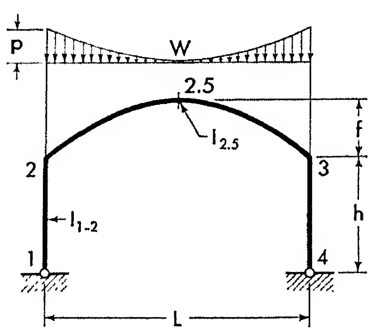
$$M_x = \frac{Wx}{2} - \frac{2W}{L}\left(x - \frac{3L}{8}\right)^2 - H_1(h + y_2)$$

$$N_x = H_1 \cos \varphi + 2W\left(1 - \frac{2x}{L}\right) \sin \varphi \qquad (11\text{-}17)$$

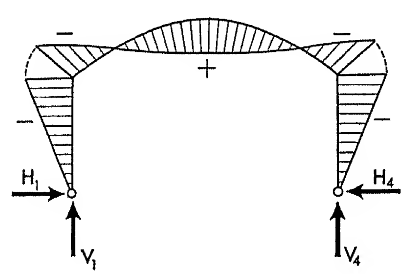
$$Q_x = 2W\left(1 - \frac{2x}{L}\right) \cos \varphi - H_1 \sin \varphi$$

Apply Eq. (11-1) to obtain the moment at any section of the left column. Moments and axial forces at corresponding sections in the right half of the frame are identical to those in the left half. Shearing forces in the right half have the same numerical values as those in the left half, but are of the opposite sign.

11-8. Vertical Complementary Parabolic Load over Entire Span



Total load $W = \frac{pL}{3}$



$$H_1 = H_4 = \frac{WL\phi}{Ah}\left(\frac{21 + 16\psi}{35}\right)$$

Members of Constant Section

$$M_2 = M_3 = -H_1h \qquad V_1 = V_4 = \frac{W}{2}$$

$$M_{2.5} = \frac{WL}{16} - H_1(h + f)$$

When $x \leqq \frac{L}{2}$

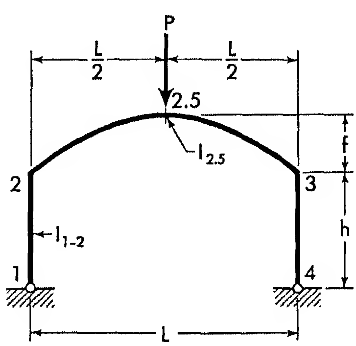
$$M_x = \frac{WL}{16}\left[1 - \left(\frac{L - 2x}{L}\right)^4\right] - H_1(h + y_2)$$

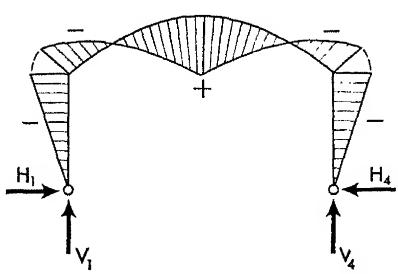
$$N_x = \frac{W}{2}\left(\frac{L - 2x}{L}\right)^3 \sin\varphi + H_1 \cos\varphi \qquad (11\text{-}18)$$

$$Q_x = \frac{W}{2}\left(\frac{L - 2x}{L}\right)^3 \cos\varphi - H_1 \sin\varphi$$

Moments and axial forces at corresponding sections in the right half of the arched frame are identical to those in the left half. Shearing forces in the right half have the same numerical values as those in the left half, but are of the opposite sign.

11-9. Vertical Concentrated Load at Crown





$$H_1 = H_4 = \frac{PL\phi}{Ah}\left(\frac{6 + 5\psi}{4}\right)$$

$$M_2 = M_3 = -H_1h \qquad V_1 = V_4 = \frac{P}{2}$$

$$M_{2.5} = \frac{PL}{4} - H_1(h + f)$$

$$N_x = H_1 \cos\varphi + \frac{P}{2} \sin\varphi \qquad (11\text{-}19)$$

For Notations and Constants, see Arts. 11-1 and 11-2

When $x \leqq \frac{L}{2}$

$$M_x = \frac{Px}{2} - H_1(h + y_2)$$

$$Q_x = -H_1 \sin\varphi + \frac{P}{2}\cos\varphi \qquad (11\text{-}20)$$

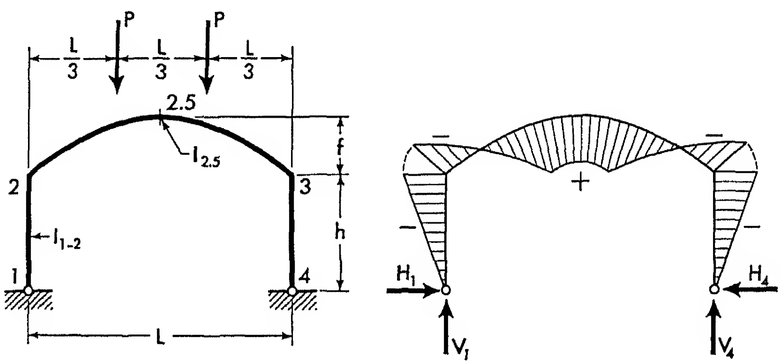
When $x > \frac{L}{2}$

$$M_x = \frac{P}{2}(L - x) - H_1(h + y_2)$$

$$Q_x = H_1 \sin\varphi - \frac{P}{2}\cos\varphi \qquad (11\text{-}21)$$

Apply Eqs. (11-1) and (11-3) to obtain the moment at any section of the frame columns.

11-10. Two Vertical Concentrated Loads on Arched Girder



$$H_1 = H_4 = \frac{PL\phi}{3Ah}\left(8 + \frac{176\psi}{27}\right)$$

$$M_2 = M_3 = -H_1 h \qquad V_1 = V_4 = P$$

When $x \leqq \frac{L}{3}$

$$M_x = Px - H_1(h + y_2)$$

$$N_x = H_1 \cos\varphi + P \sin\varphi$$

$$Q_x = -H_1 \sin\varphi + P \cos\varphi$$

When $x > \frac{L}{3}$, but $\leqq \frac{L}{2}$

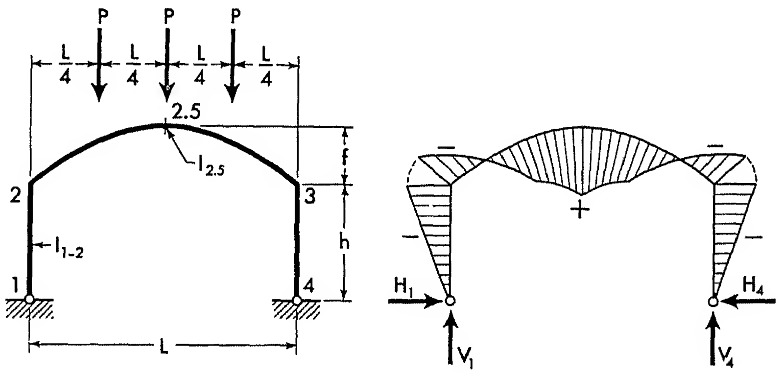
$$M_x = \frac{PL}{3} - H_1(h + y_2)$$

$$N_x = H_1 \cos \varphi$$

$$Q_x = -H_1 \sin \varphi$$

Apply Eq. (11-1) to obtain the moment at any section of the left column. Moments and axial forces at corresponding sections in the right half of the frame are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

11-11. Three Vertical Concentrated Loads on Arched Girder



$$H_1 = H_4 = \frac{PL\phi}{4Ah}\left(15 + \frac{97\psi}{8}\right)$$

$$M_2 = M_3 = -H_1h \qquad V_1 = V_4 = \frac{3}{2}P$$

When $x \leqq \frac{L}{4}$

$$M_x = \frac{3Px}{2} - H_1(h + y_2)$$

$$N_x = H_1 \cos \varphi + \frac{3P}{2} \sin \varphi$$

$$Q_x = -H_1 \sin \varphi + \frac{3P}{2} \cos \varphi$$

For Notations and Constants, see Arts. 11-1 and 11-2

When $x > \frac{L}{4}$, but $\leqq \frac{L}{2}$

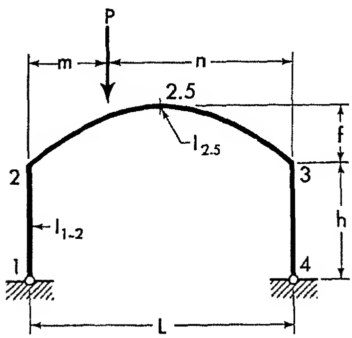
$$M_x = \frac{P(L + 2x)}{4} - H_1(h + y_2)$$

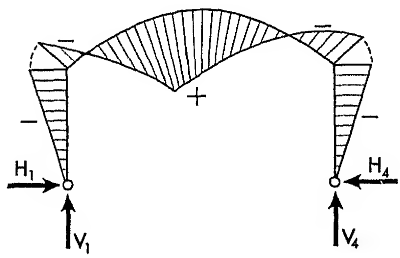
$$N_x = H_1 \cos \varphi + \frac{P}{2} \sin \varphi$$

$$Q_x = - H_1 \sin \varphi + \frac{P}{2} \cos \varphi$$

Apply Eq. (11-1) to obtain the moment at any section of the left column. Moments and axial forces at corresponding sections in the right half of the frame are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

11-12. Vertical Concentrated Load on Arched Girder





Obtain value of K from Table 11-1.

$$H_1 = H_4 = \frac{PLK\phi}{Ah} \qquad M_2 = M_3 = - H_1 h$$

$$V_1 = P\left(1 - \frac{m}{L}\right) \qquad V_4 = \frac{Pm}{L}$$

When $x \leqq m$

$$M_x = \frac{Pnx}{L} - H_1(h + y_2)$$

Table 11-1. Values of K for Various m

Values of m	Values of K
0	0
0.1L	$0.54 + 0.3924\psi$
0.2L	$0.96 + 0.7424\psi$
0.3L	$1.26 + 1.0160\psi$
0.4L	$1.44 + 1.1900\psi$
0.5L	$1.50 + 1.2500\psi$
0.6L	$1.44 + 1.1900\psi$
0.7L	$1.26 + 1.0160\psi$
0.8L	$0.96 + 0.7424\psi$
0.9L	$0.54 + 0.3924\psi$
1.0L	0

When $x > m$

$$M_x = Pm\left(1 - \frac{x}{L}\right) - H_1(h + y_2)$$

When $x < m$ and $\frac{L}{2}$

$$N_x = H_1 \cos\varphi + \frac{Pn}{L}\sin\varphi$$
$$Q_x = -H_1 \sin\varphi + \frac{Pn}{L}\cos\varphi \qquad (11\text{-}22)$$

When $x \leqq m$, but $\geqq \frac{L}{2}$

$$N_x = H_1 \cos\varphi - \frac{Pn}{L}\sin\varphi$$
$$Q_x = H_1 \sin\varphi + \frac{Pn}{L}\cos\varphi \qquad (11\text{-}23)$$

When $x \geqq m$, but $\leqq \frac{L}{2}$

$$N_x = H_1 \cos\varphi - \frac{Pm}{L}\sin\varphi$$
$$Q_x = -H_1 \sin\varphi - \frac{Pm}{L}\cos\varphi \qquad (11\text{-}24)$$

For Notations and Constants, see Arts. 11-1 and 11-2

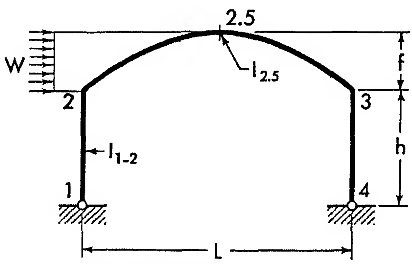
When $x \geqq m$ and $\frac{L}{2}$

$$N_x = H_1 \cos \varphi + \frac{Pm}{L} \sin \varphi$$

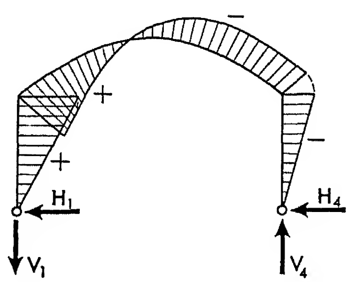
$$Q_x = H_1 \sin \varphi - \frac{Pm}{L} \cos \varphi \qquad (11\text{-}25)$$

Apply Eqs. (11-1) and (11-3) to obtain the moment at any section of the frame columns.

11-13. Horizontal Uniform Load on Left Half of Arched Girder



W = total load



$$K = \frac{1}{5}\left(12 + \frac{749\psi}{82}\right)$$

$$H_4 = W\left(B + \frac{K\phi\psi}{A}\right) \qquad H_1 = -(W - H_4)$$

$$V_4 = \frac{W}{2L}(2h + f) \qquad V_1 = -V_4$$

$$M_2 = -H_1 h \qquad M_3 = -H_4 h$$

When $x \leqq \frac{L}{2}$

$$M_x = V_1 x - \frac{Wy_2^2}{2f} - H_1(h + y_2)$$

$$N_x = \left(\frac{Wy_2}{f} + H_1\right) \cos \varphi + V_1 \sin \varphi \qquad (11\text{-}26)$$

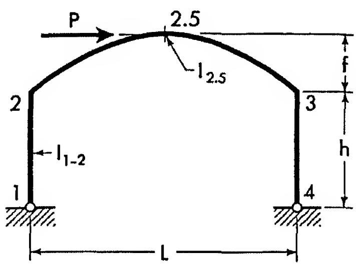
$$Q_x = -\left(\frac{Wy_2}{f} + H_1\right) \sin \varphi + V_1 \cos \varphi$$

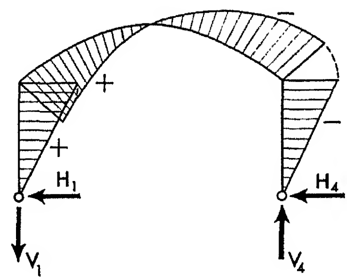
When $x > \frac{L}{2}$

$$\begin{aligned} M_x &= V_4(L - x) - H_4(h + y_2) \\ N_x &= (W + H_1) \cos \varphi - V_1 \sin \varphi \\ Q_x &= V_1 \cos \varphi + (W + H_1) \sin \varphi \end{aligned} \qquad (11\text{-}27)$$

Apply Eqs. (11-1) and (11-3) to obtain the moment at any section of the frame columns.

11-14. Horizontal Concentrated Load at Crown





$$H_1 = -\frac{P}{2} \qquad H_4 = \frac{P}{2}$$

$$M_2 = \frac{Ph}{2} \qquad M_3 = -\frac{Ph}{2}$$

$$V_4 = \frac{P(h + f)}{L} \qquad V_1 = -V_4 \qquad M_{2.5} = 0$$

$$Q_x = \frac{P}{2} \sin \varphi + V_1 \cos \varphi \qquad (11\text{-}28)$$

When $x \leqq \frac{L}{2}$

$$\begin{aligned} M_x &= \frac{P}{2}\left[h + y_2 - 2x\left(\frac{h + f}{L}\right)\right] \\ N_x &= -\frac{P}{2} \cos \varphi + V_1 \sin \varphi \end{aligned} \qquad (11\text{-}29)$$

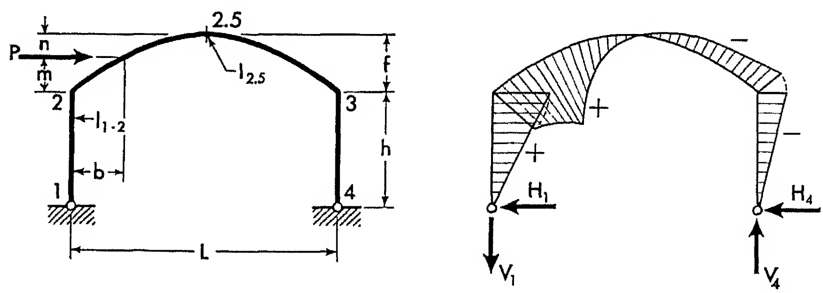
When $x > \frac{L}{2}$

$$M_x = \frac{P}{2}\left[2(h+f)\left(1-\frac{x}{L}\right)-(h+y_2)\right]$$

$$N_x = \frac{P}{2}\cos\varphi - V_1 \sin\varphi \qquad (11\text{-}30)$$

Apply Eqs. (11-1) and (11-3) to obtain the moment at any section of the frame columns.

11-15. Horizontal Concentrated Load at Any Point of Left Half of Arched Girder



Obtain value of K from Table 11-2.

$$H_4 = P\left(B + \frac{K\phi\psi_1}{A}\right) \qquad H_1 = -(P - H_4)$$

$$V_4 = \frac{P(h+m)}{L} \qquad V_1 = -V_4$$

$$M_2 = -H_1 h \qquad M_3 = -H_4 h$$

When $x \leqq b$

$$M_x = -\frac{Px(h+m)}{L} - H_1(h+y_2)$$

When $x > b$

$$M_x = P(h+m)\left(1-\frac{x}{L}\right) - H_4(h+y_2)$$

When $x \leqq b$

$$N_x = H_1 \cos\varphi + V_1 \sin\varphi$$
$$Q_x = -H_1 \sin\varphi + V_1 \cos\varphi \qquad (11\text{-}31)$$

Members of Constant Section

When $x > b$, but $< \frac{L}{2}$

$$N_x = (P + H_1)\cos\varphi + V_1 \sin\varphi \qquad (11\text{-}32)$$
$$Q_x = -(P + H_1)\sin\varphi + V_1 \cos\varphi$$

When $x \geqq \frac{L}{2}$

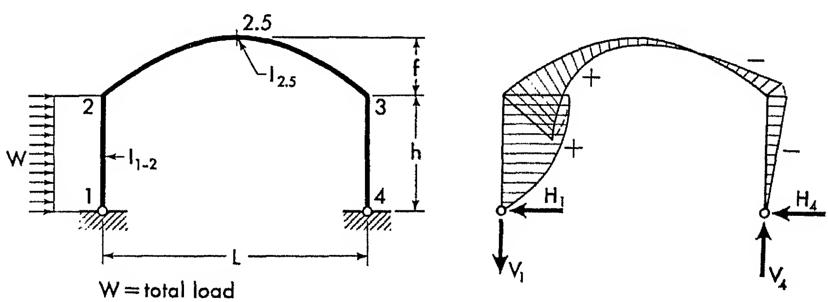
$$N_x = (P + H_1)\cos\varphi - V_1 \sin\varphi \qquad (11\text{-}33)$$
$$Q_x = (P + H_1)\sin\varphi + V_1 \cos\varphi$$

Apply Eqs. (11-1) and (11-3) to obtain the moment at any section of the frame columns.

Table 11-2. Values of K for Various m

Value of m	Value of K
0	0
0.2f	$1.138 + 0.796\psi$
0.4f	$2.142 + 1.564\psi$
0.6f	$2.988 + 2.269\psi$
0.8f	$3.642 + 2.857\psi$
1.0f	$4.000 + 3.200\psi$

11-16. Horizontal Uniform Load on Column



$$H_4 = \frac{W}{2A}(1 + AB) \qquad H_1 = -(W - H_4)$$

$$V_4 = \frac{Wh}{2L} \qquad V_1 = -V_4$$

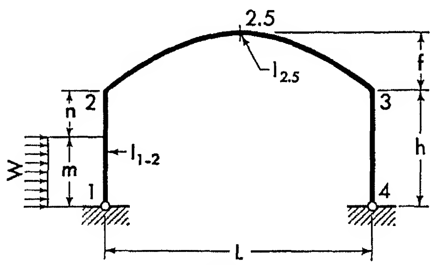
$$M_2 = \frac{Wh}{2} - H_4 h \qquad M_3 = -H_4 h$$

For Notations and Constants, see Arts. 11-1 and 11-2

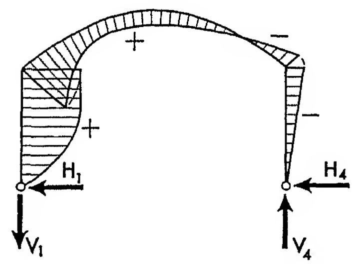
$$M_{y_1} = \frac{Wy_1}{2}\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (11-2) and (11-3) to obtain the moment at any section of frame members 2-3 and 3-4, and use Eqs. (11-4) and (11-5) to determine the axial and shearing forces in the arched girder.

11-17. Horizontal Uniform Load over Part of Column



W = total load



$$g = \frac{m}{h} \qquad H_4 = \frac{Wg}{2A}(2 + AB - g^2)$$

$$H_1 = -(W - H_4) \qquad V_4 = \frac{Wm}{2L}$$

$$V_1 = -V_4 \qquad M_2 = \frac{Wm}{2} - H_4h$$

$$M_3 = -H_4h$$

When $y_1 \leqq m$

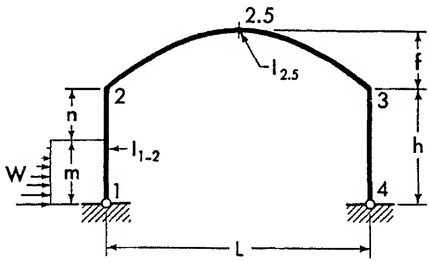
$$M_{y_1} = M_2\frac{y_1}{h} + \frac{Wy_1}{2}\left(\frac{n}{h} + \frac{m - y_1}{m}\right)$$

When $y_1 > m$

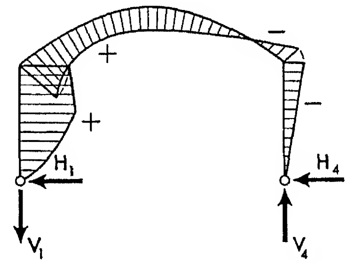
$$M_{y_1} = M_2\frac{y_1}{h} + \frac{Wm}{2h}(h - y_1)$$

Apply Eqs. (11-2) and (11-3) to obtain the moment at any section of frame members 2-3 and 3-4, and use Eqs. (11-4) and (11-5) to determine the axial and shearing forces in the arched girder.

11-18. Horizontal Triangular Load over Part of Column



W = total load



$$g = \frac{m}{h} \qquad H_4 = \frac{Wg}{15A}(10 + 5AB - 3g^2)$$

$$H = -(W - H_4) \qquad V_4 = \frac{Wm}{3L}$$

$$V_1 = -V_4 \qquad M_2 = \frac{Wm}{3} - H_4 h \qquad M_3 = -H_4 h$$

When $y_1 \leqq m$

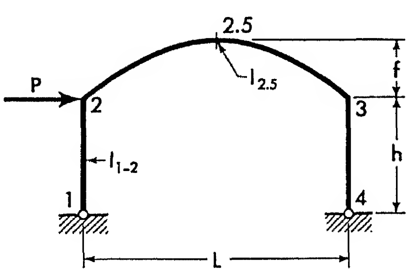
$$M_{y_1} = M_2 \frac{y_1}{h} + \frac{Wy_1}{3}\left[\frac{n}{h} + \frac{(m - y_1)(2m - y_1)}{m^2}\right]$$

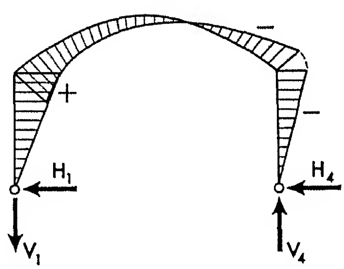
When $y_1 > m$

$$M_{y_1} = M_2 \frac{y_1}{h} + \frac{Wm(h - y_1)}{3h}$$

Apply Eqs. (11-2) and (11-3) to obtain the moment at any section of frame members 2-3 and 3-4, and use Eqs. (11-4) and (11-5) to determine the axial and shearing forces in the arched girder.

11-19. Horizontal Concentrated Load at Joint 2





$$H_4 = PB \qquad H_1 = -(P - H_4)$$

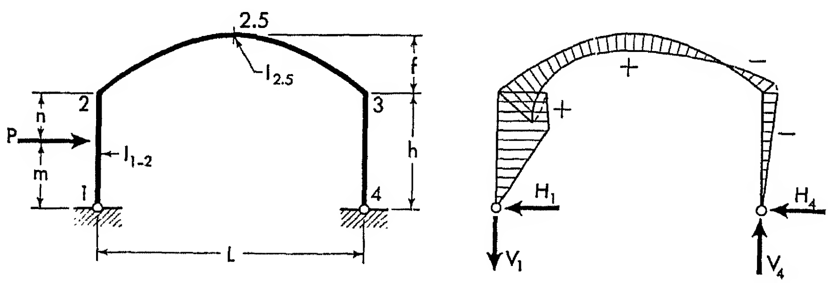
For Notations and Constants, see Arts. 11-1 and 11-2

$$V_4 = \frac{Ph}{L} \qquad V_1 = -V_4 \qquad M_2 = h(P - H_4)$$

$$M_3 = -H_4h \qquad M_{y_1} = M_2\frac{y_1}{h}$$

Apply Eqs. (11-2) and (11-3) to obtain the moment at any section of frame members 2-3 and 3-4, and use Eqs. (11-4) and (11-5) to determine the axial and shearing forces in the arched girder.

11-20. Horizontal Concentrated Load at Any Point of Column



$$H_4 = \frac{Pm}{h}\left[B + \frac{2n(h+m)}{Ah^2}\right]$$

$$H_1 = -(P - H_4) \qquad V_4 = \frac{Pm}{L}$$

$$V_1 = -V_4 \qquad M_2 = Pm - H_4h$$

$$M_3 = -H_4h$$

When $y_1 \leqq m$

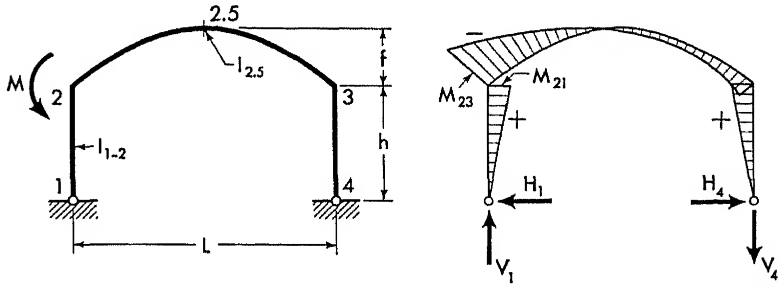
$$M_{y_1} = (Pn + M_2)\frac{y_1}{h}$$

When $y_1 > m$

$$M_{y_1} = M_2\frac{y_1}{h} + \frac{Pm(h - y_1)}{h}$$

Apply Eqs. (11-2) and (11-3) to obtain the moment at any section of frame members 2-3 and 3-4, and use Eqs. (11-4) and (11-5) to determine the axial and shearing forces in the arched girder.

11-21. Moment Applied at Joint 2



$$H_1 = H_4 = -\frac{2M\phi}{Ah}(3 + 2\psi)$$

$$V_1 = \frac{M}{L} \qquad V_4 = -V_1$$

$$M_{21} = -H_1 h \qquad M_{23} = -(M - M_{21})$$

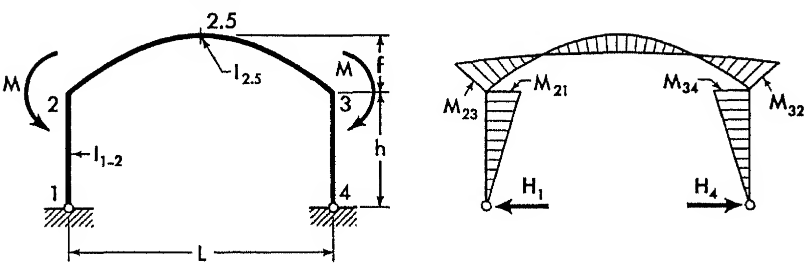
$$M_3 = -H_4 h \qquad M_{y_1} = M_{21}\frac{y_1}{h}$$

$$M_x = M_{23}\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L} - H_4 y_2$$

$$M_{y_4} = M_3\frac{y_4}{h}$$

Apply Eqs. (11-4) and (11-5) to determine the axial and shearing forces in the arched girder.

11-22. Two Equal Moments Applied at Joints 2 and 3



$$H_1 = H_4 = -\frac{4M\phi}{Ah}(3 + 2\psi)$$

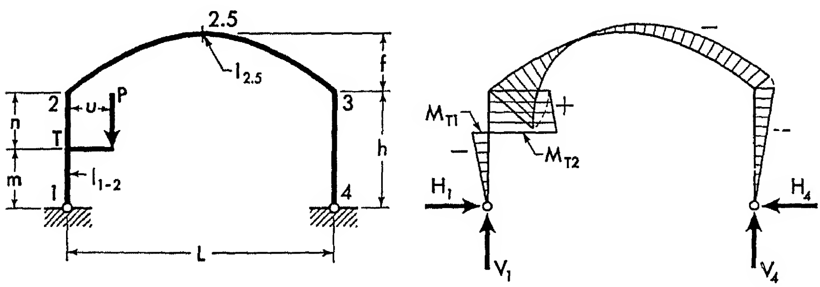
$$V_1 = V_4 = 0 \qquad M_{21} = M_{34} = -H_4h$$

$$M_{23} = M_{32} = -(M - M_{21}) \qquad M_{y_1} = M_{21}\frac{y_1}{h}$$

$$M_x = M_{23} - H_1y_2 \qquad M_{y_4} = M_{34}\frac{y_4}{h}$$

Apply Eqs. (11-4) and (11-5) to determine the axial and shearing forces in the arched girder.

11-23. Vertical Concentrated Load Applied at Bracket



Bracket acts as a simple cantilever and its maximum moment is Pu at point T. The moment diagram of the cantilever is intentionally not shown so that the frame's bending moment diagram may be more clearly illustrated.

$$M = Pu$$

$$K = \frac{2(h^2 - 3m^2)}{Ah^2} \qquad H_1 = H_4 = \frac{M}{h}(B + K)$$

$$V_4 = \frac{M}{L} \qquad V_1 = P - \frac{M}{L} \qquad M_2 = M - H_4h$$

$$M_3 = -H_4h$$

When $y_1 < m$

$$M_{y_1} = -(M - M_2)\frac{y_1}{h}$$

When $y_1 = m$

$$M_{T1} = -(M - M_2)\frac{m}{h}$$

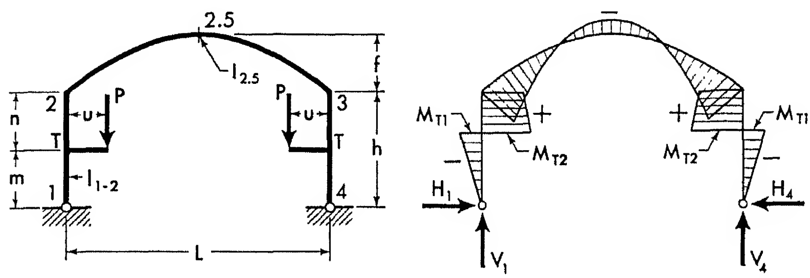
$$M_{T2} = M_2\frac{m}{h} + M\frac{n}{h}$$

When $y_1 > m$

$$M_{y_1} = M_2 \frac{y_1}{h} + M\left(1 - \frac{y_1}{h}\right)$$

Apply Eqs. (11-2) and (11-3) to obtain the moment at any section of frame members 2-3 and 3-4 and use Eqs. (11-4) and (11-5) to determine the axial and shearing forces in the arched girder.

11-24. Two Equal Vertical Concentrated Loads Symmetrically Applied at Brackets



Brackets act as simple cantilevers with the maximum moments of Pu at points T. The moment diagrams of these cantilevers are intentionally not shown so that the frame's bending moment diagram may be more clearly illustrated.

$$M = Pu$$

$$K = \frac{2(h^2 - 3m^2)}{Ah^2} \qquad H_1 = H_4 = \frac{2M}{h}(B + K)$$

$$V_1 = V_4 = P \qquad M_2 = M_3 = M - H_4 h$$

$$M_x = M_2 - H_4 y_2$$

When $y_1 < m$

$$M_{y_1} = -(M - M_2)\frac{y_1}{h}$$

When $y_1 = m$

$$M_{T1} = -(M - M_2)\frac{m}{h}$$

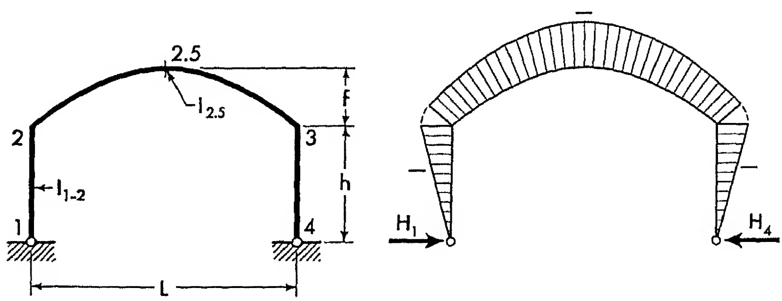
$$M_{T2} = M_2 \frac{m}{h} + M \frac{n}{h}$$

When $y_1 > m$

$$M_{y_1} = M_2 \frac{y_1}{h} + M\left(1 - \frac{y_1}{h}\right)$$

Apply Eqs. (11-4) and (11-5) to determine the axial and shearing forces of the arched girder. Moments at corresponding sections in the right half of the frame are identical to those in the left half.

11-25. Effect of Temperature Rise. Range t° for entire frame.



$$H_1 = H_4 = \frac{12L\varepsilon t^\circ}{Ah^3} EI_{1\text{-}2} \qquad V_1 = V_4 = 0$$

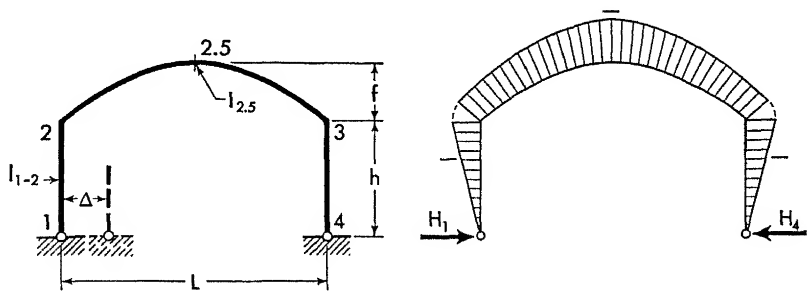
$$M_2 = M_3 = -H_4 h \qquad M_{y_1} = -H_1 y_1$$

$$M_x = M_2 - H_1 y_2 \qquad M_{y_4} = -H_4 y_4$$

Apply Eqs. (11-4) and (11-5) to determine the axial and shearing forces in the arched girder.

Note: For temperature drop, introduce the value of t° with a negative sign.

11-26. Horizontal Displacement of One Support



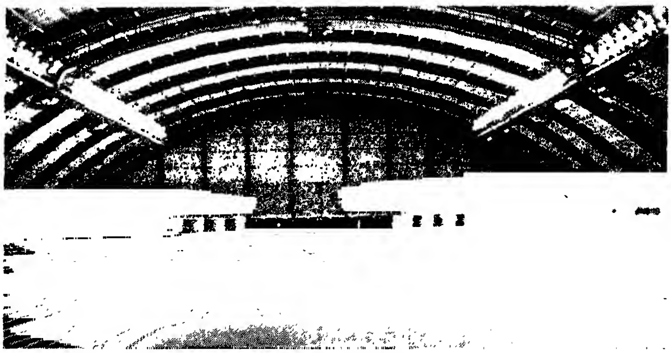
$$H_1 = H_4 = \frac{12\Delta}{Ah^3} EI_{1-2} \qquad V_1 = V_4 = 0$$

$$M_2 = M_3 = -H_4 h \qquad M_{y_1} = -H_1 y_1$$

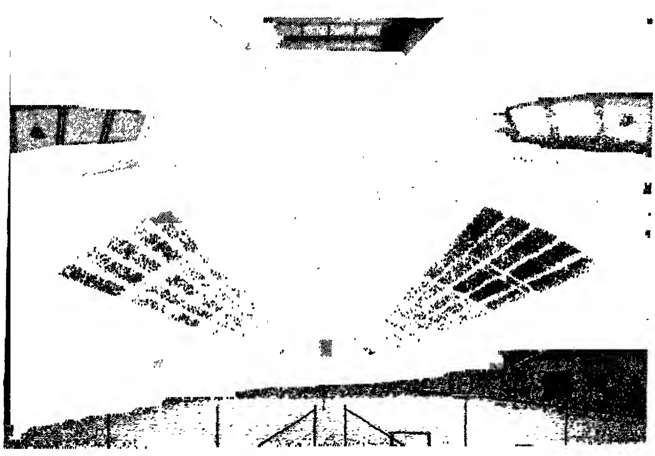
$$M_x = M_2 - H_1 y_2 \qquad M_{y_4} = -H_4 y_4$$

Apply Eqs. (11-4) and (11-5) to determine the axial and shearing forces in the arched girder.

Note: If the direction of the frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.



An interior view of the Activities Building of the University of Maryland, at College Park, Maryland. This magnificent and spacious building was built using steel arched frames of 241-foot span. The uniform curvilinear girders, made of 36-inch rolled steel beams, contrasted with the light aluminum panels provide an extremely neat and impressive appearance. Structural framework was designed by the Arch Roof Construction Co. of New York. (Courtesy of the Arch Roof Construction Co.)

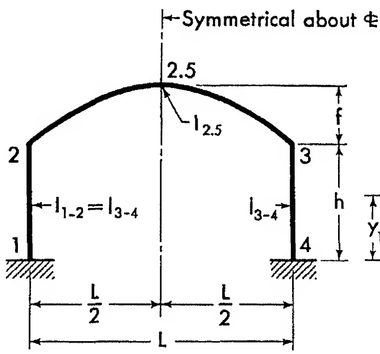


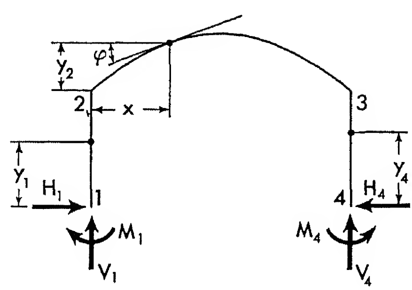
An interior view of the Field House of Swarthmore College at Swarthmore, Pennsylvania. Steel, wood, and glass are ingeniously combined to enhance the modern and unusual structural and architectural features of this field house. The curvilinear girders are made of 21-inch rolled steel beams with riveted joints and span the 125-foot distance between walls. Cupolas at the end of the field house and the continuous skylights impart a highly aesthetic appearance. Structural framework was designed by the Arch Roof Construction Co. of New York. (Courtesy of the Arch Roof Construction Co.)

SECTION 12

SYMMETRICAL PARABOLIC FRAMES WITH FIXED SUPPORTS

12-1. Notations, Coordinates, and Frame Constants





The sketch appearing on the left, above, explains notations for a representative arched frame with columns of constant section and slightly haunched girder, as defined by Eq. (8-2).[1] The axis of the arched girder is a symmetrical parabola conforming to Eq. (8-1).

The sketch appearing on the right, above, shows positive directions of the moments and the vertical and horizontal components of the frame reactions. It also defines the angle of inclination and the coordinates at any section of the frame. Angles of inclination and coordinates are to be considered only in the positive sense.

Frame Constants: $\phi = \dfrac{I_{1\text{-}2}}{I_{2.5}} \cdot \dfrac{L}{h}$

[1] The application of the condensed solutions of analysis given in this section may be extended to frames with arched members of constant section, by the reasons given in Art. 8-3.

$$\psi = \frac{f}{h} \qquad A = \frac{1.5 - \phi\psi}{1 + 0.8\phi\psi^2}$$

$$C = \frac{3 + 1.5\phi}{1 + 0.8\phi\psi^2} \qquad D = 2(6 + \phi)$$

$$F = 12(2 + \phi) - 4A(3 - 2\phi\psi)$$

12-2. Equations of Frame Reactions and Moments. The equations for the redundant moments and the vertical and horizontal components of frame reactions are given on the following pages for a number of loading conditions. Corresponding expressions for the moment and forces at any section of a member with applied load are also provided.

The equations for the moments and forces of load-free members are listed below for reference.

1. The bending moment equations are:

$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h} \tag{12-1}$$

$$M_x = M_2\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L} - H_4y_2 \tag{12-2}$$

$$M_{y_4} = M_3\frac{y_4}{h} + M_4\left(1 - \frac{y_4}{h}\right) \tag{12-3}$$

2. The equations for axial and shearing forces in the arched member are:

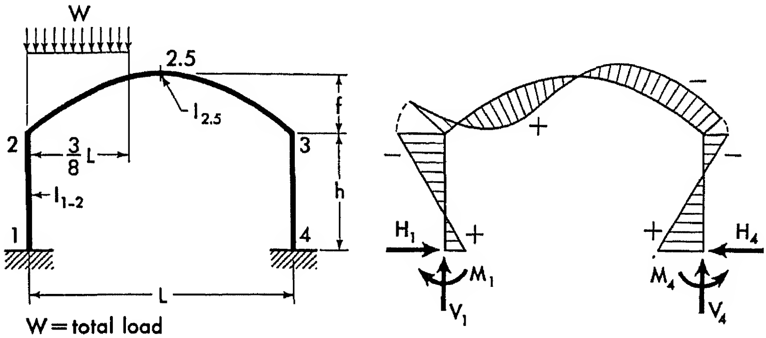
When $x \leqq \frac{L}{2}$

$$\left.\begin{aligned} N_x &= V_1 \sin \varphi + H_4 \cos \varphi \\ Q_x &= V_1 \cos \varphi - H_4 \sin \varphi \end{aligned}\right. \tag{12-4}$$

When $x > \frac{L}{2}$

$$\left.\begin{aligned} N_x &= H_4 \cos \varphi - V_1 \sin \varphi \\ Q_x &= V_1 \cos \varphi + H_4 \sin \varphi \end{aligned}\right. \tag{12-5}$$

12-3. Vertical Uniform Load over Three-eighths of Span



$$G = \frac{27}{32} + \frac{2{,}705}{4{,}096} A\psi \qquad K = \frac{27}{32} A + \frac{2{,}705}{4{,}096} C\psi$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -WL\phi\left(\frac{G}{F} \pm \frac{75}{1{,}024D}\right)$$

$$H_1 = H_4 = \frac{WLK\phi}{Fh} \qquad V_1 = W\left(\frac{13}{16} + \frac{75\phi}{512D}\right)$$

$$V_4 = W - V_1 \qquad M_1 = M_2 + H_1 h$$

$$M_4 = M_3 + H_4 h$$

When $x \leqq \frac{3}{8} L$

$$M_x = Wx\left(\frac{13}{16} - \frac{4x}{3L}\right) + M_2\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L} - H_4 y_2$$

When $x > \frac{3}{8} L$

$$M_x = \frac{3W}{16}(L - x) + M_2\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L} - H_4 y_2$$

When $x \leqq \frac{3}{8} L$

$$\left.\begin{aligned} N_x &= H_1 \cos\varphi - \left(\frac{8Wx}{3L} - V_1\right)\sin\varphi \\ Q_x &= -H_1 \sin\varphi - \left(\frac{8Wx}{3L} - V_1\right)\cos\varphi \end{aligned}\right. \qquad (12\text{-}6)$$

For Notations and Constants, see Arts. 12-1 and 12-2

When $x > \frac{3}{8}L$, but $\leqq \frac{L}{2}$

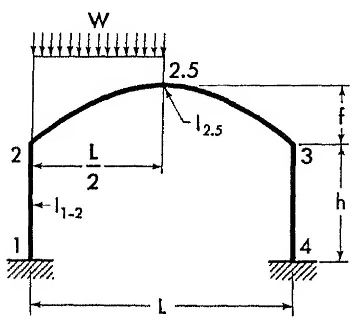
$$\begin{aligned} N_x &= H_1 \cos\varphi - (W - V_1)\sin\varphi \\ Q_x &= -H_1 \sin\varphi - (W - V_1)\cos\varphi \end{aligned} \qquad (12\text{-}7)$$

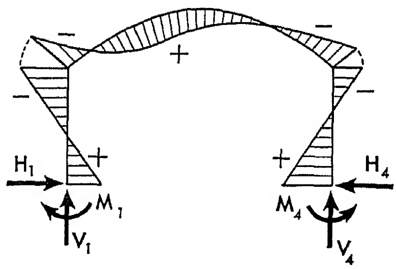
When $x > \frac{L}{2}$

$$\begin{aligned} N_x &= H_1 \cos\varphi + (W - V_1)\sin\varphi \\ Q_x &= H_1 \sin\varphi - (W - V_1)\cos\varphi \end{aligned} \qquad (12\text{-}8)$$

Apply Eqs. (12-1) and (12-3) to obtain the moment at any section of the frame columns.

12-4. Vertical Uniform Load over Left Half of Span





$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\} = -WL\phi\left(\frac{5 + 4A\psi}{5F} \pm \frac{1}{16D}\right)$$

$$H_1 = H_4 = \frac{WL\phi}{5Fh}(5A + 4C\psi)$$

$$V_1 = W\left(\frac{3}{4} + \frac{\phi}{8D}\right) \qquad V_4 = W - V_1$$

$$M_1 = M_2 + H_1 h \qquad M_4 = M_3 + H_4 h$$

When $x \leqq \frac{L}{2}$

$$M_x = Wx\left(\frac{3}{4} - \frac{x}{L}\right) + M_2\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L} - H_4 y_2$$

When $x > \frac{L}{2}$

$$M_x = \frac{W}{4}(L - x) + M_2\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L} - H_4y_2$$

When $x \leqq \frac{L}{2}$

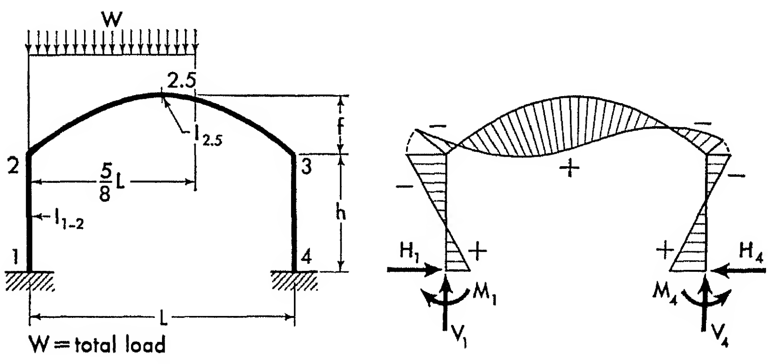
$$\begin{aligned} N_x &= H_1 \cos\varphi + \left(V_1 - \frac{2Wx}{L}\right)\sin\varphi \\ Q_x &= -H_1 \sin\varphi + \left(V_1 - \frac{2Wx}{L}\right)\cos\varphi \end{aligned} \qquad (12\text{-}9)$$

When $x > \frac{L}{2}$

$$\begin{aligned} N_x &= H_1 \cos\varphi + (W - V_1)\sin\varphi \\ Q_x &= H_1 \sin\varphi - (W - V_1)\cos\varphi \end{aligned} \qquad (12\text{-}10)$$

Apply Eqs. (12-1) and (12-3) to obtain the moment at any section of the frame columns.

12-5. Vertical Uniform Load over Five-eighths of Span



$$G = \frac{35}{32} + \frac{905}{1{,}024}A\psi \qquad K = \frac{35}{32}A + \frac{905}{1{,}024}C\psi$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -WL\phi\left(\frac{G}{F} \pm \frac{45}{1{,}024D}\right)$$

$$H_1 = H_4 = \frac{WLK\phi}{Fh} \qquad V_1 = W\left(\frac{11}{16} + \frac{45\phi}{512D}\right)$$

For Notations and Constants, see Arts. 12-1 and 12-2

$$V_4 = W - V_1 \qquad M_1 = M_2 + H_1 h$$

$$M_4 = M_3 + H_4 h$$

When $x \leqq \frac{5}{8} L$

$$M_x = Wx\left(\frac{11}{16} - \frac{4x}{5L}\right) + M_2\left(1 - \frac{x}{L}\right) + M_3 \frac{x}{L} - H_4 y_2$$

When $x > \frac{5}{8} L$

$$M_x = \frac{5W}{16}(L - x) + M_2\left(1 - \frac{x}{L}\right) + M_3 \frac{x}{L} - H_4 y_2$$

When $x \leqq \frac{L}{2}$

$$\begin{aligned} N_x &= H_1 \cos\varphi - \left(\frac{8Wx}{5L} - V_1\right)\sin\varphi \\ Q_x &= -H_1 \sin\varphi - \left(\frac{8Wx}{5L} - V_1\right)\cos\varphi \end{aligned} \qquad (12\text{-}11)$$

When $x > \frac{L}{2}$, but $\leqq \frac{5}{8} L$

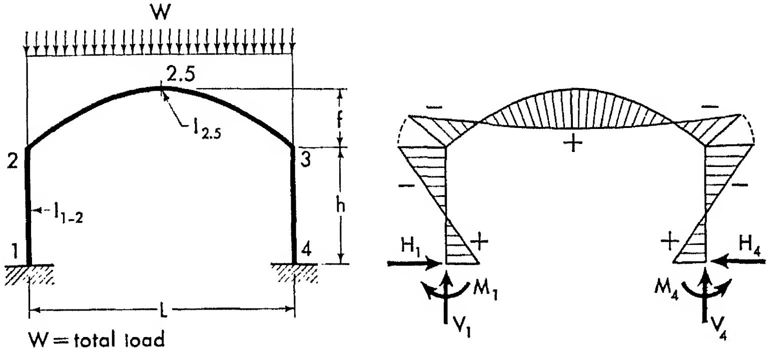
$$\begin{aligned} N_x &= H_1 \cos\varphi + \left(\frac{8Wx}{5L} - V_1\right)\sin\varphi \\ Q_x &= H_1 \sin\varphi - \left(\frac{8Wx}{5L} - V_1\right)\cos\varphi \end{aligned} \qquad (12\text{-}12)$$

When $x > \frac{5}{8} L$

$$\begin{aligned} N_x &= H_1 \cos\varphi + (W - V_1)\sin\varphi \\ Q_x &= H_1 \sin\varphi - (W - V_1)\cos\varphi \end{aligned} \qquad (12\text{-}13)$$

Apply Eqs. (12-1) and (12-3) to obtain the moment at any section of the frame columns.

12-6. Vertical Uniform Load over Entire Span



$$M_2 = M_3 = -\frac{WL\phi}{5F}(5 + 4A\psi)$$

$$H_1 = H_4 = \frac{WL\phi}{5Fh}(5A + 4C\psi)$$

$$V_1 = V_4 = \frac{W}{2} \qquad M_1 = M_4 = M_2 + H_1 h$$

$$M_x = \frac{Wx}{2}\left(1 - \frac{x}{L}\right) + M_2 - H_4 y_2$$

When $x \leqq \frac{L}{2}$

$$\begin{aligned} N_x &= H_1 \cos\varphi + \frac{W}{2}\left(1 - \frac{2x}{L}\right)\sin\varphi \\ Q_x &= -H_1 \sin\varphi + \frac{W}{2}\left(1 - \frac{2x}{L}\right)\cos\varphi \end{aligned} \qquad (12\text{-}14)$$

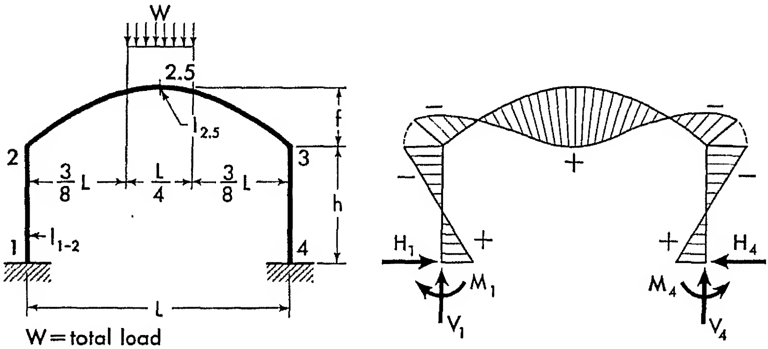
When $x > \frac{L}{2}$

$$\begin{aligned} N_x &= H_1 \cos\varphi - \frac{W}{2}\left(1 - \frac{2x}{L}\right)\sin\varphi \\ Q_x &= H_1 \sin\varphi + \frac{W}{2}\left(1 - \frac{2x}{L}\right)\cos\varphi \end{aligned} \qquad (12\text{-}15)$$

Apply Eqs. (12-1) and (12-3) to obtain the moment at any section of the frame columns.

For Notations and Constants, see Arts. 12-1 and 12-2

12-7. Vertical Uniform Load over Center Quarter of Span



$$M_2 = M_3 = -\frac{WL\phi}{F}\left(\frac{47 + 39A\psi}{32}\right)$$

$$H_1 = H_4 = \frac{WL\phi}{Fh}\left(\frac{47A + 39C\psi}{32}\right)$$

$$V_1 = V_4 = \frac{W}{2}$$

$$M_1 = M_4 = M_2 + H_1h$$

When $x \leqq \frac{3}{8}L$

$$M_x = \frac{Wx}{2} + M_2 - H_4y_2$$

$$N_x = \frac{W}{2}\sin\varphi + H_1\cos\varphi \qquad (12\text{-}16)$$

$$Q_x = \frac{W}{2}\cos\varphi - H_1\sin\varphi$$

When $x > \frac{3}{8}L$, but $\leqq \frac{L}{2}$

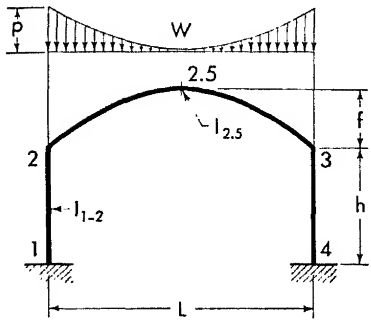
$$M_x = \frac{Wx}{2} - \frac{2W}{L}\left(x - \frac{3L}{8}\right)^2 + M_2 - H_4y_2$$

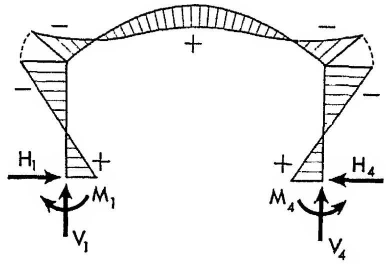
$$N_x = H_1\cos\varphi + 2W\left(1 - \frac{2x}{L}\right)\sin\varphi \qquad (12\text{-}17)$$

$$Q_x = 2W\left(1 - \frac{2x}{L}\right)\cos\varphi - H_1\sin\varphi$$

Apply Eqs. (12-1) and (12-3) to obtain the moment at any section of the frame columns. Moments and axial forces at corresponding sections in the right half of the arched girder are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

12-8. Vertical Complementary Parabolic Load over Entire Span





Total load $W = \dfrac{pL}{3}$

$$M_2 = M_3 = -\frac{WL\phi}{F}\left(\frac{21 + 16A\psi}{35}\right)$$

$$H_1 = H_4 = \frac{WL\phi}{Fh}\left(\frac{21A + 16C\psi}{35}\right)$$

$$V_1 = V_4 = \frac{W}{2} \qquad M_1 = M_4 = M_2 + H_1h$$

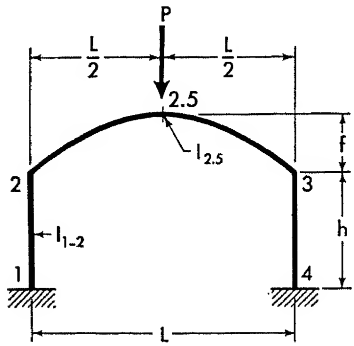
When $x \leqq \dfrac{L}{2}$

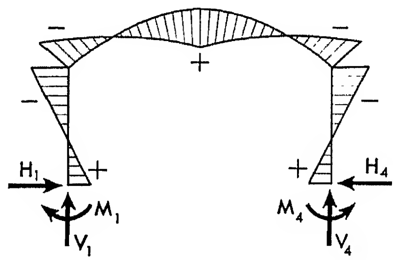
$$\left.\begin{aligned} M_x &= M_2 + \frac{WL}{16}\left[1 - \left(\frac{L-2x}{L}\right)^4\right] - H_4y_2 \\ N_x &= \frac{W}{2}\left(\frac{L-2x}{L}\right)^3 \sin\varphi + H_1\cos\varphi \\ Q_x &= \frac{W}{2}\left(\frac{L-2x}{L}\right)^3 \cos\varphi - H_1\sin\varphi \end{aligned}\right\} \qquad (12\text{-}18)$$

Apply Eqs. (12-1) and (12-3) to obtain the moment at any section of the frame columns. Moments and axial forces at corresponding sections in the right half of the arched girder are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

For Notations and Constants, see Arts. 12-1 and 12-2

12-9. Vertical Concentrated Load at Crown





$$M_2 = M_3 = -\frac{PL\phi}{F}\left(\frac{6 + 5A\psi}{4}\right)$$

$$H_1 = H_4 = \frac{PL\phi}{Fh}\left(\frac{6A + 5C\psi}{4}\right)$$

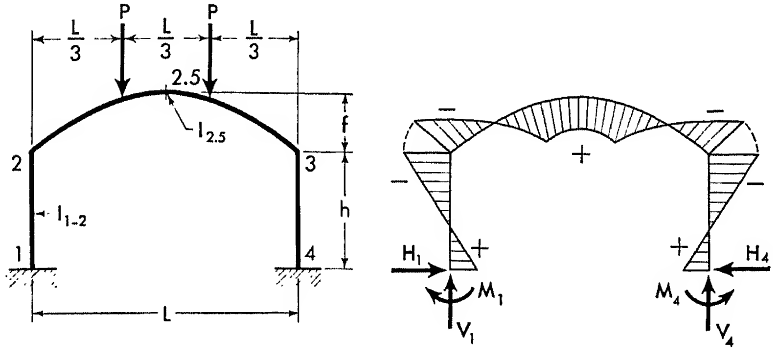
$$V_1 = V_4 = \frac{P}{2} \qquad M_1 = M_4 = M_2 + H_1 h$$

When $x \leqq \frac{L}{2}$

$$\begin{aligned} M_x &= \frac{Px}{2} + M_2 - H_1 y_2 \\ N_x &= H_1 \cos\varphi + \frac{P}{2}\sin\varphi \\ Q_x &= -H_1 \sin\varphi + \frac{P}{2}\cos\varphi \end{aligned} \tag{12-19}$$

Apply Eqs. (12-1) and (12-3) to obtain the moment at any section of the frame columns. Moments and axial forces at corresponding sections in the right half of the arched girder are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

12-10. Two Vertical Concentrated Loads on Arched Girder



$$M_2 = M_3 = -\frac{PL\phi}{F}\left(\frac{216 + 176A\psi}{81}\right)$$

$$H_1 = H_4 = \frac{PL\phi}{Fh}\left(\frac{216A + 176C\psi}{81}\right)$$

$$V_1 = V_4 = P \qquad M_1 = M_4 = M_2 + H_1 h$$

When $x \leqq \frac{L}{3}$

$$M_x = Px + M_2 - H_1 y_2$$

$$N_x = H_1 \cos \varphi + P \sin \varphi$$

$$Q_x = -H_1 \sin \varphi + P \cos \varphi$$

When $x > \frac{L}{3}$, but $\leqq \frac{L}{2}$

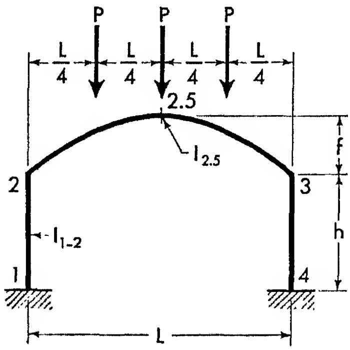
$$M_x = \frac{PL}{3} + M_2 - H_1 y_2$$

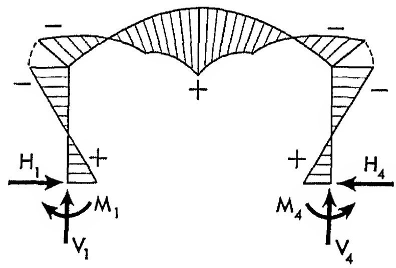
$$N_x = H_1 \cos \varphi$$

$$Q_x = -H_1 \sin \varphi$$

Apply Eqs. (12-1) and (12-3) to obtain the moment at any section of the frame columns. Moments and axial forces at corresponding sections in the right half of the arched girder are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values at those in the left half, but are of the opposite sign.

12-11. Three Vertical Concentrated Loads on Arched Girder





$$M_2 = M_3 = -\frac{PL\phi}{F}\left(\frac{120 + 97A\psi}{32}\right)$$

$$H_1 = H_4 = \frac{PL\phi}{Fh}\left(\frac{120A + 97C\psi}{32}\right)$$

$$V_1 = V_4 = \frac{3P}{2} \qquad M_1 = M_4 = M_2 + H_1h$$

When $x \leqq \frac{L}{4}$

$$M_x = \frac{3Px}{2} + M_2 - H_1y_2$$

$$N_x = H_1 \cos\varphi + \frac{3P}{2}\sin\varphi$$

$$Q_x = -H_1 \sin\varphi + \frac{3P}{2}\cos\varphi$$

When $x > \frac{L}{4}$, but $\leqq \frac{L}{2}$

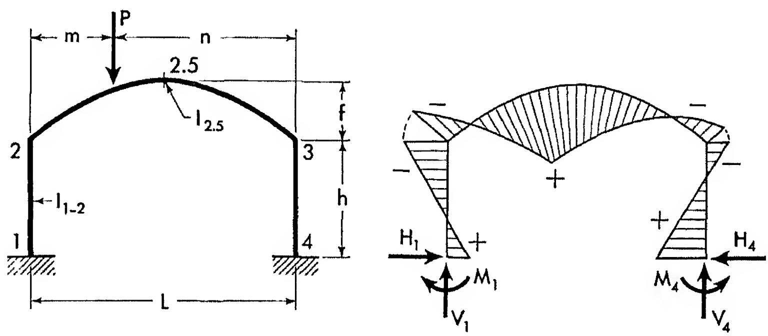
$$M_x = \frac{P(L + 2x)}{4} + M_2 - H_1y_2$$

$$N_x = H_1 \cos\varphi + \frac{P}{2}\sin\varphi$$

$$Q_x = -H_1 \sin\varphi + \frac{P}{2}\cos\varphi$$

Apply Eqs. (12-1) and (12-3) to obtain the moment at any section of the frame columns. Moments and axial forces at corresponding sections in the right half of the arched girder are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

12-12. Vertical Concentrated Load on Arched Girder



Obtain values of G, J, and K from Table 12-1.

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -PL\phi\left(\frac{G}{F} \pm \frac{J}{10D}\right)$$

$$H_1 = H_4 = \frac{PLK\phi}{Fh}$$

$$V_1 = P\left(1 - \frac{m}{L}\right) + \frac{PJ\phi}{5D} \qquad V_4 = P - V_1$$

$$M_1 = M_2 + H_1h \qquad M_4 = M_3 + H_4h$$

When $x \leqq m$

$$M_x = (Pn + M_3)\frac{x}{L} + M_2\left(1 - \frac{x}{L}\right) - H_4y_2$$

When $x > m$

$$M_x = (Pm + M_2)\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L} - H_4y_2$$

When $x \leqq m$ and $\frac{L}{2}$

$$\begin{aligned} N_x &= H_1 \cos\varphi + V_1 \sin\varphi \\ Q_x &= -H_1 \sin\varphi + V_1 \cos\varphi \end{aligned} \qquad (12\text{-}20)$$

For Notations and Constants, see Arts. 12-1 and 12-2

Table 12-1. Values of G, J, and K for Various m

Value of m	Values of G, J, and K
0	$G = 0$ $J = 0$ $K = 0$
0.1L	$G = 0.54 + 0.392A\psi$ $J = 0.72$ $K = 0.54A + 0.392C\psi$
0.2L	$G = 0.96 + 0.742A\psi$ $J = 0.96$ $K = 0.96A + 0.742C\psi$
0.3L	$G = 1.26 + 1.016A\psi$ $J = 0.84$ $K = 1.26A + 1.016C\psi$
0.4L	$G = 1.44 + 1.190A\psi$ $J = 0.48$ $K = 1.44A + 1.190C\psi$
0.5L	$G = 1.50 + 1.250A\psi$ $J = 0$ $K = 1.50A + 1.250C\psi$
0.6L	$G = 1.44 + 1.190A\psi$ $J = -0.48$ $K = 1.44A + 1.190C\psi$
0.7L	$G = 1.26 + 1.016A\psi$ $J = -0.84$ $K = 1.26A + 1.016C\psi$
0.8L	$G = 0.96 + 0.742A\psi$ $J = -0.96$ $K = 0.96A + 0.742C\psi$
0.9L	$G = 0.54 + 0.392A\psi$ $J = -0.72$ $K = 0.54A + 0.392C\psi$
1.0L	$G = 0$ $J = 0$ $K = 0$

Intermediate values may be obtained by interpolation.

When $x \leqq m$, but $\geqq \frac{L}{2}$

$$N_x = H_1 \cos \varphi - V_1 \sin \varphi \qquad (12\text{-}21)$$
$$Q_x = H_1 \sin \varphi + V_1 \cos \varphi$$

When $x \geqq m$, but $\leqq \frac{L}{2}$

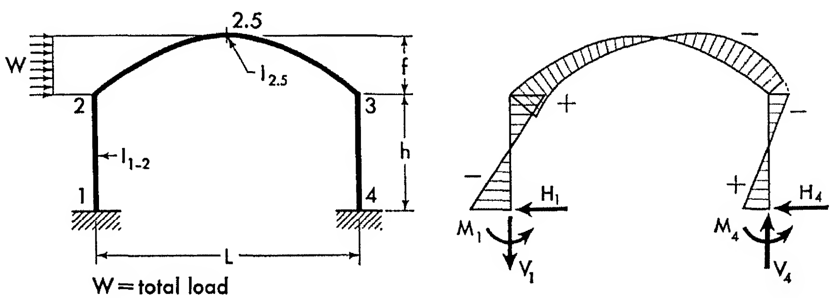
$$N_x = H_1 \cos \varphi - (P - V_1) \sin \varphi \qquad (12\text{-}22)$$
$$Q_x = - H_1 \sin \varphi - (P - V_1) \cos \varphi$$

When $x \geqq m$ and $\frac{L}{2}$

$$N_x = H_1 \cos \varphi + (P - V_1) \sin \varphi \qquad (12\text{-}23)$$
$$Q_x = H_1 \sin \varphi - (P - V_1) \cos \varphi$$

Apply Eqs. (12-1) and (12-3) to obtain the moment at any section of the frame columns.

12-13. Horizontal Uniform Load on Left Half of Arched Girder



$$G = \frac{\phi\psi}{5}\left(12 + \frac{749A\psi}{82}\right)$$

$$K = \frac{1}{5}\left(12A + \frac{749C\psi}{82}\right)$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = \frac{Wh}{F}(6 - 4A - G) \pm \frac{Wh}{D}\left(3 - \frac{\phi\psi}{4}\right)$$

$$H_4 = \frac{W}{F}(K\phi\psi + 4C - 6A)$$

$$H_1 = -(W - H_4)$$

For Notations and Constants, see Arts. 12-1 and 12-2

$$V_4 = \frac{Wh}{2DL}(12 + D\psi - \phi\psi) \qquad V_1 = -V_4$$

$$M_1 = M_2 + H_1 h$$

$$M_4 = M_3 + H_4 h$$

$$M_{2.5} = \frac{Wf}{4} + \frac{M_2 + M_3}{2} - H_4 f$$

When $x \leqq \frac{L}{2}$

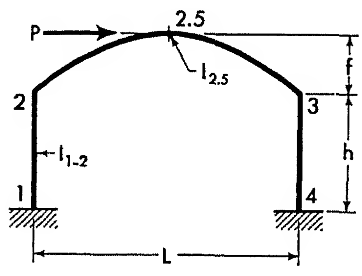
$$\left.\begin{aligned} M_x &= M_2 - V_4 x - \frac{Wy_2^2}{2f} - H_1 y_2 \\ N_x &= \left(H_1 + \frac{Wy_2}{f}\right)\cos\varphi + V_1 \sin\varphi \\ Q_x &= -\left(\frac{Wy_2}{f} + H_1\right)\sin\varphi + V_1\cos\varphi \end{aligned}\right\} \tag{12-24}$$

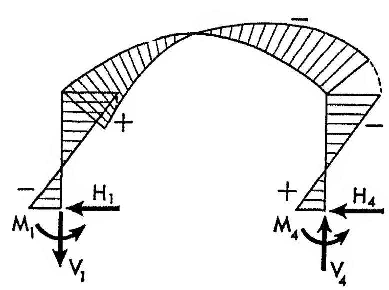
When $x > \frac{L}{2}$

$$\left.\begin{aligned} M_x &= M_3 + V_4(L - x) - H_4 y_2 \\ N_x &= (W + H_1)\cos\varphi - V_1 \sin\varphi \\ Q_x &= (W + H_1)\sin\varphi + V_1\cos\varphi \end{aligned}\right\} \tag{12-25}$$

Apply Eqs. (12-1) and (12-3) to obtain the moment at any section of the frame columns.

12-14. Horizontal Concentrated Load at Crown





$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\} = \pm\frac{Ph}{4D}(12 - \phi\psi) \qquad H_1 = -\frac{P}{2}$$

$$H_4 = \frac{P}{2} \qquad V_4 = \frac{Pf}{L} + \frac{Ph}{2DL}(12 - \phi\psi)$$

$$V_1 = -V_4 \qquad M_1 = M_2 + H_1 h$$

$$M_4 = M_3 + H_4 h \qquad M_{2.5} = 0$$

$$Q_x = \frac{P}{2}\sin\varphi + V_1\cos\varphi \tag{12-26}$$

When $x \leqq \frac{L}{2}$

$$M_x = M_2 + \frac{Py_2}{2} - V_4 x$$

$$N_x = V_1 \sin\varphi - \frac{P}{2}\cos\varphi \tag{12-27}$$

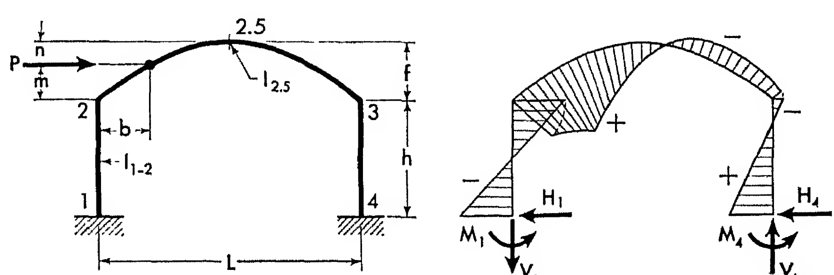
When $x > \frac{L}{2}$

$$M_x = M_3 + V_4(L - x) - \frac{Py_2}{2}$$

$$N_x = -V_1 \sin\varphi + \frac{P}{2}\cos\varphi \tag{12-28}$$

Apply Eqs. (12-1) and (12-3) to obtain the moment at any section of the frame columns.

12-15. Horizontal Concentrated Load at Any Point of Left Half of Arched Girder



Obtain values of G, J, and K from Table 12-2.

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\} = \frac{Ph}{F}(6 - 4A - G\phi\psi) \pm \frac{Ph}{D}(3 - J\phi\psi)$$

Table 12-2. Values of G, J, and K for Various m

Value of m	Values of G, J, and K
0	$G = 0$ $J = 0$ $K = 0$
0.2f	$G = 1.138 + 0.796A\psi$ $J = 0.17$ $K = 1.138A + 0.796C\psi$
0.4f	$G = 2.142 + 1.564A\psi$ $J = 0.28$ $K = 2.142A + 1.564C\psi$
0.6f	$G = 2.988 + 2.269A\psi$ $J = 0.33$ $K = 2.988A + 2.269C\psi$
0.8f	$G = 3.642 + 2.857A\psi$ $J = 0.32$ $K = 3.642A + 2.857C\psi$
1.0f	$G = 4.0 + 3.2A\psi$ $J = 0.25$ $K = 4.0A + 3.2C\psi$

Intermediate values may be obtained by interpolation.

$$H_4 = \frac{P}{F}(K\phi\psi + 4C - 6A)$$

$$H_1 = -(P - H_4)$$

$$V_4 = \frac{Pm}{L} + \frac{2Ph}{DL}(3 - J\phi\psi)$$

$$V_1 = -V_4 \qquad M_1 = M_2 + H_1h$$

$$M_4 = M_3 + H_4h$$

When $x \leqq b$

$$M_x = M_2 - V_4x - H_1y_2$$

When $x > b$

$$M_x = M_3 + V_4(L - x) - H_4y_2$$

When $x \leqq b$

$$N_x = H_1 \cos\varphi + V_1 \sin\varphi$$

$$Q_x = -H_1 \sin\varphi + V_1 \cos\varphi \qquad (12\text{-}29)$$

Members of Constant Section

When $x \geqq b$, but $< \frac{L}{2}$

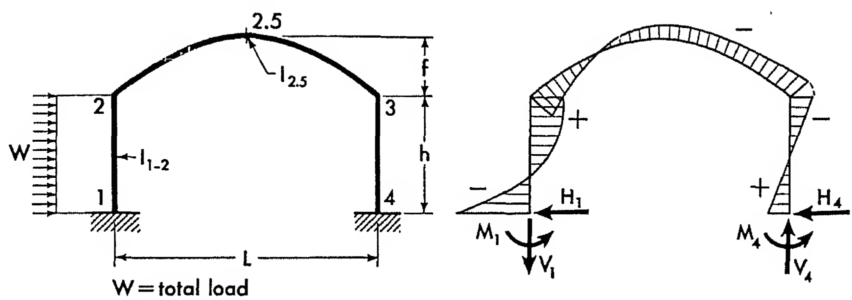
$$N_x = (P + H_1)\cos\varphi + V_1 \sin\varphi$$
$$Q_x = -(P + H_1)\sin\varphi + V_1\cos\varphi \qquad (12\text{-}30)$$

When $x > b$ and $\frac{L}{2}$

$$N_x = (P + H_1)\cos\varphi - V_1 \sin\varphi$$
$$Q_x = (P + H_1)\sin\varphi + V_1\cos\varphi \qquad (12\text{-}31)$$

Apply Eqs. (12-1) and (12-3) to obtain the moment at any section of the frame columns.

12-16. Horizontal Uniform Load on Column



$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = \frac{Wh}{2F}(4 - 3A) \pm \frac{Wh}{D}$$

$$H_4 = \frac{W}{2F}(3C - 4A) \qquad H_1 = -(W - H_4)$$

$$V_4 = \frac{2Wh}{DL} \qquad V_1 = -V_4$$

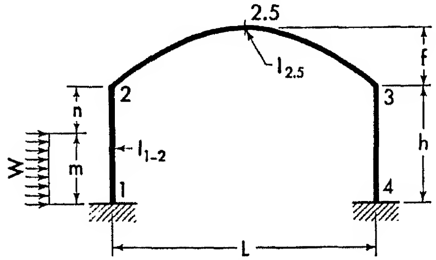
$$M_1 = M_2 + H_4 h - \frac{Wh}{2} \qquad M_4 = M_3 + H_4 h$$

$$M_{y_1} = \left(\frac{Wy_1}{2} + M_1\right)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

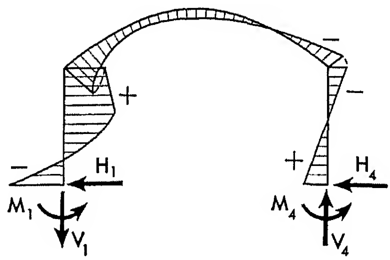
Apply Eqs. (12-2) and (12-3) to obtain the moment at any section of frame members 2-3 and 3-4, and use Eqs. (12-4) and (12-5) to determine the axial and shearing forces in the arched girder.

For Notations and Constants, see Arts. 12-1 and 12-2

12-17. Horizontal Uniform Load over Part of Column



W = total load



$$g = \left(2 - \frac{m}{h}\right)^2$$

$$G = 1 + \frac{2n}{h} - \frac{Ag}{2}$$

$$K = \frac{Cg}{2} - \frac{A(2n+h)}{h}$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = \frac{Wm}{F}(3 - 2A - G) \pm \frac{Wm^2}{Dh}$$

$$H_4 = \frac{Wm}{Fh}(2C - 3A - K) \qquad H_1 = -(W - H_4)$$

$$V_4 = \frac{2Wm^2}{DLh} \qquad V_1 = -V_4$$

$$M_1 = M_2 + H_4h - \frac{Wm}{2}$$

$$M_4 = M_3 + H_4h$$

When $y_1 \leqq m$

$$M_{y_1} = \frac{Wy_1}{2}\left(\frac{n}{h} + \frac{m - y_1}{m}\right) + M_1\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

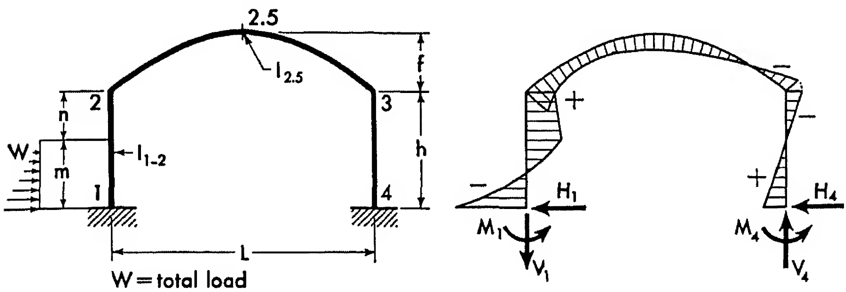
When $y_1 > m$

$$M_{y_1} = \left(\frac{Wm}{2} + M_1\right)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (12-2) and (12-3) to obtain the moment at any section of frame members 2-3 and 3-4, and use Eqs. (12-4) and (12-5) to determine the axial and shearing forces in the arched girder.

Members of Constant Section

12-18. Horizontal Triangular Load over Part of Column



$$g = \frac{4}{3} - \frac{m}{h} + \frac{m^2}{5h^2}$$

$$G = 1 + \frac{n}{h} - Ag$$

$$K = Cg - A\left(1 + \frac{n}{h}\right)$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = \frac{Wm}{3F}(6 - 4A - 3G) \pm \frac{Wm^2}{2Dh}$$

$$H_4 = \frac{Wm}{3Fh}(4C - 6A - 3K)$$

$$H_1 = -(W - H_4) \qquad V_4 = \frac{Wm^2}{DLh} \qquad V_1 = -V_4$$

$$M_1 = M_2 + H_4h - \frac{Wm}{3} \qquad M_4 = M_3 + H_4h$$

When $y_1 \leqq m$

$$M_{y_1} = \frac{Wy_1}{3}\left[\frac{n}{h} + \frac{(m - y_1)(2m - y_1)}{m^2}\right] + M_1\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

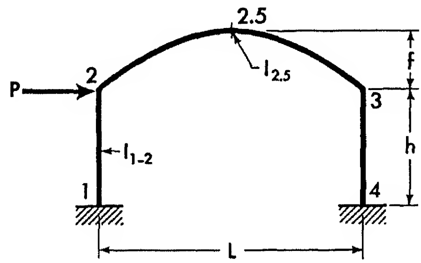
When $y_1 > m$

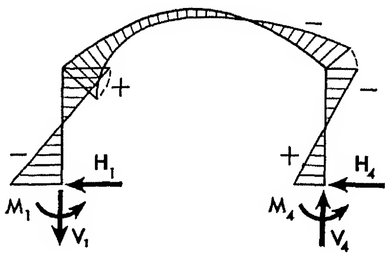
$$M_{y_1} = \left(\frac{Wm}{3} + M_1\right)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (12-2) and (12-3) to obtain the moment at any section of frame members 2-3 and 3-4, and use Eqs. (12-4) and (12-5) to determine the axial and shearing forces in the arched girder.

For Notations and Constants, see Arts. 12-1 and 12-2

12-19. Horizontal Concentrated Load at Joint 2





$$\left.\begin{matrix}M_2\\M_3\end{matrix}\right\rangle = \frac{Ph}{F}(6-4A) \pm \frac{3Ph}{D}$$

$$H_4 = \frac{2P}{F}(2C-3A) \qquad H_1 = -(P-H_4)$$

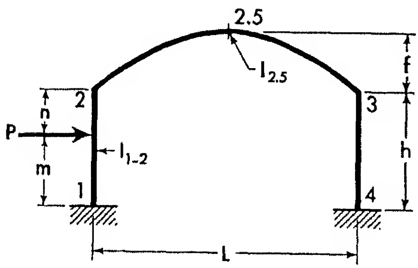
$$V_4 = \frac{6Ph}{DL} \qquad V_1 = -V_4$$

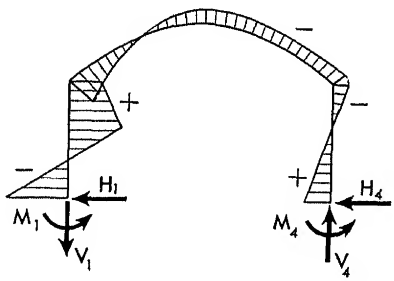
$$M_1 = M_2 - h(P-H_4) \qquad M_4 = M_3 + H_4h$$

$$M_{y_1} = M_1\left(1-\frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (12-2) and (12-3) to obtain the moment at any section of frame members 2-3 and 3-4, and use Eqs. (12-4) and (12-5) to determine the axial and shearing forces in the arched girder.

12-20. Horizontal Concentrated Load at Any Point of Column





$$g = 2 - \frac{n}{h} - \left(\frac{n}{h}\right)^2 \qquad G = \frac{6m}{h} - 2Ag$$

$$K = 2Cg - \frac{6Am}{h}$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = Pm\left(\frac{G}{F} \pm \frac{3m}{Dh}\right) \qquad H_4 = \frac{PKm}{Fh}$$

$$H_1 = -(P - H_4) \qquad V_4 = \frac{6Pm^2}{DLh}$$

$$V_1 = -V_4$$

$$M_1 = M_2 + H_4h - Pm$$

$$M_4 = M_3 + H_4h$$

When $y_1 \leqq m$

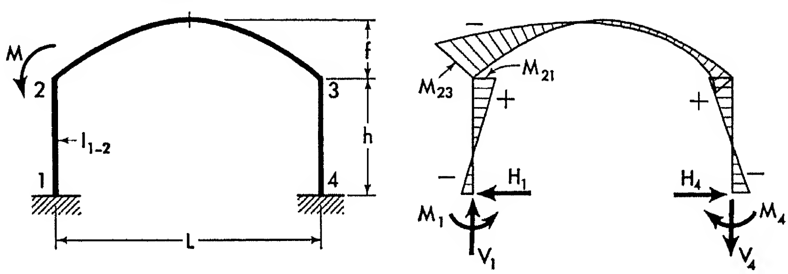
$$M_{y_1} = (Pn + M_2)\frac{y_1}{h} + M_1\left(1 - \frac{y_1}{h}\right)$$

When $y_1 > m$

$$M_{y_1} = (Pm + M_1)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (12-2) and (12-3) to obtain the moment at any section of frame members 2-3 and 3-4, and use Eqs. (12-4) and (12-5) to determine the axial and shearing forces in the arched girder.

12-21. Moment Applied at Joint 2



$$\left.\begin{matrix} M_{21} \\ M_3 \end{matrix}\right\rangle = \frac{M\phi}{F}(6 + 4A\psi) \pm \frac{M\phi}{D}$$

$$H_1 = H_4 = -\frac{M\phi}{Fh}(6A + 4C\psi)$$

$$V_1 = \frac{M}{DL}(D - 2\phi) \qquad V_4 = -V_1$$

For Notations and Constants, see Arts. 12-1 and 12-2

$$M_{23} = -(M - M_{21}) \qquad M_1 = M_{21} + H_4 h$$

$$M_4 = M_3 + H_4 h$$

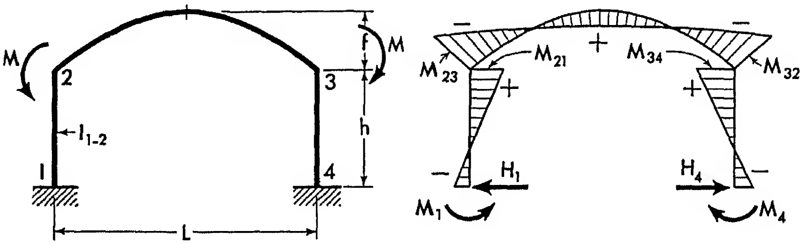
$$M_{y_1} = M_1 \left(1 - \frac{y_1}{h}\right) + M_{21} \frac{y_1}{h}$$

$$M_x = M_{23} \left(1 - \frac{x}{L}\right) + M_3 \frac{x}{L} - H_4 y_2$$

$$M_{y_4} = M_4 \left(1 - \frac{y_4}{h}\right) + M_3 \frac{y_4}{h}$$

Apply Eqs. (12-4) and (12-5) to determine the axial and shearing forces at any section of the arched girder.

12-22. Two Equal Moments Applied at Joints 2 and 3



$$M_{21} = M_{34} = \frac{4M\phi}{F}(3 + 2A\psi)$$

$$H_1 = H_4 = -\frac{4M\phi}{Fh}(3A + 2C\psi)$$

$$M_1 = M_4 = M_{21} + H_4 h$$

$$M_{23} = M_{32} = -(M - M_{21}) \qquad V_1 = V_4 = 0$$

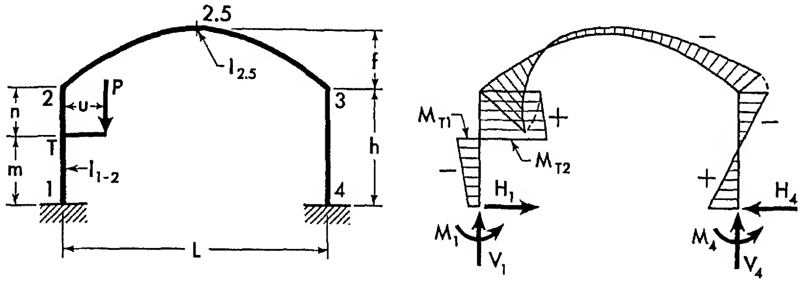
$$M_{y_1} = M_1 \left(1 - \frac{y_1}{h}\right) + M_{21} \frac{y_1}{h}$$

$$M_x = M_{23} - H_4 y_2$$

$$M_{y_4} = M_4 \left(1 - \frac{y_4}{h}\right) + M_{34} \frac{y_4}{h}$$

Apply Eqs. (12-4) and (12-5) to determine the axial and shearing forces at any section of the arched girder.

12-23. Vertical Concentrated Load Applied at Bracket



Bracket acts as a simple cantilever and its maximum moment is Pu at point T. The moment diagram of the cantilever is intentionally not shown so that the frame's bending moment diagram may be more clearly illustrated.

$$M = Pu$$

$$g = 1 + \left(\frac{m}{h}\right)^2 - \left(\frac{n}{h}\right)^2$$

$$G = 6A\left[1 - \left(\frac{n}{h}\right)^2\right] - 6g$$

$$K = 6C\left[1 - \left(\frac{n}{h}\right)^2\right] - 6Ag$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -\frac{MG}{F} \pm \frac{3Mg}{D}$$

$$H_1 = H_4 = \frac{MK}{Fh} \qquad V_4 = \frac{6Mg}{DL}$$

$$V_1 = P - V_4$$

$$M_1 = M_2 + H_4h - M$$

$$M_4 = M_3 + H_4h$$

When $y_1 < m$

$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) - (M - M_2)\frac{y_1}{h}$$

When $y_1 = m$

$$M_{T1} = M_1\frac{n}{h} - (M - M_2)\frac{m}{h} \quad \text{and} \quad M_{T2} = (M + M_1)\frac{n}{h} + M_2\frac{m}{h}$$

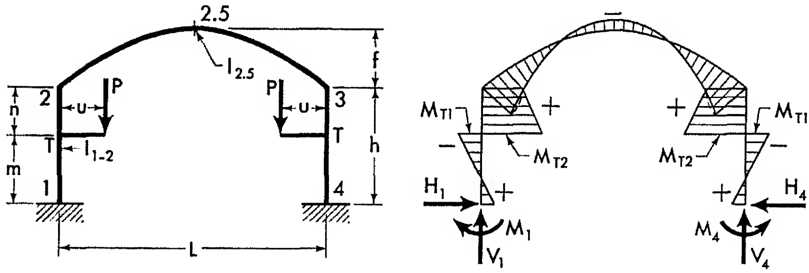
For Notations and Constants, see Arts. 12-1 and 12-2

When $y_1 > m$

$$M_{y_1} = (M + M_1)\left(1 - \frac{y_1}{h}\right) + M_2 \frac{y_1}{h}$$

Apply Eqs. (12-2) and (12-3) to obtain the moment at any section of frame members 2-3 and 3-4, and use Eqs. (12-4) and (12-5) to determine the axial and shearing forces in the arched girder.

12-24. Two Equal Vertical Concentrated Loads Symmetrically Applied at Brackets



Brackets act as simple cantilevers with the maximum moments of Pu at points T. The moment diagrams of these cantilevers are intentionally not shown so that the frame's bending moment diagram may be more clearly illustrated.

$$M = Pu$$

$$g = 1 + \left(\frac{m}{h}\right)^2 - \left(\frac{n}{h}\right)^2$$

$$G = 6A\left[1 - \left(\frac{n}{h}\right)^2\right] - 6g$$

$$K = 6C\left[1 - \left(\frac{n}{h}\right)^2\right] - 6Ag$$

$$M_2 = M_3 = -\frac{2MG}{F} \qquad H_1 = H_4 = \frac{2MK}{Fh}$$

$$V_1 = V_4 = P$$

$$M_1 = M_4 = M_2 + H_4h - M$$

When $x \leqq \frac{L}{2}$

$$M_x = M_2 - H_4y_2$$

Members of Constant Section

When $y_1 < m$

$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) - (M - M_2)\frac{y_1}{h}$$

When $y_1 = m$

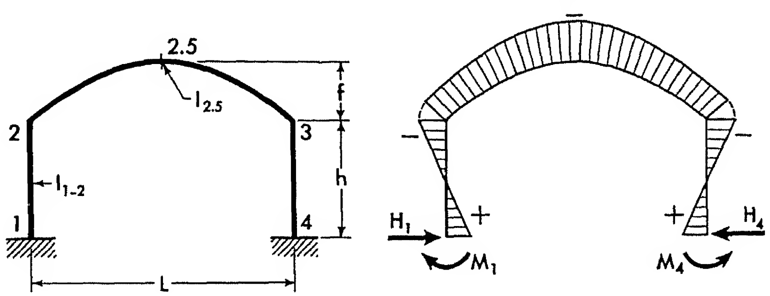
$$M_{T1} = M_1\frac{n}{h} - (M - M_2)\frac{m}{h} \qquad \text{and} \qquad M_{T2} = (M + M_1)\frac{n}{h} + M_2\frac{m}{h}$$

When $y_1 > m$

$$M_{y_1} = (M + M_1)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (12-4) and (12-5) to determine the axial and shearing forces in the arched girder. Moments at corresponding sections in the right half of the frame are identical to those in the left half.

12-25. Effect of Temperature Rise. Range t° for entire frame.



$$K = \frac{12L\varepsilon t^\circ}{Fh^2} EI_{1\text{-}2}$$

$$M_2 = M_3 = -AK$$

$$M_1 = M_4 = K(C - A)$$

$$H_1 = H_4 = \frac{CK}{h} \qquad V_1 = V_4 = 0$$

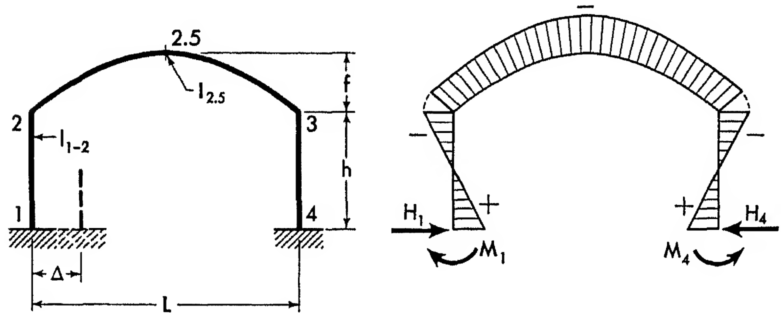
$$M_x = M_2 - H_4 y_2 \qquad M_{2.5} = M_2 - H_4 f$$

Apply Eqs. (12-1) and (12-3) to obtain the moment at any section of the frame columns, and use Eqs. (12-4) and (12-5) to determine the axial and shearing forces in the arched girder.

Note: For temperature drop, introduce the value of t° with a negative sign.

For Notations and Constants, see Arts. 12-1 and 12-2

12-26. Horizontal Displacement of One Support



$$K = \frac{12\Delta}{Fh^2} EI_{1-2}$$

$$M_2 = M_3 = -AK$$

$$M_1 = M_4 = K(C - A)$$

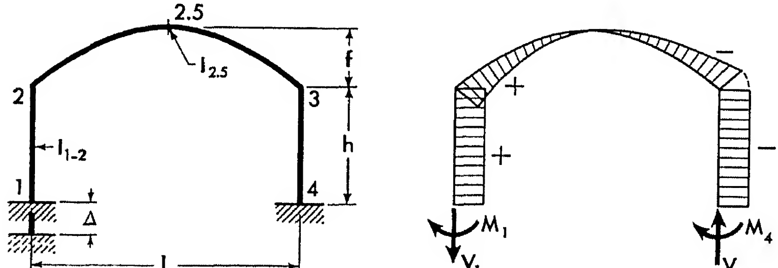
$$H_1 = H_4 = \frac{CK}{h} \qquad V_1 = V_4 = 0$$

$$M_x = M_2 - H_4 y_2 \qquad M_{2.5} = M_2 - H_4 f$$

Apply Eqs. (12-1) and (12-3) to obtain the moment at any section of the frame columns, and use Eqs. (12-4) and (12-5) to determine the axial and shearing forces in the arched member.

Note: If the direction of the frame displacement is opposite to that shown in the sketch, introduce the value of Δ with a negative sign.

12-27. Vertical Settlement of One Support



$$K = \frac{12\Delta}{DLh} EI_{1-2}$$

$$M_1 = M_2 = K \qquad M_3 = M_4 = -K$$

$M_{2.5} = 0 \qquad H_1 = H_4 = 0$

$$V_1 = -\frac{2K}{L} \qquad V_4 = -V_1$$

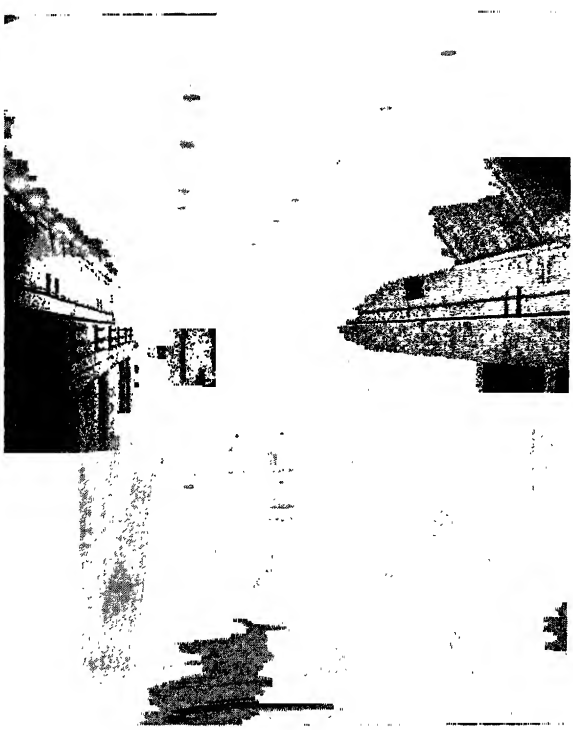
$M_x = M_2 + V_1 x \qquad M_{y_1} = M_1 \qquad M_{y_4} = M_4$

Apply Eqs. (12-4) and (12-5) to determine the axial and shearing forces in the arched member.

Note: If the direction of the frame displacement is opposite to that shown in the sketch, introduce the value of Δ with a negative sign.

PART TWO

FRAMES AND ARCHES WITH MEMBERS OF VARIABLE CROSS SECTION



Gigantic underground chamber of the Kemano Powerhouse built in British Columbia, Canada, for the Aluminum Company of Canada, Ltd. This cavern, the largest man-made underground chamber in the world, shelters a huge powerhouse situated on four levels. The chamber is lined with arched frames 139 feet high and 82 feet wide. The photograph illustrates the initial stage of the powerhouse construction. Until the four floors were constructed, the arched frames alone held the enormous load exerted by shattered and dislocated rocks. The frames demonstrate exceptional functional utility in this application. Designed by the International Engineering Company of Canada, subsidiary of Morrison-Knudsen Co. (Courtesy of Aluminum Company of Canada and International Engineering Co., San Francisco, Calif.)

SECTION 13

INTRODUCTION TO ANALYSIS OF FRAMES WITH STRAIGHT MEMBERS

13-1. General. The condensed solutions of structural analysis given in this part of the text are based on the author's concept of elastic parameters [1] and afford the opportunity of performing mechanically the analysis of frames with members of variable cross section. This analysis is called mechanical because it is confined to algebraic operations and relieves the designer of the necessity of having advanced knowledge of involved methods of frame analysis.

All equations have been formulated in general terms and are applicable to symmetrical frames with members of various shapes and proportions. Expressions for forces and moments produced by important vertical and horizontal loads are given in the text. In addition, forces and moments produced by impressed distortions, such as settlement or displacement of the support, or applied moment, are also provided.

The presentation of analysis in a generalized form, applicable to the members of various shapes and proportions, creates insurmountable difficulties in respect to diagrammatic representation of moment curves. The shape and the moment of inertia of individual members have a pronounced influence on the magnitude of redundants, and therefore the bending moments of the frame may vary over a wide range in magnitude and direction. Since it is impossible to outline even representative bending moment diagrams for the considered types of frames and arches, such diagrams have been omitted entirely from this part of the text.

[1] See author's paper "Concept of Elastic Parameters," *Proc. Am. Concrete Inst.*, vol. 54, pp. 987–1008, 1958.

13-2. Axes of Members. No definition of the longitudinal axes for members of variable cross section, with due regard to practical application, has yet gained unanimous acceptance among structural engineers.

However, the Portland Cement Association, as well as other recognized authorities in the field,[1] recommends that the longitudinal axis of a straight member with a variable cross section be taken as the line parallel to the straight edge of the member and passing through the center of gravity of the smallest cross section. This definition is adopted for the present text.

13-3. Frame Members. The analysis of the frame is predicated on the use of the physical and elastic properties of the individual members of the frame. The first step of analysis, therefore, is the reduction of the frame to its constituent members. The lengths of these members are defined by considering the distances between the intersections of the axes to be the lengths of the members. The shapes of the members are defined by extending the haunches to the lines drawn normally to the axes of the members through the above-mentioned points of intersections. The application of this rule is shown in detail in Fig. 13-1, where the reduction of a gable frame to individual members is demonstrated.

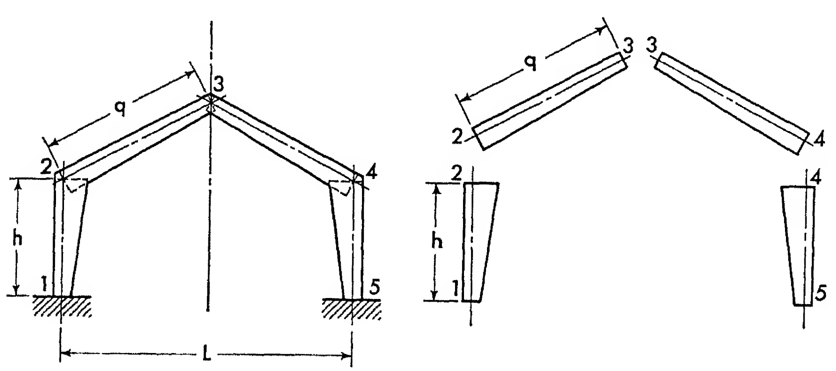


FIG. 13-1. Reduction of the frame to elementary members

Since frames symmetrical about their vertical center lines are considered in this text, only properties of the girder and one column need to be used in the analysis of a three-member frame. Similarly, the properties of only one half of the frame need to be considered in the analysis of a four-member frame.

[1] See, for example, Richard Gulden, *Rahmentragwerke und Durchlaufträger*, Springer-Verlag OHG, Vienna, 1943.

13-4. Elastic Parameters and Load Constants. To develop the analysis of frames with members of variable cross section in a general form applicable to various types and shapes of members, the elastic properties of the members must be defined in a suitable manner and introduced into the analysis.

To allow for various loadings on the frames, the properties of the moment areas for various loads must also be defined in a suitable form and introduced into the analysis.

The elastic properties of the straight member of variable cross section are defined by the three parameters α_n, α_m, and β_n, which completely characterize all elastic properties of the member.[1] The numerical values of these parameters for a variety of members are given, in graphical form, in the Appendix.

The properties of the moment area of the loaded member with variable cross section are defined by the two load constants R_n and R_m. The numerical values of these load constants for a number of principal loadings are also given in the Appendix.

The numerical values of the parameters are dependent only upon the shape of the member, while the values of the load constants are dependent upon the shape of the member and the manner of loading. The values of the parameters and the load constants are equally applicable to members having either hinged or fixed supports.

It is imperative that the designer should not err in selection of the numerical values of elastic parameters and load constants or in the performance of calculations. To assist him, the terminology for charts and tables has been carefully selected to minimize the chances for error. When a straight nonsymmetrical member is considered separately from the frame for the purpose of selecting the numerical values of the elastic parameters or load constants from the tables and charts, distinction between the ends is made by use of the terms *large end* and *small end.* Accordingly, all values in the tables and charts are given for the large and small ends of the member.

After selecting the numerical values of the parameters or load constants, proper numerical subscripts should be assigned to the symbols expressing them. Since the haunched member is oriented in the structure in a definite manner the subscripts should identify the large or small end as it is framed in the structure. For this purpose, the subscripts are formed in cross-reference manner, employing the joints' numerals which appear at the ends of the member.

[1] See author's development, *op. cit.*, p. 221.

Thus, for example, with reference to member 2-3, shown in Fig. 13-1, the elastic parameters α and β, being referred to the large end of the member, are denoted as α_{23} and β_{23}. The same parameters, being referred to the small end of the member, are denoted as α_{32} and β_{32}. The same rule applies to the load constants.

13-5. Condensed Solutions of Analysis. After the numerical values of the elastic parameters and load constants have been found, the redundant quantities of the frame may be readily calculated, employing the equations of the condensed solutions of analysis. In performing these calculations only simple algebraic operations are required. No solutions of simultaneous equations are needed and no balancing operations are required. The results are obtained directly and constitute the redundant quantities of the frame.

Having determined the redundant quantities of the frame, the moment and axial and shearing forces at any section of the frame may be obtained without difficulty. As additional aid, the condensed solutions also provide the expressions for the moment at any section of the frame.

13-6. Frames with Inclined Members. The equations given in Sections 16 through 19 pertain to frames with inclined members subjected to vertical or horizontal load applied to these members. The values of the load constants for these loading conditions may be obtained directly from the same tables and charts used for horizontal and vertical members. Similarly, for any orientation of the member, the appropriate values of the elastic parameters may be selected directly from the same tables and charts used for horizontal and vertical members, and may be applied, without modification, in the equations. This holds true since the equations in this text have been developed to permit the use of standard values of load constants and elastic parameters for the inclined members.[1] The practical application of the described procedure is illustrated in Examples 13-2 and 13-7.

13-7. Determination of Uncommon Elastic Constants. With this text, the analysis of frames with members of variable cross section becomes relatively simple, provided that numerical values of elastic constants are available. Occasionally, it is necessary to analyze a frame with members of uncommon shape. Since the variation of members is limitless, it is evident that tables of constants can by no means cover all possible variations.

[1] See author's development, *op. cit.*, p. 221.

Furthermore, derivation of new constants by the method currently in use is tedious and time consuming.

The problem of obtaining new constants is substantially reduced when the author's system of elastic parameters and load constants is used because the principle of superposition may be applied. The procedure will be demonstrated in Example 13-7.

ILLUSTRATIVE EXAMPLES

Example 13-1. A rigid portal frame of the shape and dimensions shown in Fig. 13-2 carries a 12-kip vertical concentrated load on the girder. Assuming the width of the members normal to the plane of the drawing is 1.5 ft, determine the magnitude of the bending moments at joints 2 and 3.

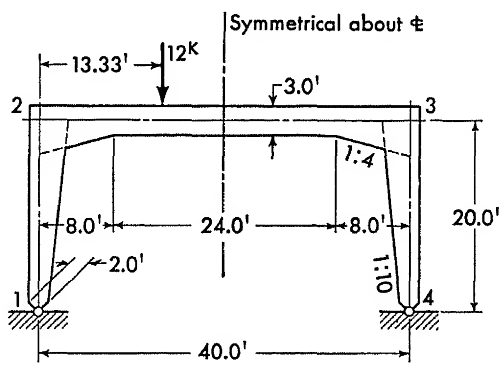


FIG. 13-2. Portal frame with applied vertical load

The first step of the analysis is the assignment of the numerals 1, 2, 3, and 4 to the frame joints, beginning at the left support. The frame should then be reduced to its elementary members, using the procedure described in Art. 13-3. These isolated elementary members are shown in Fig. 13-3.

The elastic parameters are next to be evaluated, using the charts of the Appendix; and because of the symmetry of the frame, only the parameters for members 1-2 and 2-3 need be determined. In conformity with the notation used in the charts, the length of the haunched part of member 1-2 is denoted as l_h and its entire length is termed l; the ratio of these lengths, v, is seen to be unity:

$$v = \frac{l_h}{l} = \frac{20}{20} = 1$$

The term, t, representing the cube of the ratio of the member's minimum to maximum depth, is

$$t = \left(\frac{\text{min d}}{\text{max d}}\right)^3 = \left(\frac{2}{4}\right)^3 = 0.125$$

Entering Chart 6 with the values of v and t as determined above, the value of the elastic parameter α at the large end of the member is found to be 0.82. The large end of member 1-2 is defined in the established system of frame notation by the subscript 21; therefore $\alpha_{21} = 0.82$.

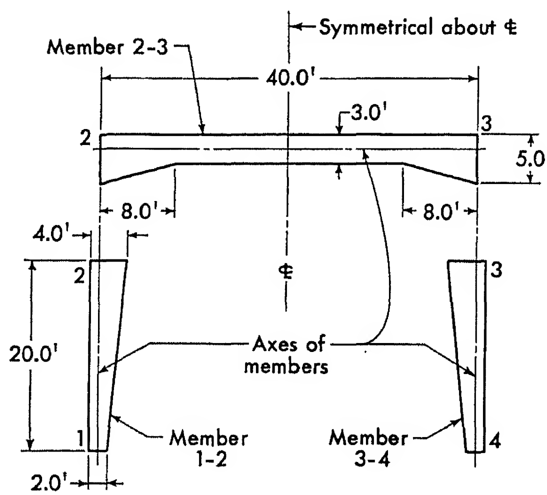


FIG. 13-3. Shape and dimensions of the elementary members of the frame

Using the same procedure for member 2-3, its elastic parameters may be evaluated. The quantities v and t are

$$v = \frac{l_h}{l} = \frac{8}{40} = 0.2$$

and

$$t = \left(\frac{\text{min d}}{\text{max d}}\right)^3 = \left(\frac{3}{5}\right)^3 = 0.216$$

For these values, the following parameters are determined from Charts 1 and 2 in the Appendix:

$$\alpha_{23} = \alpha_{32} = 2.92 \qquad \beta_{23} = \beta_{32} = 1.82$$

Note that $\alpha_{23} = \alpha_{32}$ because of the symmetry of member 2-3 about the center line of the frame.

The load constants for the loaded member are to be determined in the next step. Tables 1 and 2 of the Appendix provide values of load constants for members of variable cross section for the unit concentrated load on the member. For a load applied at the third-point of the span, measured from the left support and for $v = 0.2$ and $t = 0.216$, the following numerical values of load constants are obtained by interpolation:

For the left end, $R = 0.680 \therefore R_{23} = 0.680$

For the right end, $R = 0.557 \therefore R_{32} = 0.557$

This point marks the completion of compilation of preliminary data and the beginning of the performance of the solution of structural analysis. With the aid of the condensed solutions of analysis given in this text the ordinary involved analysis reduces to simple algebraic computations.

Observing that the minimum moments of inertia of members 1-2 and 2-3 are

$$\min I_{1\text{-}2} = \frac{2^3 \times 1.5}{12} = 1$$

and

$$\min I_{2\text{-}3} = \frac{3^3 \times 1.5}{12} = 3.375$$

the frame constants ϕ, Θ_{23}, and A may be determined by the use of the equations given in Art. 14-1; they are

$$\phi = \frac{\min I_{1\text{-}2}}{\min I_{2\text{-}3}} \cdot \frac{L}{h} = \frac{1}{3.375} \cdot \frac{40}{20} = 0.593$$

$$\Theta_{23} = 2(\alpha_{23} + \beta_{23}) = 2(2.92 + 1.82) = 9.48$$

and

$$A = \Theta_{23} + \frac{2\alpha_{21}}{\phi} = 9.48 + \frac{2 \times 0.82}{0.593} = 12.25$$

Inserting the numerical values of frame and load constants into equations of the condensed solution given in Art. 14-5, the term K and the horizontal and vertical components of frame reactions are

$$K = \frac{L}{A}(R_{23} + R_{32}) = \frac{40}{12.25}(0.680 + 0.557) = 4.04$$

$$H_1 = H_4 = \frac{PK}{h} = \frac{12 \times 4.04}{20} = 2.42 \text{ kip}$$

$$V_1 = P\left(1 - \frac{m}{L}\right) = 12\left(1 - \frac{13.33}{40}\right) = 8 \text{ kip}$$

$$V_4 = P - V_1 = 12 - 8 = 4 \text{ kip}$$

And, finally, the moments at joints 2 and 3 may be determined. They are

$$M_2 = M_3 = -PK = -(12 \times 4.04) = -48.48 \text{ ft-kip}$$

Example 13-2. Determine the magnitude of the bending moments at the principal sections of the two-hinged trapezoidal frame subjected to the action of a 20-kip concentrated load shown in Fig. 13-4a. The dimension of all members normal to the plane of drawing is 1.5 ft.

To illustrate the expediency and brevity of the condensed analysis in application to frames with inclined members, the shapes and dimensions of individual members of the above frame are selected to correspond to those of the frame of Example 13-1. Consequently, the elastic parameters and load constants of the members, being dependent upon the shape of the members only, may be taken directly from Example 13-1, and only the solution of the problem need be presented.

(It should be restated that condensed solutions of analysis given in Sections 13 through 24 of this text are applicable to members of any shape. The shape of members shown on sketches is arbitrary and for illustrative purposes only.)

The numerical values of elastic parameters and load constants, as determined in Example 13-1, are as follows:

$$\alpha_{21} = 0.82 \qquad \alpha_{23} = 2.92 \qquad \beta_{23} = 1.82$$

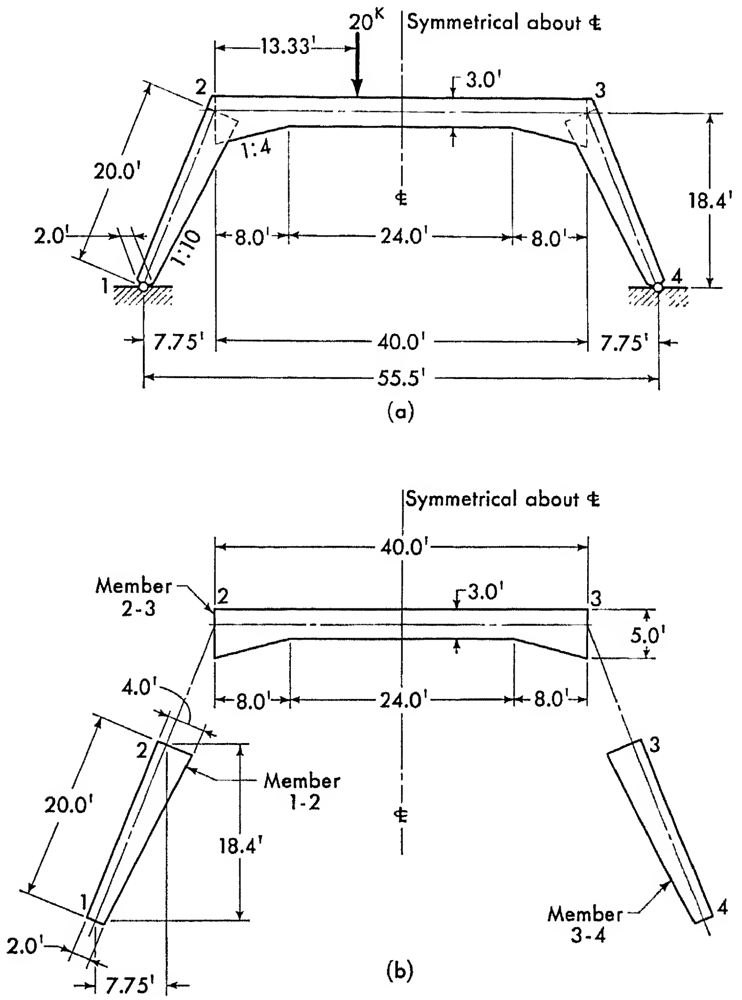
$$R_{23} = 0.680 \qquad \text{and} \qquad R_{32} = 0.557$$

Again, applying the equations of condensed solution of analysis given in Arts. 16-1 and 16-5, the solution of the problem is readily obtained.

Observing that $\min I_{1\text{-}2} = 1 \text{ ft}^4$ and $\min I_{2\text{-}3} = 3.375 \text{ ft}^4$, the frame constants ϕ, Θ_{23}, and A may be determined by the use of equations given in Art. 16-1; they are

$$\phi = \frac{\min I_{1\text{-}2}}{\min I_{2\text{-}3}} \cdot \frac{b}{q} = \frac{1}{3.375} \cdot \frac{40}{20} = 0.593$$

$$\Theta_{23} = 2(\alpha_{23} + \beta_{23}) = 2(2.92 + 1.82) = 9.48$$



(a) Frame with applied load (b) Elementary members of the frame

FIG. 13-4. Analysis of trapezoidal frame

$$A = \Theta_{23} + \frac{2\alpha_{21}}{\phi} = 9.48 + \frac{2 \times 0.82}{0.593} = 12.25$$

Inserting the numerical values of frame and load constants into equations of the condensed solution given in Art. 16-5, the terms J and K and the horizontal and vertical components of frame reactions are

$$J = \frac{b}{A}(R_{23} + R_{32}) = \frac{40}{12.25}(0.680 + 0.557) = 4.04$$

$$K = \frac{b - 2m}{2L} = \frac{40 - (2 \times 13.33)}{2 \times 55.5} = 0.12$$

$$H_1 = H_4 = \frac{P}{2h}(2J + a) = \frac{20}{2 \times 18.4}(2 \times 4.04 + 7.75) = 8.6 \text{ kip}$$

$$V_1 = \frac{P}{2}(1 + 2K) = \frac{20}{2}(1 + 2 \times 0.12) = 12.4 \text{ kip}$$

$$V_4 = \frac{P}{2}(1 - 2K) = \frac{20}{2}(1 - 2 \times 0.12) = 7.6 \text{ kip}$$

And, finally, the moments at joints 2 and 3 may be determined. They are

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -P(J \mp Ka) = -20(4.04 \mp 0.12 \times 7.75) = \begin{matrix} -62.2 \text{ ft-kip} \\ -99.4 \text{ ft-kip} \end{matrix}$$

The results of the analysis are shown in a diagrammatic form in Fig. 13-5.

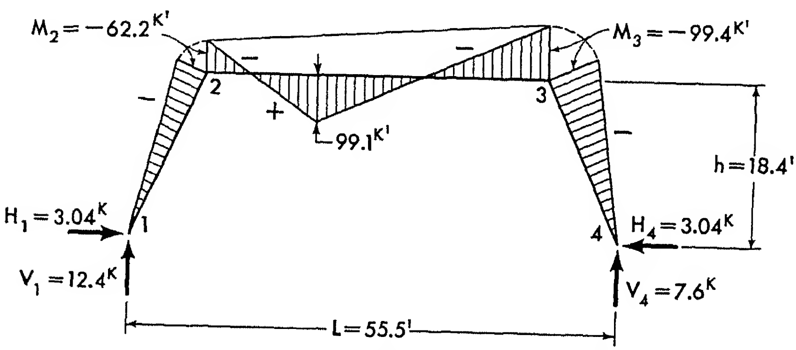


FIG. 13-5. Bending moment diagram and frame reactions

Example 13-3. A rigid portal frame of the shape and dimensions shown in Fig. 13-6 carries a vertical uniform load of 2 kip/ft. Assuming the width of the members normal to the plane of drawing is 1.5 ft, find the magnitude of bending moments of the frame.

Observing that the shape and dimensions of the members of this frame are the same as those of the frame considered in Example 13-1, the geometric constants of member 1-2 are

$v = 1$ and $t = 0.125$

and the value of the elastic parameter α for the small end of the member is found from Chart 5 to be 2.32. Since the small end of the member is defined by subscript 12, α_{12} then is 2.32. The value of the elastic parameter α_{21} for the large end of the member is found from Chart 6 to be 0.82. Similarly, β_{12} is determined from Chart 7 as 0.69.

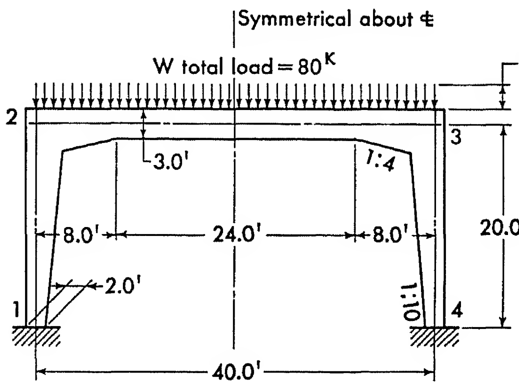


FIG. 13-6. Portal frame with applied vertical uniform load

Applying the same reasoning, the geometric constants of member 2-3 are

$v = 0.2$ and $t = 0.216$

and the values of the elastic parameters are found from Charts 1 and 2 as follows:

$\alpha_{23} = \alpha_{32} = 2.92$ and $\beta_{23} = 1.82$

The load constants should be determined, in the next step, for the given load on the girder. Since the member 2-3 and its loading are symmetrical about the member's center line, the load constants are identical for both ends of the member. For $v = 0.2$ and $t = 0.216$ the values of load constants are found from Chart 11 as

$R_{23} = R_{32} = 0.457$

From this point on, the solution of the problem is readily obtained by the use of the condensed solution of analysis given in Section 15.

Observing that min $I_{1\text{-}2} = 1\ \text{ft}^4$, and min $I_{2\text{-}3} = 3.375\ \text{ft}^4$, the values Θ_{12} and Θ_{23} and frame constants ϕ, A, B, D, and F may be determined by the use of the equations given in Art. 15-1; they are

$$\Theta_{12} = \alpha_{12} + \alpha_{21} + 2\beta_{12} = 2.32 + 0.82 + 2 \times 0.69 = 4.52$$

$$\Theta_{23} = 2(\alpha_{23} + \beta_{23}) = 2(2.92 + 1.82) = 9.48$$

$$\phi = \frac{\min I_{1\text{-}2}}{\min I_{2\text{-}3}} \cdot \frac{L}{h} = \frac{1}{3.375} \cdot \frac{40}{20} = 0.593$$

$$A = \frac{\beta_{12}}{\alpha_{12}} = \frac{0.69}{2.32} = 0.297$$

$$B = 2\Theta_{12} + \Theta_{23}\phi = 2 \times 4.52 + 9.48 \times 0.593 = 14.66$$

$$D = \frac{B}{2} - 2\phi\beta_{23} = \frac{14.66}{2} - 2 \times 0.593 \times 1.82 = 5.17$$

$$F = B - \frac{2(\alpha_{12} + \beta_{12})^2}{\alpha_{12}} = 14.66 - \frac{2(2.32 + 0.69)^2}{2.32} = 6.85$$

Inserting the numerical values of frame and load constants into the equations of the condensed solution given in Art. 15-3, the term K and the horizontal and vertical components of frame redundant reactions are

$$K = \frac{2L\phi R_{23}}{F} = \frac{2 \times 40 \times 0.593 \times 0.457}{6.85} = 3.16$$

$$H_1 = H_4 = \frac{WK\,(1 + A)}{h} = \frac{80 \times 3.16\,(1 + 0.297)}{20} = 16.39\ \text{kip}$$

$$V_1 = V_4 = \frac{W}{2} = \frac{80}{2} = 40\ \text{kip}$$

And finally, the moments at the joints of the frame are

$$M_2 = M_3 = -\,WK = -\,(80 \times 3.16) = -\,252.8\ \text{ft-kip}$$

$$M_1 = M_4 = WAK = 80 \times 0.297 \times 3.16 = 75.08\ \text{ft-kip}$$

Having determined the numerical values of redundant moments and forces, the bending moment at any section of the frame may be found without difficulty. For example, the bending moment in the girder, 10 ft from the left support, is

$$M_x = \frac{Wx}{2}\left(1 - \frac{x}{L}\right) + M_2 = \frac{80 \times 10}{2}\left(1 - \frac{10}{40}\right) + (-\,252.8) = 47.2\ \text{ft-kip}$$

Similarly, the bending moment in the left column, 8 ft above the joint 1, is

$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

$$= 75.08\left(1 - \frac{8}{20}\right) + (-252.8)\frac{8}{20} = -56.08 \text{ ft-kip}$$

The results of the analysis are given in Fig. 13-7.

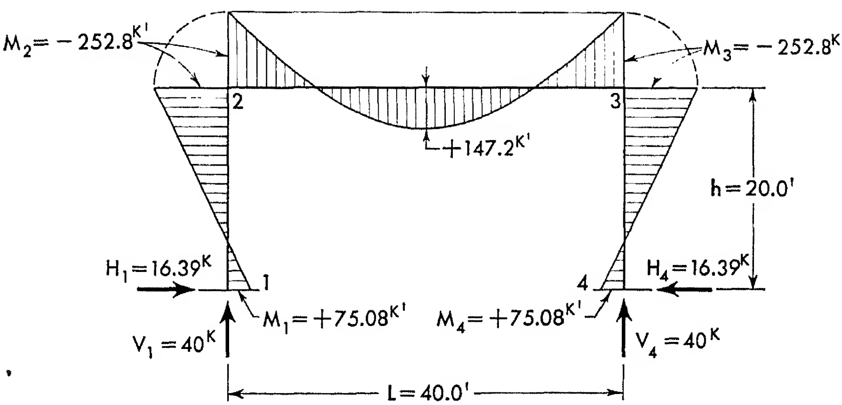


FIG. 13-7. Bending moment diagram and frame reactions

Static Check. It is always advisable to make a static check as a precautionary measure against numerical error. With reference to Fig. 13-8, the magnitude of the bending moment at joint 2 may be found by

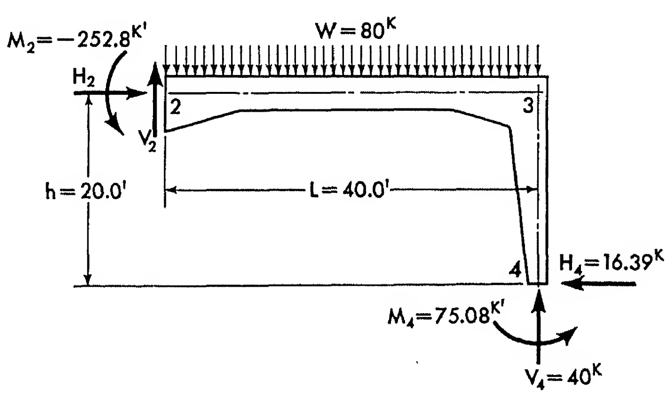


FIG. 13-8. Free body diagram

applying the principle of equilibrium to a free body comprised of members 2-3 and 3-4.

The equation of the sum of moments about joint 2 may be written as

$$M_2 = M_4 + V_4L - H_4h - \frac{WL}{2}$$

and substituting numerical values

$$M_2 = 75.08 + 40 \times 40 - 16.39 \times 20 - \frac{80 \times 40}{2} = -252.7 \text{ ft-kip}$$

The resulting value of M_2 is in good agreement with that previously computed (−252.8 ft-kip).

Example 13-4. Find the redundant quantities of the rigid portal frame with dimensions shown in Fig. 13-9, carrying a 20-kip vertical concentrated load on the girder. The width of all members normal to the plane of the drawing is 1.5 ft.

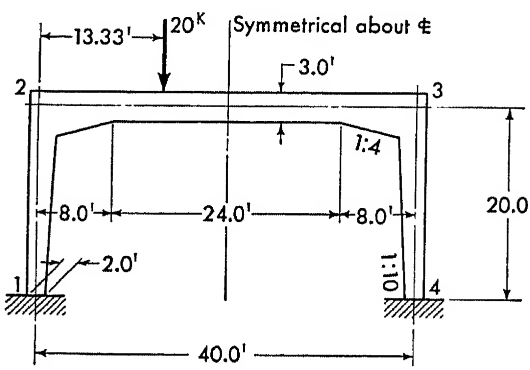


FIG. 13-9. Portal frame with applied vertical concentrated load

Since the frame of this example is identical to the frame analyzed in Example 13-3, the numerical values of the elastic parameters and frame constants are the same as determined in Example 13-3. They are

$\Theta_{12} = 4.52$ $\quad\Theta_{23} = 9.48$ $\quad\phi = 0.593$

$A = 0.297$ $\quad B = 14.66$ $\quad D = 5.17$

$F = 6.85$

The numerical values of the load constants for the concentrated load on the girder may be obtained from Tables 1 and 2, in the Appendix, using the geometric constants of the girder. For $v = 0.2$ and $t = 0.216$, the

values of these load constants, being determined by the interpolation, are as follows:

For the left end $R = 0.680 \therefore R_{23} = 0.680$

For the right end $R = 0.557 \therefore R_{32} = 0.557$

Applying equations of the condensed solution of analysis given in Art. 15-5, the terms J and K and the components of redundant reactions are readily obtained.

$$J = \frac{L\phi}{F}(R_{23} + R_{32}) = \frac{40 \times 0.593}{6.85}(0.680 + 0.557) = 4.29$$

$$K = \frac{L\phi}{2D}(R_{23} - R_{32}) = \frac{40 \times 0.593}{2 \times 5.17}(0.680 - 0.557) = 0.28$$

$$H_1 = H_4 = \frac{PJ}{h}(1 + A) = \frac{20 \times 4.29}{20}(1 + 0.297) = 5.56 \text{ kip}$$

$$V_1 = P\left(1 - \frac{m}{L}\right) + \frac{2PK}{L} = 20\left(1 - \frac{13.33}{40}\right) + \frac{2 \times 20 \times 0.28}{40} = 13.61 \text{ kip}$$

$$V_4 = P - V_1 = 20 - 13.61 = 6.39 \text{ kip}$$

Likewise, the moments at the joints of the frame are

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -P(J \pm K) = -20(4.29 \pm 0.28) = \begin{matrix} -91.4 \text{ ft-kip} \\ -80.2 \text{ ft-kip} \end{matrix}$$

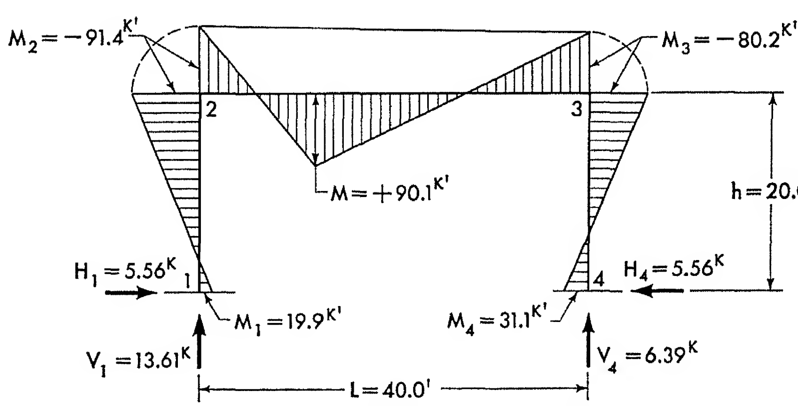


FIG. 13-10. Bending moment diagram and frame reactions

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = P\,(AJ \mp K) = 20\,(0.297 \times 4.29 \mp 0.28) = \begin{matrix} 19.9 \text{ ft-kip} \\ 31.1 \text{ ft-kip} \end{matrix}$$

In Fig. 13-10, the results of the analysis are given.

Static Check. As mentioned previously, it is advisable to check the results of the analysis by considering a part of the frame in equilibrium. With reference to Fig. 13-11, the moment at joint 2 is

$$M_2 = 31.1 + 6.39 \times 40 - 5.56 \times 20 - 20 \times 13.33 = -91.2 \text{ ft-kip}$$

The check shows a good agreement with the previously computed value (−91.4 ft-kip).

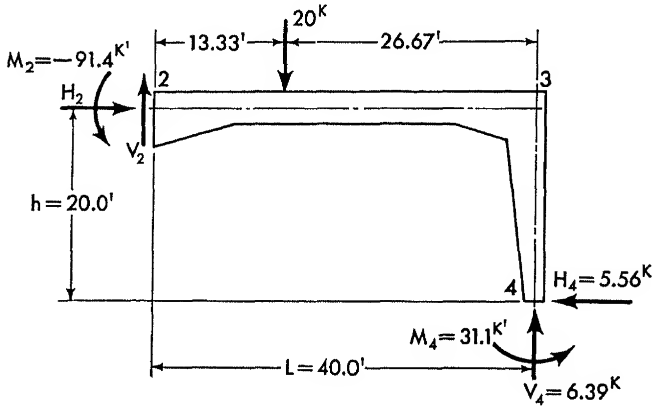


FIG. 13-11. Free body diagram

Example 13-5. Find the numerical values of the redundant reactions and moments of the loaded frame shown in Fig. 13-12.

Since the frame is identical to that analyzed in Example 13-3, it is apparent that the elastic parameters and frame constants are the same as determined in the above-mentioned example. The frame analysis thus reduces to the determination of the load constants and redundants of the frame. Isolating member 1-2, and noting that the geometric constants of the member are

$$v = \frac{l_h}{l} = 1 \qquad t = \left(\frac{\min d}{\max d}\right)^3 = 0.125$$

the load constants are determined from Tables 5 and 6 in the Appendix. The numerical value of the load constant for the small end is $0.148 + 0.25(0.212 - 0.148) = 0.164$, and for the large end is 0.154; or

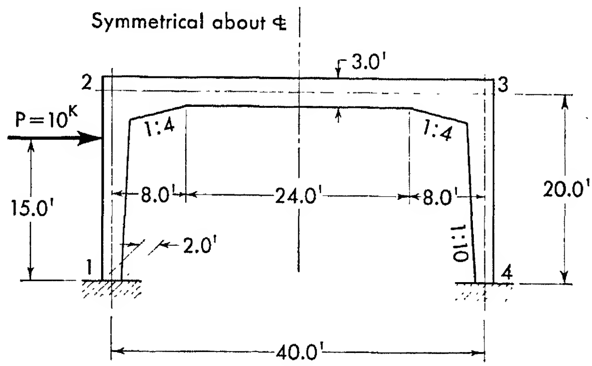


FIG. 13-12. Portal frame with applied horizontal concentrated load

$R_{12} = 0.164$ and $R_{21} = 0.154$

Recap: $\alpha_{12} = 2.32$ $\beta_{12} = 0.69$ $\alpha_{21} = 0.82$

$\phi = 0.593$ $A = 0.297$ $B = 14.66$ $D = 5.17$

$F = 6.85$ $\Theta_{12} = 4.52$ $\Theta_{23} = 9.48$ $R_{12} = 0.164$

$R_{21} = 0.154$

The solution of the frame subjected to lateral load is no more complicated than that for vertical load. Upon substituting the numerical values of elastic constants and dimensional quantities into equations, the

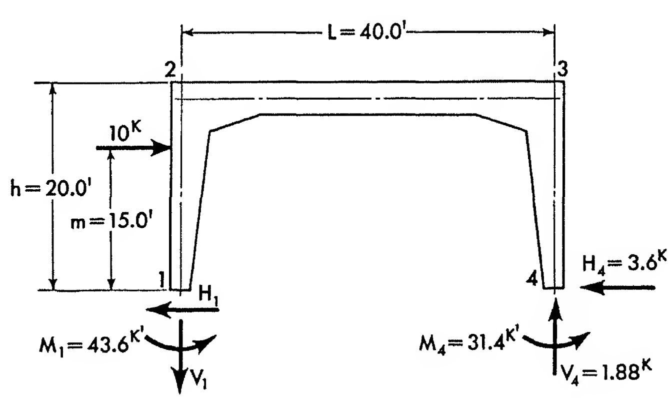


FIG. 13-13. Free body diagram

solution may be easily obtained. Using equations given in Arts. 15-1 and 15-8, the terms G, J, K, and N are

$$G = \frac{\phi\Theta_{23} + 2\alpha_{21}}{2\alpha_{12}} = \frac{0.593 \times 9.48 + 2 \times 0.82}{2 \times 2.32} = 1.56$$

$$J = h\,(AR_{21} - GR_{12}) = 20\,(0.297 \times 0.154 - 1.56 \times 0.164) = -4.2$$

$$K = h\,(R_{21} - AR_{12}) = 20\,(0.154 - 0.297 \times 0.164) = 2.11$$

$$\begin{aligned} N &= m\,(\alpha_{12} + \beta_{12}) - h\,(R_{12} + R_{21}) \\ &= 15\,(2.32 + 0.69) - 20\,(0.164 + 0.154) = 38.8 \end{aligned}$$

In the next step, the moments at the joints of the frame may be obtained. They are

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -P\left(\frac{K}{F} \mp \frac{N}{2D}\right) = -10\left(\frac{2.11}{6.85} \mp \frac{38.8}{2 \times 5.17}\right) = \begin{matrix} +34.4 \text{ ft-kip} \\ -40.6 \text{ ft-kip} \end{matrix}$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = P\left(\frac{J}{F} \pm \frac{Dm - N}{2D}\right)$$

$$= 10\left[\frac{(-4.2)}{6.85} \mp \frac{(5.17 \times 15) - 38.8}{2 \times 5.17}\right] = \begin{matrix} -43.6 \text{ ft-kip} \\ +31.4 \text{ ft-kip} \end{matrix}$$

And, finally, the horizontal and vertical components of frame reactions are

$$H_4 = \frac{P}{h}\left(\frac{J+K}{F} + \frac{m}{2}\right) = \frac{10}{20}\left[\frac{(-4.2) + 2.11}{6.85} + \frac{15}{2}\right] = 3.60 \text{ kip}$$

$$H_1 = -(P - H_4) = -(10 - 3.60) = -6.40 \text{ kip}$$

$$V_4 = \frac{PN}{DL} = \frac{10 \times 38.8}{5.17 \times 40} = 1.88 \text{ kip}$$

and

$$V_1 = -V_4 = -1.88 \text{ kip}$$

Static Check. Using the free body diagram shown in Fig. 13-13, the bending moment M_1 at joint 1 is

$$M_1 = 31.40 + (1.88 \times 40) - (10 \times 15) = -43.40 \text{ ft-kip}$$

Since the previously computed value of M_1 is −43.60 ft-kip, the check confirms the accuracy of numerical calculations.

Example 13-6. The rigid gable frame shown in Fig. 13-14 is hinged at the supports and carries a single vertical concentrated load of 10 kip. Calculate numerical values of the bending moments and horizontal and vertical reactions at the joints of the frame. Assume the width of all members normal to the plane of drawing is 2 ft, and the columns' cross section at the base is 2 by 2 ft.

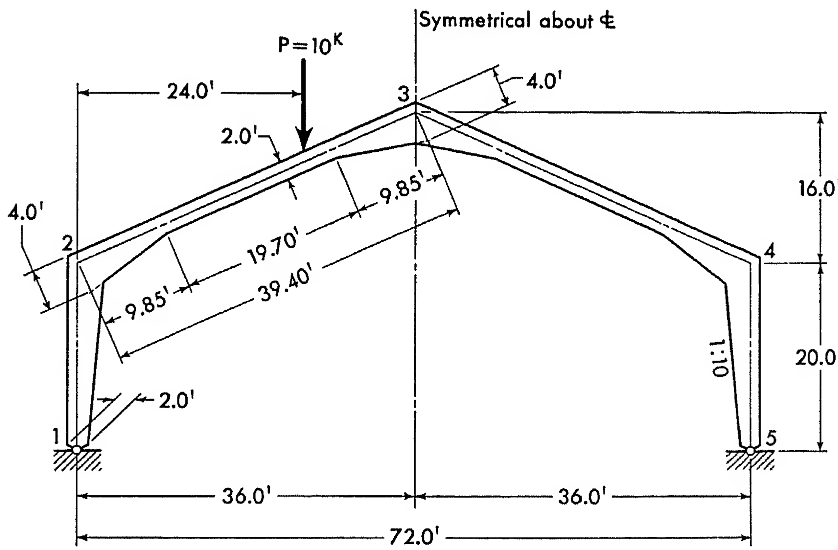


FIG. 13-14. Gable frame with applied vertical concentrated load

Following the procedure outlined in the previous examples, and noting that for member 1-2,

$$v = \frac{l_h}{l} = \frac{20}{20} = 1 \qquad \text{and} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3 = \left(\frac{2}{4}\right)^3 = 0.125$$

the following values of the elastic parameters are obtained from Charts 5 to 7:

$$\alpha_{12} = 2.32 \qquad \alpha_{21} = 0.82 \qquad \text{and} \qquad \beta_{12} = 0.69$$

Similarly, observing that for member 2-3

$$v = \frac{l_h}{l} = \frac{9.85}{39.4} = 0.25 \qquad \text{and} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3 = \left(\frac{2}{4}\right)^3 = 0.125$$

the following values of the elastic parameters are obtained from Charts

1 and 2:

$$\alpha_{23} = \alpha_{32} = 2.45 \qquad \text{and} \qquad \beta_{23} = 1.67$$

Further, using the geometric relations of member 2-3, determined above, the numerical values of the load constants for the given position of the load are obtained from Tables 1 and 2 in the Appendix. They are

For the left end $R = 0.521 \;\therefore\; R_{23} = 0.521$

For the right end $R = 0.623 \;\therefore\; R_{32} = 0.623$

Noting that the minimum moments of inertia of the column and girder are identical (1.333 ft^4), the frame constants being determined by the use of equations of Art. 18-1 are as follows:

$$\phi = \frac{\min I_{1\text{-}2}}{\min I_{2\text{-}3}} \cdot \frac{q}{h} = \frac{1.333}{1.333} \cdot \frac{39.4}{20} = 1.97$$

$$\psi = \frac{f}{h} = \frac{16}{20} = 0.8$$

$$\Theta_{23} = \alpha_{23} + \alpha_{32} + 2\beta_{23} = 2.45 + 2.45 + 2 \times 1.67 = 8.24$$

$$A = \Theta_{23} + \psi^2\alpha_{32} + 2\psi(\alpha_{32} + \beta_{23}) + \frac{\alpha_{21}}{\phi}$$

$$= 8.24 + 0.8^2 \times 2.45 + 2 \times 0.8(2.45 + 1.67) + \frac{0.82}{1.97} = 16.82$$

$$B = \alpha_{32}\,(1 + \psi) + \beta_{23} = 2.45(1 + 0.8) + 1.67 = 6.08$$

Applying the equations of Art. 18-5, the term K and horizontal reaction H are

$$K = R_{23} + R_{32}\,(1 + \psi) = 0.521 + 0.623(1 + 0.8) = 1.642$$

$$H_1 = H_5 = \frac{P}{4Ah}\,(2Bm + KL)$$

$$= \frac{10}{4 \times 16.82 \times 20}\,(2 \times 6.08 \times 24 + 1.642 \times 72) = 3.04 \text{ kip}$$

And finally, the bending moments at joints 2 and 3 are

$$M_2 = -H_5h = -3.04 \times 20 = -60.8 \text{ ft-kip}$$

$$M_3 = \frac{Pm}{2} - H_5h\,(1 + \psi)$$

$$= \frac{10 \times 24}{2} - 3.04 \times 20\,(1 + 0.8) = 10.56 \text{ ft-kip}$$

Example 13-7. An unsymmetrical member of a shape not covered in the tables and charts of elastic parameters and load constants is illustrated in Fig. 13-15. Assuming that this member is of a constant width, find numerical values of the elastic parameters and the load constants for the condition of a uniformly distributed load over the entire length of the member.

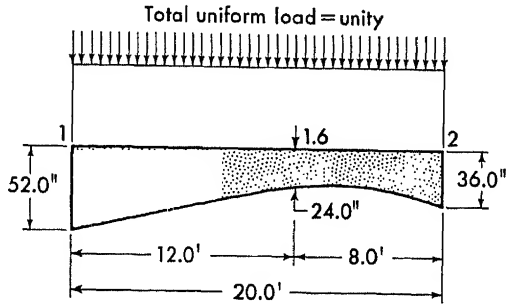


FIG. 13-15. Member of uncommon shape

As stated in Art. 13-7, this type of problem may be conveniently solved by the use of the charts and tables of the Appendix and the application of the principle of superposition. The procedure[1] commences by considering a hypothetical member which is of the same length, width, and reference moment of inertia as the given member possesses, and is comprised of the identical part 1-1.6 of the given member and the prismatic body of the member's minimum section for the remaining length. The elastic parameters for this hypothetical member may be found from charts. Symbolically, they are

α'_{12}, α'_{21}, and β'_{12}

In the next step, a second hypothetical member is considered of the same length, width, and reference moment of inertia as the given member possesses, but comprised of the identical part 1.6-2 of the given member and the prismatic body of the member's minimum section for the remaining length. The elastic parameters for this hypothetical member also may be found from charts. Symbolically, they are

α''_{12}, α''_{21}, and β''_{12}

The summation of corresponding parameters (those carrying the

[1] First introduced by Vaclav Dasek, *Beton und Eisen*, 1936.

same subscript) gives the numerical values of the elastic parameters for a member comprised of the two hypothetical members, described above, laterally combined. It is obvious that this resultant member differs from the given member only in the duplication of the prismatic body of the latter's minimum section, over the entire length.

To compute the elastic parameters of the given member, it is then necessary to subtract from the quantities determined above the values of the parameters of the duplicated body, that is, a straight prismatic member having a depth corresponding to the member's smallest depth and possessing the reference moment of inertia of the given member. Designating the parameters of the prismatic member as

$$\alpha'''_{12},\ \alpha'''_{21},\ \text{and}\ \beta'''_{12}$$

then the mathematical expressions for the required parameters of the given member are

$$\alpha_{12} = \alpha'_{12} + \alpha''_{12} - \alpha'''_{12}$$

$$\alpha_{21} = \alpha'_{21} + \alpha''_{21} - \alpha'''_{21}$$

$$\beta_{12} = \beta'_{12} + \beta''_{12} - \beta'''_{12}$$

In Fig. 13-16, a numerical solution for the elastic parameters of member 1-2 is presented in diagrammatic form.

Using the same procedure, the numerical values of load constants for member 1-2 may be determined. The member is uniformly loaded over the entire length; for such a loading case Charts 13 to 16, in the Appendix, provide all the numerical values needed for the calculation. The resulting values of the load constants are

$$R_{12} = 0.238 + 0.488 - 0.50 = 0.226$$

$$R_{21} = 0.380 + 0.440 - 0.50 = 0.320$$

$$v = \frac{l_h}{l} = \frac{12}{20} = 0.6$$

$$t = \left(\frac{\text{min d}}{\text{max d}}\right)^3 = \left(\frac{24}{52}\right)^3 = 0.099 \cong 0.1$$

From Charts 5 to 7, in the Appendix,

$$v = \frac{l_h}{l} = \frac{8}{20} = 0.4$$

$$t = \left(\frac{\text{min d}}{\text{max d}}\right)^3 = \left(\frac{24}{36}\right)^3 = 0.296$$

From Charts 8 to 10, in the Appendix,

Uniform member

From Charts 5 to 7, in the Appendix,

The resulting values represent the elastic parameters of the member shown in Fig. 13-15.

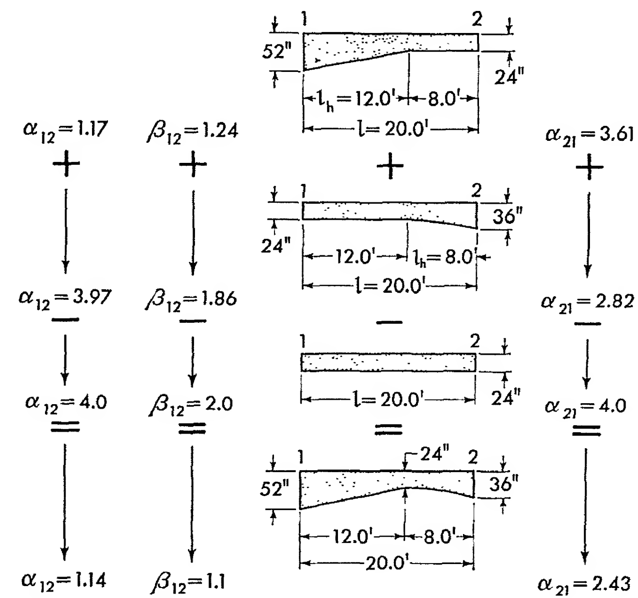
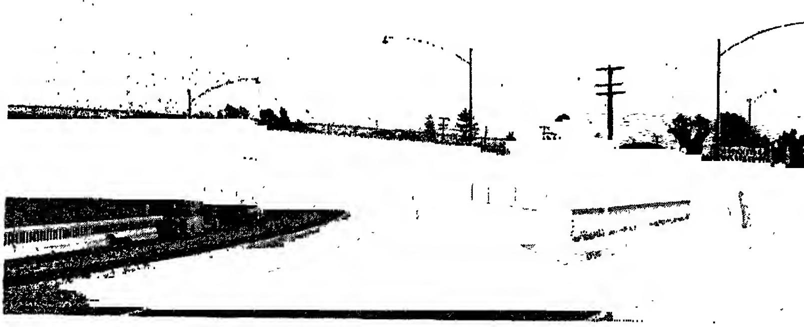


FIG. 13-16. Calculation of elastic parameters for the member of uncommon shape

A concrete rigid frame underpass on the Hollywood Freeway in the city of Los Angeles, California. The portal rigid frames, integrated with the concrete abutments and wing walls, give the impression of massiveness. The girders, with parabolic haunches, are functional and economical, and effectively contribute to the pleasant appearance of the bridge. (Courtesy of the California State Department of Public Works, Division of Highways.)

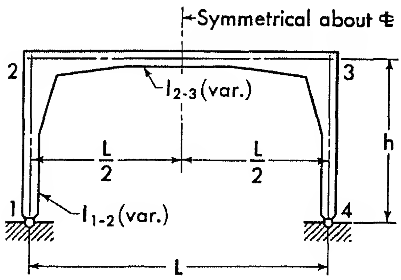


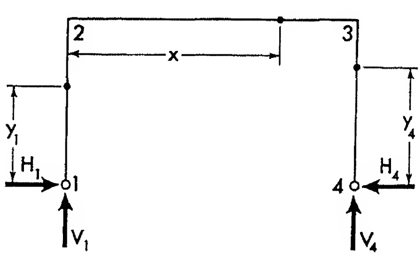
A concrete rigid frame underpass on U.S. Highway 101, near San Diego, California. This portal frame underpass illustrates a strict and plain style of contemporary bridge architecture, which speaks for itself. (Courtesy of the California State Department of Public Works, Division of Highways.)

SECTION 14

SYMMETRICAL PORTAL FRAMES WITH HINGED SUPPORTS

14-1. Notations, Coordinates, and Frame Constants





The sketch appearing on the left, above, explains notations for a representative portal frame with members of variable cross section.

The solutions of analysis given on the following pages are not limited to the shapes of the members shown, but are applicable to any shape, provided only that the frame is symmetrical.

The sketch appearing on the right, above, shows positive directions of the vertical and horizontal components of the frame reactions. It also defines the coordinates for any section of the frame. Coordinates to be considered only in the positive sense.

Frame Constants. Obtain numerical values of column and girder elastic parameters α_{21}, α_{23}, and β_{23} from applicable Charts 1 to 10 in the Appendix.

$$\phi = \frac{\min I_{1\text{-}2}}{\min I_{2\text{-}3}} \cdot \frac{L}{h}$$

$$\Theta_{23} = 2(\alpha_{23} + \beta_{23})$$

$$A = \Theta_{23} + \frac{2\alpha_{21}}{\phi}$$

14-2. Equations of Frame Reactions and Moments. The equations for the vertical and the redundant horizontal components of frame reactions are given on the following pages for a number of loading conditions. Corresponding expressions for the moment and forces at any section of a member with applied load are also provided.

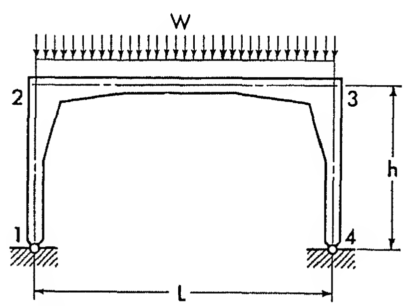
The equations for the moments of load-free members are listed below for reference.

$$M_{y_1} = M_2 \frac{y_1}{h} \tag{14-1}$$

$$M_x = M_2 \left(1 - \frac{x}{L}\right) + M_3 \frac{x}{L} \tag{14-2}$$

$$M_{y_4} = M_3 \frac{y_4}{h} \tag{14-3}$$

14-3. Vertical Uniform Load on Girder



W = total load

Obtain value of load constant R_{23} from Chart 11 or 12.

$$K = \frac{2LR_{23}}{A}$$

$$M_2 = M_3 = -WK$$

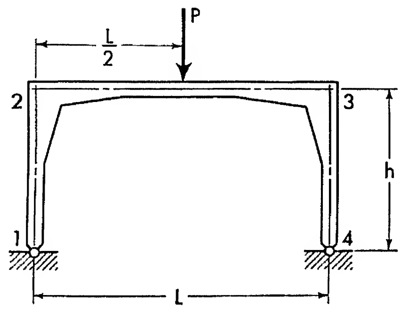
$$H_1 = H_4 = \frac{WK}{h}$$

$$V_1 = V_4 = \frac{W}{2}$$

$$M_x = \frac{Wx}{2}\left(1 - \frac{x}{L}\right) + M_2$$

Apply Eq. (14-1) to obtain the moment at any section of the left column. Moments at corresponding sections in the right half of the frame are identical to those in the left half.

14-4. Vertical Concentrated Load at Mid-point of Girder



Obtain value of load constant R_{23} from Table 1 or 3.

$$K = \frac{2LR_{23}}{A}$$

$$M_2 = M_3 = -PK$$

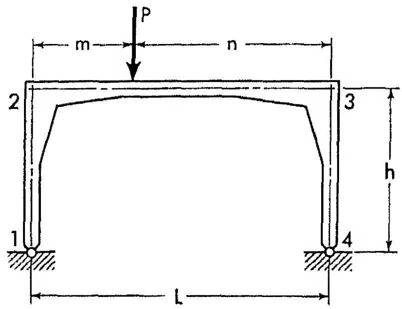
$$H_1 = H_4 = \frac{PK}{h}$$

$$V_1 = V_4 = \frac{P}{2}$$

When $x \leqq \frac{L}{2}$ $\qquad M_x = \frac{Px}{2} + M_2$

Apply Eq. (14-1) to obtain the moment at any section of the left column. Moments at corresponding sections in the right half of the frame are identical to those in the left half.

14-5. Vertical Concentrated Load at Any Point of Girder



Obtain values of load constants R_{23} and R_{32} from applicable Tables 1 to 4.

$$K = \frac{L}{A}(R_{23} + R_{32})$$

$$M_2 = M_3 = -PK \qquad H_1 = H_4 = \frac{PK}{h}$$

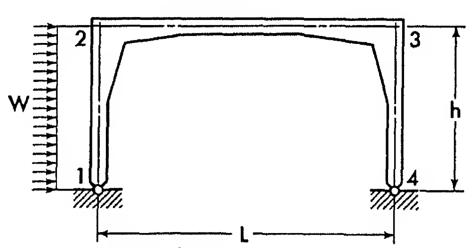
$$V_1 = P\left(1 - \frac{m}{L}\right) \qquad V_4 = \frac{Pm}{L}$$

When $x \leqq m$ $\qquad M_x = \frac{Pnx}{L} + M_2$

When $x > m$ $\qquad M_x = Pm\left(1 - \frac{x}{L}\right) + M_2$

Apply Eqs. (14-1) and (14-3) to obtain the moment at any section of the frame columns.

14-6. Horizontal Uniform Load on Column



Obtain value of load constant R_{21} from applicable Charts 11 to 16.

$$K = \frac{R_{21}}{A\phi}$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -Wh\left(K \mp \frac{1}{4}\right)$$

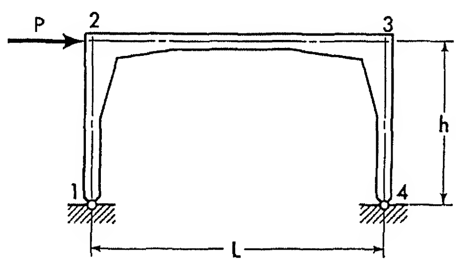
$$H_4 = W\left(K + \frac{1}{4}\right) \qquad H_1 = -(W - H_4)$$

$$V_4 = \frac{Wh}{2L} \qquad V_1 = -V_4$$

$$M_{y_1} = \frac{Wy_1}{2}\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (14-2) and (14-3) to obtain the moment at any section of frame members 2-3 and 3-4.

14-7. Horizontal Concentrated Load at Joint 2



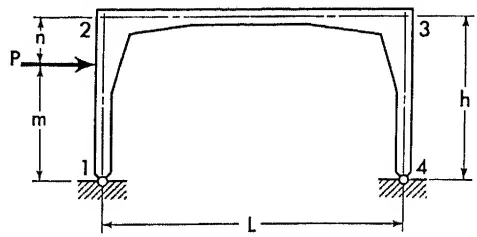
$$M_2 = \frac{Ph}{2} \qquad M_3 = -\frac{Ph}{2}$$

$$H_1 = -\frac{P}{2} \qquad H_4 = \frac{P}{2}$$

$$V_4 = \frac{Ph}{L} \qquad V_1 = -V_4$$

Apply Eqs. (14-1) through (14-3) to obtain the moment at any section of the frame members.

14-8. Horizontal Concentrated Load at Any Point of Column



Obtain value of load constant R_{21} from applicable Tables 1 to 8.

$$K = \frac{R_{21}}{A\phi}$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -P\left(Kh \mp \frac{m}{2}\right)$$

$$H_4 = P\left(K + \frac{m}{2h}\right) \qquad H_1 = -(P - H_4)$$

$$V_4 = \frac{Pm}{L} \qquad V_1 = -V_4$$

When $y_1 \leqq m$

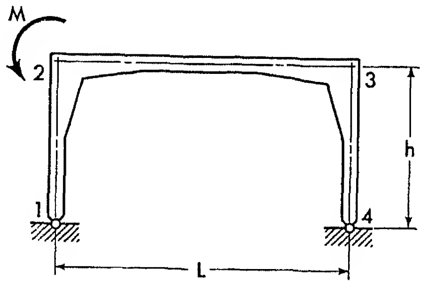
$$M_{y_1} = (M_2 + Pn)\frac{y_1}{h}$$

When $y_1 > m$

$$M_{y_1} = Pm\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (14-2) and (14-3) to obtain the moment at any section of frame members 2-3 and 3-4.

14-9. Moment Applied at Joint 2



$$K = \frac{\Theta_{23}}{2A} \qquad M_{21} = M_3 = MK$$

$$M_{23} = -(M - M_{21})$$

$$H_1 = H_4 = -\frac{MK}{h} \qquad V_1 = \frac{M}{L}$$

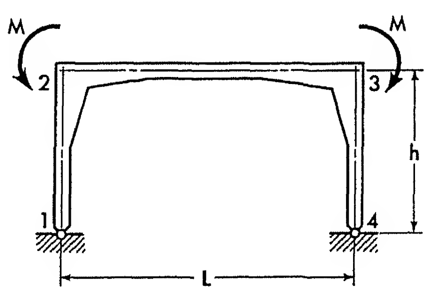
$$V_4 = -\frac{M}{L}$$

$$M_{y_1} = M_{21}\frac{y_1}{h}$$

$$M_x = M_{23}\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L}$$

Apply Eq. (14-3) to obtain the moment at any section of member 3-4.

14-10. Two Equal Moments Applied at Joints 2 and 3



$$K = \frac{\Theta_{23}}{2A} \qquad M_{21} = M_{34} = 2MK$$

$$M_{23} = M_{32} = -(M - M_{21})$$

$$H_1 = H_4 = -\frac{2MK}{h}$$

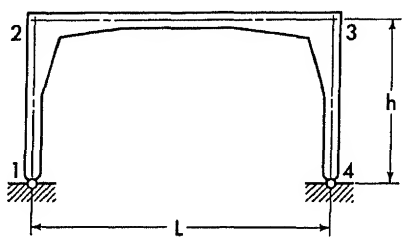
$$V_1 = V_4 = 0$$

$$M_{y_1} = M_{21}\frac{y_1}{h}$$

$$M_x = M_{23}$$

$$M_{y_4} = M_{34}\frac{y_4}{h}$$

14-11. Effect of Temperature Rise. Range t° for entire frame.



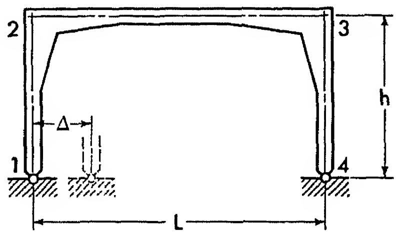
$$H_1 = H_4 = \frac{12L\varepsilon t^\circ}{Ah^3\phi}E(\min I_{1\text{-}2})$$

$$M_2 = M_3 = -H_4h \qquad V_1 = V_4 = 0$$

Apply Eqs. (14-1) through (14-3) to obtain the moment at any section of the frame members.

Note: For temperature drop, introduce the value of t° with a negative sign.

14-12. Horizontal Displacement of One Support

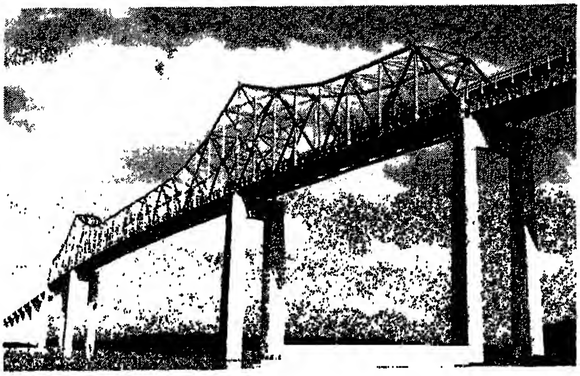


$$H_1 = H_4 = \frac{12\Delta}{Ah^3\phi} E(\min I_{1\text{-}2})$$

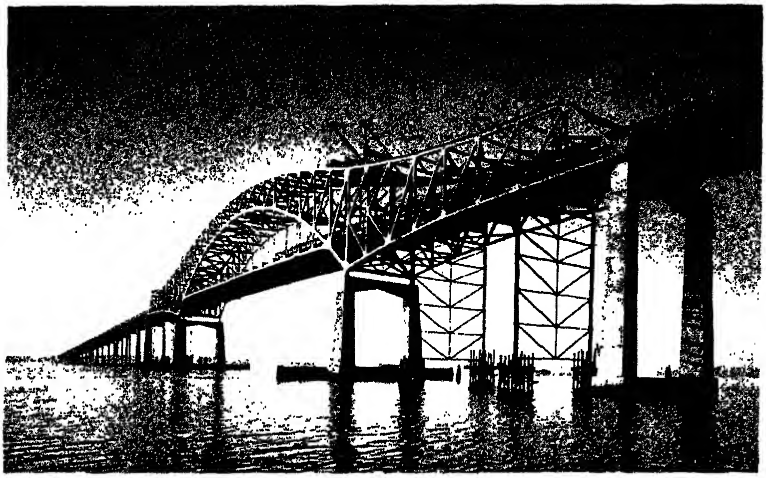
$$M_2 = M_3 = -H_4 h \qquad V_1 = V_4 = 0$$

Apply Eqs. (14-1) through (14-3) to obtain the moment at any section of the frame members.

Note: If the direction of the frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.



The Eugene Talmadge Memorial Bridge in Savannah, Georgia. This is an example of a majestic cantilever bridge having portal rigid frames as supporting elements. Noting that the central span is 710 feet long, anchor spans are 289 feet each, and the vertical clearance under bridge is 135 feet, one may realize the magnitude of lateral, vertical, and transverse forces transmitted from the superstructure to the frames. The portal frames, with an average section of 13 by 15 feet, effectively demonstrate their utility in this application. Parsons, Brinckerhoff, Hall and MacDonald, consulting engineers of New York, are designers. (Courtesy of United States Steel Corporation, builders of the superstructure.)

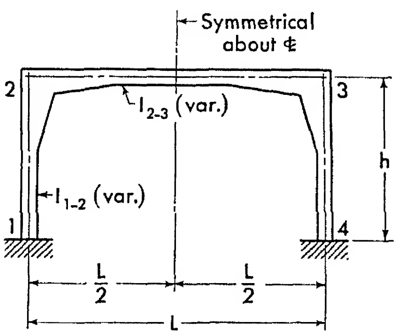


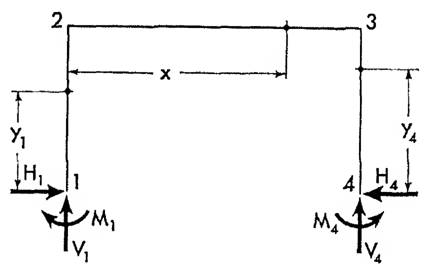
The grandiose Newark Bridge over the Newark Bay also illustrates an application of rigid portal frames in bridge construction. The superstructure of this bridge is even longer than that of the Eugene Talmadge Memorial Bridge, and consequently, even greater forces might be realized on the frames. Again, the frames demonstrate their effective functional service in this application. Howard, Needles, Tammen & Bergenhoff, of New York, were consulting engineers for the project. (Courtesy of Bethlehem Steel Co.)

SECTION 15

SYMMETRICAL PORTAL FRAMES WITH FIXED SUPPORTS

15-1. Notations, Coordinates, and Frame Constants





The sketch appearing on the left, above, explains notations for a representative portal frame with members of variable cross section.

The solutions of analysis given on the following pages are not limited to the shapes of the members shown, but are applicable to any shape, provided only that the frame is symmetrical.

The sketch appearing on the right, above, shows positive directions of the moments and the vertical and horizontal components of the frame reactions. It also defines the coordinates for any section of the frame. Coordinates to be considered only in the positive sense.

General Frame Constants. Obtain numerical values of column and girder elastic parameters α_{12}, α_{21}, β_{12}, α_{23}, and β_{23} from applicable Charts 1 to 10 in the Appendix.

$$\phi = \frac{\min I_{1\text{-}2}}{\min I_{2\text{-}3}} \cdot \frac{L}{h}$$

$$\Theta_{12} = \alpha_{12} + \alpha_{21} + 2\beta_{12}$$

$$\Theta_{23} = 2(\alpha_{23} + \beta_{23})$$

$$A = \frac{\beta_{12}}{\alpha_{12}} \qquad B = 2\Theta_{12} + \phi\Theta_{23} \qquad D = \frac{B}{2} - 2\phi\beta_{23}$$

$$F = B - \frac{2(\alpha_{12} + \beta_{12})^2}{\alpha_{12}}$$

Constant G. To be used only in cases of horizontal load on column.

$$G = \frac{\phi\Theta_{23} + 2\alpha_{21}}{2\alpha_{12}}$$

15-2. Equations of Frame Reactions and Moments. The equations for the redundant moments and the vertical and horizontal components of frame reactions are given on the following pages for a number of loading conditions. Corresponding expressions for the moment and forces at any section of a member with applied load are also provided.

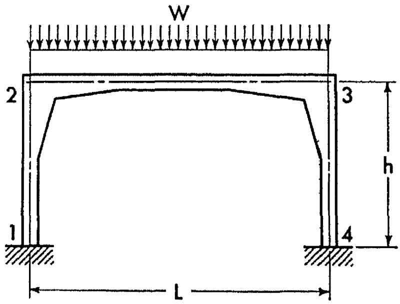
The equations for the moments of load-free members are listed below for reference.

$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h} \tag{15-1}$$

$$M_x = M_2\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L} \tag{15-2}$$

$$M_{y_4} = M_3\frac{y_4}{h} + M_4\left(1 - \frac{y_4}{h}\right) \tag{15-3}$$

15-3. Vertical Uniform Load on Girder



W = Total load

Obtain value of load constant R_{23} from Chart 11 or 12.

$$K = \frac{2L\phi}{F} R_{23}$$

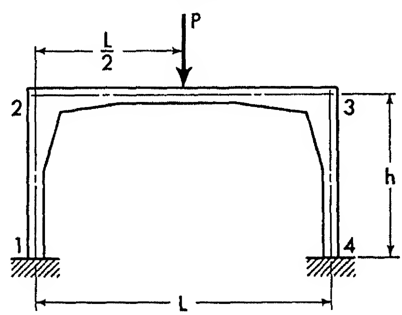
$$M_2 = M_3 = -WK \qquad M_1 = M_4 = WAK$$

$$H_1 = H_4 = \frac{WK}{h}(1 + A) \qquad V_1 = V_4 = \frac{W}{2}$$

$$M_x = \frac{Wx}{2}\left(1 - \frac{x}{L}\right) + M_2$$

Apply Eqs. (15-1) and (15-3) to obtain the moment at any section of the frame columns.

15-4. Vertical Concentrated Load at Mid-point of Girder



Obtain value of load constant R_{23} from Table 1 or 3.

$$K = \frac{2L\phi}{F} R_{23} \qquad M_2 = M_3 = -PK$$

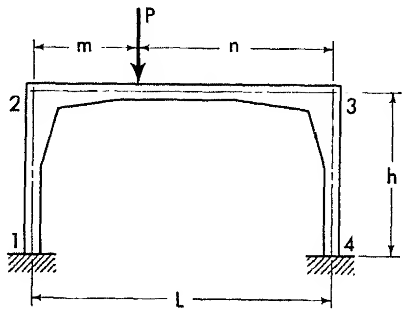
$$M_1 = M_4 = PAK$$

$$H_1 = H_4 = \frac{PK}{h}(1 + A) \qquad V_1 = V_4 = \frac{P}{2}$$

When $x \leqq \frac{L}{2}$ $\qquad M_x = \frac{Px}{2} + M_2$

Apply Eq. (15-1) to obtain the moment at any section of left column. Moments and forces at corresponding sections in the right half of the frame are identical to those in the left half.

15-5. Vertical Concentrated Load at Any Point of Girder



Obtain values of load constants R_{23} and R_{32} from Tables 1 to 4.

$$J = \frac{L\phi}{F}(R_{23} + R_{32})$$

$$K = \frac{L\phi}{2D}(R_{23} - R_{32})$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -P(J \pm K)$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = P(AJ \mp K)$$

$$H_1 = H_4 = \frac{PJ}{h}(1 + A)$$

$$V_1 = P\left(1 - \frac{m}{L}\right) + \frac{2PK}{L}$$

$$V_4 = P - V_1$$

When $x \leqq m$

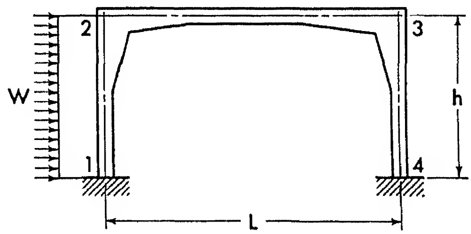
$$M_x = (Pn + M_3)\frac{x}{L} + M_2\left(1 - \frac{x}{L}\right)$$

When $x > m$

$$M_x = (Pm + M_2)\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L}$$

Apply Eqs. (15-1) and (15-3) to obtain the moment at any section of the frame columns.

15-6. Horizontal Uniform Load on Column



W = Total load

Obtain values of load constants R_{12} and R_{21} from applicable Charts 11 to 16.

$$J = h(AR_{21} - GR_{12})$$

$$K = h(R_{21} - AR_{12})$$

$$N = \alpha_{12} + \beta_{12} - 2(R_{12} + R_{21})$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -W\left(\frac{K}{F} \mp \frac{Nh}{4D}\right)$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = W\left[\frac{J}{F} \mp \frac{h}{4}\left(1 - \frac{N}{D}\right)\right]$$

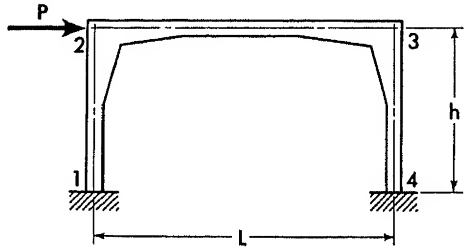
$$H_4 = \frac{W}{4}\left[1 + \frac{4(J + K)}{Fh}\right]$$

$$V_4 = \frac{WNh}{2DL} \qquad V_1 = -V_4$$

$$M_{y_1} = \left(\frac{Wy_1}{2} + M_1\right)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (15-2) and (15-3) to obtain the moment at any section of frame members 2-3 and 3-4.

15-7. Horizontal Concentrated Load at Joint 2



$$K = \frac{h}{2D}(\alpha_{12} + \beta_{12})$$

$$M_2 = PK \qquad M_3 = -PK$$

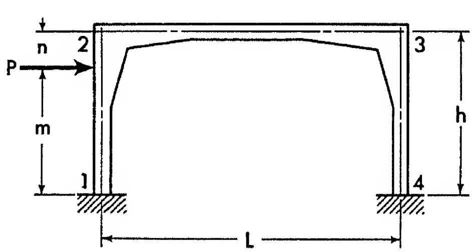
$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = \mp P\left(\frac{h}{2} - K\right)$$

$$H_1 = -\frac{P}{2} \qquad H_4 = \frac{P}{2}$$

$$V_4 = \frac{2PK}{L} \qquad V_1 = -V_4$$

Apply Eqs. (15-1) through (15-3) to obtain the moment at any section of the frame members.

15-8. Horizontal Concentrated Load at Any Point of Column



Obtain values of load constants R_{12} and R_{21} from applicable Tables 1 to 8.

$$J = h(AR_{21} - GR_{12}) \qquad K = h(R_{21} - AR_{12})$$

$$N = m(\alpha_{12} + \beta_{12}) - h(R_{12} + R_{21})$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -P\left(\frac{K}{F} \mp \frac{N}{2D}\right)$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = P\left(\frac{J}{F} \mp \frac{Dm - N}{2D}\right)$$

$$H_4 = \frac{P}{h}\left(\frac{J+K}{F} + \frac{m}{2}\right) \qquad H_1 = -(P - H_4)$$

$$V_4 = \frac{PN}{DL} \qquad V_1 = -V_4$$

When $y_1 \leqq m$

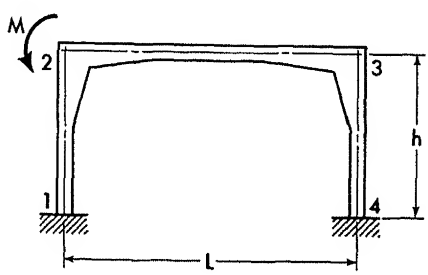
$$M_{y_1} = (M_2 + Pn)\frac{y_1}{h} + M_1\left(1 - \frac{y_1}{h}\right)$$

When $y_1 > m$

$$M_{y_1} = (M_1 + Pm)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (15-2) and (15-3) to obtain the moment at any section of frame members 2-3 and 3-4.

15-9. Moment Applied at Joint 2



$$J = \frac{\phi}{F}(\alpha_{23} + \beta_{23}) \qquad K = \frac{\phi}{2D}(\alpha_{23} - \beta_{23})$$

$$\left.\begin{matrix} M_{21} \\ M_3 \end{matrix}\right\rangle = M(J \pm K)$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = -M(AJ \mp K) \qquad M_{23} = -(M - M_{21})$$

$$H_1 = H_4 = -\frac{MJ(1 + A)}{h}$$

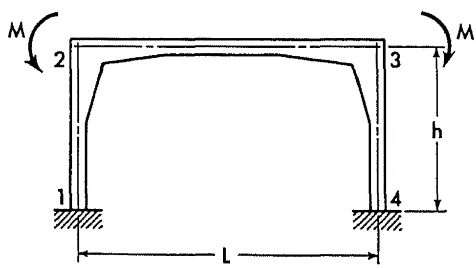
$$\left.\begin{matrix} V_1 \\ V_4 \end{matrix}\right\rangle = \pm \frac{M}{L}(1 - 2K)$$

$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) + M_{21}\frac{y_1}{h}$$

$$M_x = M_{23}\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L}$$

Apply Eq. (15-3) to obtain the moment at any section of the right column.

15-10. Two Equal Moments Applied at Joints 2 and 3



$$K = \frac{\phi\Theta_{23}}{F} \qquad M_{21} = M_{34} = MK$$

$$M_{23} = M_{32} = -(M - M_{21})$$

$$M_1 = M_4 = -MAK$$

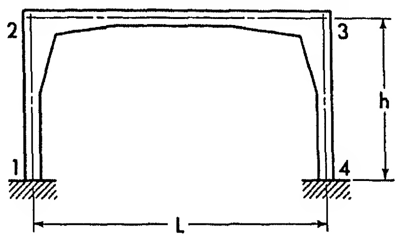
$$H_1 = H_4 = -\frac{MK(1 + A)}{h} \qquad V_1 = V_4 = 0$$

$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) + M_{21}\frac{y_1}{h}$$

$$M_x = M_{23}$$

Moments at corresponding sections in the right half of the frame are identical to those in the left half.

15-11. Effect of Temperature Rise. Range t° for entire frame.



$$K = \frac{12L\varepsilon t^\circ}{Fh^2} E(\min I_{1\text{-}2})$$

$$M_2 = M_3 = -K(1 + A)$$

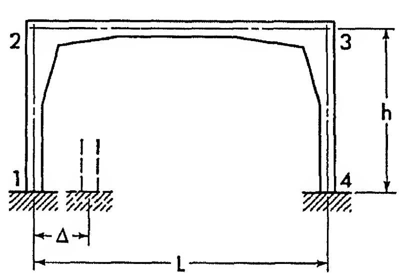
$$M_1 = M_4 = M_2 + \frac{BK}{2\alpha_{12}}$$

$$H_1 = H_4 = \frac{BK}{2h\alpha_{12}} \qquad V_1 = V_4 = 0$$

Apply Eqs. (15-1) through (15-3) to obtain the moment at any section of the frame members.

Note: For temperature drop, introduce the value of t° with a negative sign.

15-12. Horizontal Displacement of One Support



$$K = \frac{12\Delta}{Fh^2} E(\min I_{1\text{-}2})$$

$$M_2 = M_3 = -K(1 + A)$$

$$M_1 = M_4 = M_2 + \frac{BK}{2\alpha_{12}}$$

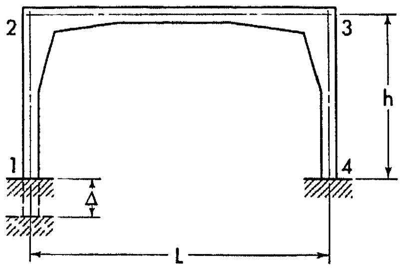
For Notations and Constants, see Arts. 15-1 and 15-2

$$H_1 = H_4 = \frac{BK}{2h\alpha_{12}} \qquad V_1 = V_4 = 0$$

Apply Eqs. (15-1) through (15-3) to obtain the moment at any section of the frame members.

Note: If the direction of the frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.

15-13. Vertical Settlement of One Support



$$K = \frac{12\Delta}{DLh} E(\min I_{1\text{-}2})$$

$$M_1 = M_2 = K$$

$$M_3 = M_4 = -K$$

$$H_1 = H_4 = 0$$

$$M_{2.5} = 0$$

$$V_1 = -\frac{2K}{L}$$

$$V_4 = -V_1$$

$$M_{y_1} = M_1$$

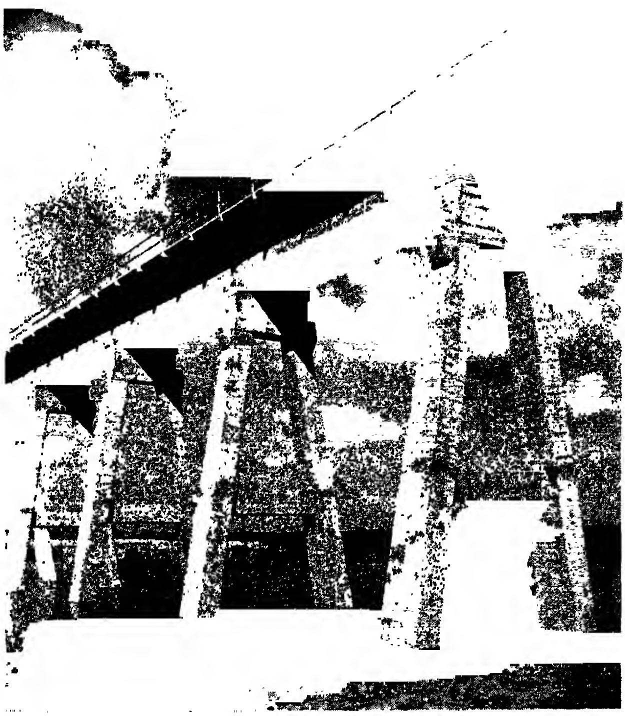
$$M_x = M_1 + V_1 x$$

$$M_{y_4} = M_4$$

Note: If the direction of the frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.

SECTION 16

SYMMETRICAL TRAPEZOIDAL FRAMES WITH HINGED SUPPORTS

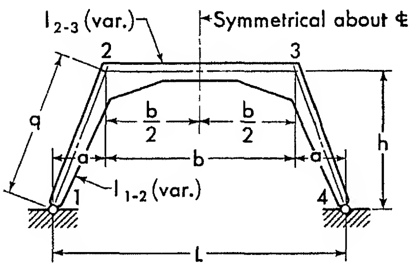


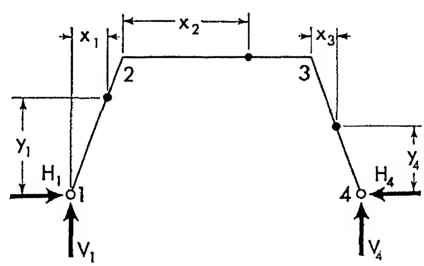
Horse Creek Viaduct, which carries the Kansas, Oklahoma & Gulf Railroad high above a narrow area of the lake created by the building of Pensacola Dam in Oklahoma. This interesting structure consists of concrete T girders, supported by 70-foot-high concrete trapezoidal frames. The ballast deck for the single track is an integral part of the T girders. Notice the simple but highly attractive appearance of the viaduct. Designed by Holway & Neuffer, engineers, and V. H. Cochrane, consulting engineer of Tulsa, Oklahoma. (Courtesy of the Portland Cement Association.)

SECTION 16

SYMMETRICAL TRAPEZOIDAL FRAMES WITH HINGED SUPPORTS

16-1. Notations, Coordinates, and Frame Constants





The sketch appearing on the left, above, explains notations for a representative trapezoidal frame with members of variable cross section.

The solutions of analysis given on the following pages are not limited to the shapes of the members shown, but are *applicable to any shape, provided only* that the frame is symmetrical.

The sketch appearing on the right, above, shows positive directions of the vertical and horizontal components of the frame reactions. It also defines the coordinates for any section of the frame. Coordinates to be considered only in the positive sense.

Frame Constants. Obtain numerical values of the column and girder parameters α_{21}, α_{23}, and β_{23} from applicable Charts 1 to 10 in the Appendix.

$$\phi = \frac{\min I_{1\text{-}2}}{\min I_{2\text{-}3}} \cdot \frac{b}{q}$$

$$\Theta_{23} = 2(\alpha_{23} + \beta_{23})$$

$$A = \Theta_{23} + \frac{2\alpha_{21}}{\phi}$$

16-2. Equations of Frame Reactions and Moments. The equations for the vertical and the redundant horizontal components of frame reactions are given on the following pages for a number of loading conditions. Corresponding expressions for the moment and forces at any section of a member with applied load are also provided.

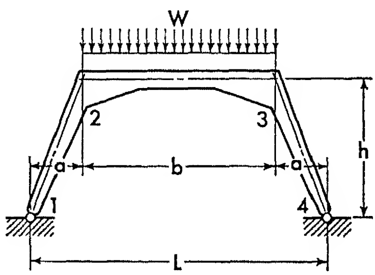
The equations for the moments of load-free members are listed below for reference.

$$M_{x_1} = M_2 \frac{x_1}{a} \tag{16-1}$$

$$M_{x_2} = M_2 \left(1 - \frac{x_2}{b}\right) + M_3 \frac{x_2}{b} \tag{16-2}$$

$$M_{x_3} = M_3 \left(1 - \frac{x_3}{a}\right) \tag{16-3}$$

16-3. Vertical Uniform Load on Girder



W = total load

Obtain value of load constant R_{23} from Chart 11 or 12.

$$K = \frac{2bR_{23}}{A} \qquad M_2 = M_3 = -WK$$

$$H_1 = H_4 = \frac{W}{h}\left(\frac{a}{2} + K\right)$$

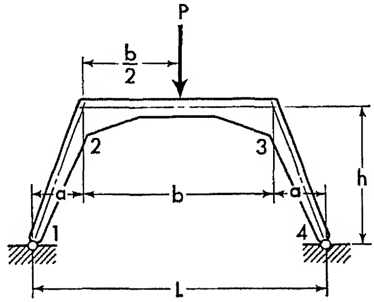
$$V_1 = V_4 = \frac{W}{2}$$

$$M_{x_2} = \frac{Wx_2}{2}\left(1 - \frac{x_2}{b}\right) + M_2$$

Apply Eq. (16-1) to obtain the moment at any section of the left column. Moments at correspondng sections in the right half of the frame are identical to those in the left half.

Members of Variable Section

16-4. Vertical Concentrated Load at Mid-point of Girder



Obtain value of load constant R_{23} from Table 1 or 3.

$$K = \frac{2bR_{23}}{A} \qquad M_2 = M_3 = -PK$$

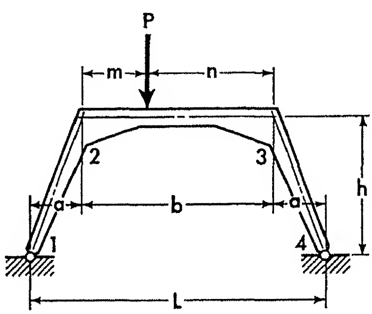
$$H_1 = H_4 = \frac{P}{h}\left(\frac{a}{2} + K\right) \qquad V_1 = V_4 = \frac{P}{2}$$

When $x_2 \leqq \frac{b}{2}$

$$M_{x_2} = \frac{Px_2}{2} + M_2$$

Apply Eq. (16-1) to obtain the moment at any section of the left column. Moments at corresponding sections in the right half of the frame are identical to those in the left half.

16-5. Vertical Concentrated Load at Any Point of Girder



Obtain values of load constants R_{23} and R_{32} from applicable Tables 1 to 4.

For Notations and Constants, see Arts. 16-1 and 16-2

$$J = \frac{b(R_{23} + R_{32})}{A} \qquad K = \frac{b - 2m}{2L}$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -P(J \mp Ka)$$

$$H_1 = H_4 = \frac{P}{2h}(2J + a)$$

$$V_1 = \frac{P}{2}(1 + 2K) \qquad V_4 = \frac{P}{2}(1 - 2K)$$

When $x_2 \leqq m$

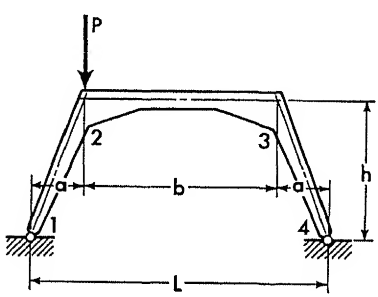
$$M_{x_2} = (Pn + M_3)\frac{x_2}{b} + M_2\left(1 - \frac{x_2}{b}\right)$$

When $x_2 > m$

$$M_{x_2} = (Pm + M_2)\left(1 - \frac{x_2}{b}\right) + M_3\frac{x_2}{b}$$

Apply Eqs. (16-1) and (16-3) to obtain the moment at any section of the frame columns.

16-6. Vertical Concentrated Load at Joint 2



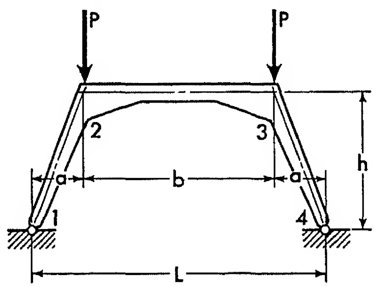
$$M_2 = \frac{Pab}{2L} \qquad M_3 = -\frac{Pab}{2L}$$

$$H_1 = H_4 = \frac{Pa}{2h}$$

$$\left.\begin{matrix} V_1 \\ V_4 \end{matrix}\right\rangle = \frac{P}{2}\left(1 \pm \frac{b}{L}\right)$$

Apply Eqs. (16-1) through (16-3) to obtain the moment at any section of frame members.

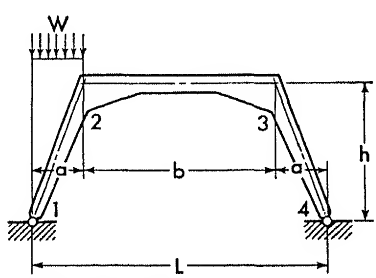
16-7. Two Equal Vertical Concentrated Loads at Joints 2 and 3



$$H_1 = H_4 = \frac{Pa}{h} \qquad V_1 = V_4 = P$$

There are no bending moments.

16-8. Vertical Uniform Load on Inclined Column



W = total load

Obtain value of load constant R_{21} from applicable Charts 11 to 16.

$$K = \frac{R_{21}}{A\phi}$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -Wa\left(K \mp \frac{b}{4L}\right)$$

$$H_1 = H_4 = \frac{Wa}{4h}(1 + 4K)$$

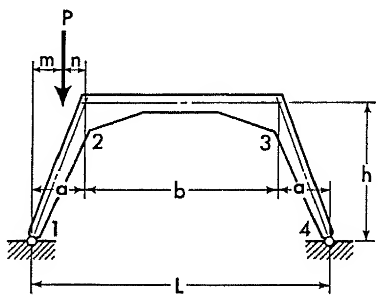
For Notations and Constants, see Arts. 16-1 and 16-2

$$V_4 = \frac{Wa}{2L} \qquad V_1 = W - V_4$$

$$M_{x_1} = \frac{Wx_1}{2}\left(1 - \frac{x_1}{a}\right) + M_2 \frac{x_1}{a}$$

Apply Eqs. (16-2) and (16-3) to obtain the moment at any section of frame members 2-3 and 3-4.

16-9. Vertical Concentrated Load at Any Point of Inclined Column



Obtain value of load constant R_{21} from applicable Tables 1 to 8.

$$K = \frac{aR_{21}}{A\phi}$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -P\left(K \mp \frac{bm}{2L}\right)$$

$$H_1 = H_4 = \frac{P}{h}\left(K + \frac{m}{2}\right) \qquad V_4 = \frac{Pm}{L}$$

$$V_1 = P - V_4$$

When $x_1 \leqq m$

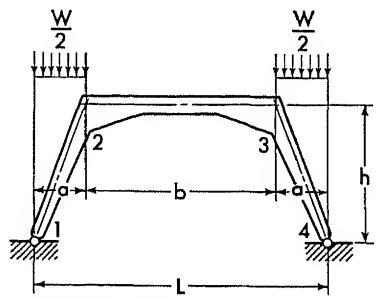
$$M_{x_1} = (Pn + M_2)\frac{x_1}{a}$$

When $x_1 > m$

$$M_{x_1} = Pm\left(1 - \frac{x_1}{a}\right) + M_2 \frac{x_1}{a}$$

Apply Eqs. (16-2) and (16-3) to obtain the moment at any section of frame members 2-3 and 3-4.

16-10. Vertical Uniform Load on Both Inclined Columns



W = total load

In view of the symmetry of the loading about the frame center line, the solution of the problem may be obtained by the use of the load constant of the left inclined column alone. Thus, obtaining the load constant R_{21} in the same way as for a uniform load on member 1-2 only (Art. 16-8), and introducing it into the equations given below, the solution of the problem is readily obtained.

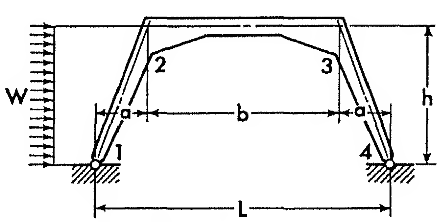
$$K = \frac{R_{21}}{A\phi} \qquad M_2 = M_3 = -WKa$$

$$H_1 = H_4 = \frac{Wa}{4h}(1 + 4K) \qquad V_1 = V_4 = \frac{W}{2}$$

$$M_{x_1} = \frac{Wx_1}{4}\left(1 - \frac{x_1}{a}\right) + M_2\frac{x_1}{a} \qquad M_{x_2} = M_2$$

Moments and forces at the corresponding sections in the right half of the frame are identical to those in the left half.

16-11. Horizontal Uniform Load on Inclined Column



W = total load

Obtain value of load constant R_{21} from applicable Charts 11 to 16.

For Notations and Constants, see Arts. 16-1 and 16-2

$$K = \frac{R_{21}}{A\phi}$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -Wh\left(K \mp \frac{b}{4L}\right)$$

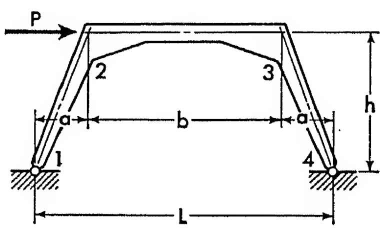
$$H_4 = W\left(\frac{1}{4} + K\right) \qquad H_1 = -(W - H_4)$$

$$V_4 = \frac{Wh}{2L} \qquad V_1 = -V_4$$

$$M_{y_1} = \frac{Wy_1}{2}\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (16-2) and (16-3) to obtain the moment at any section of frame members 2-3 and 3-4.

16-12. Horizontal Concentrated Load at Joint 2



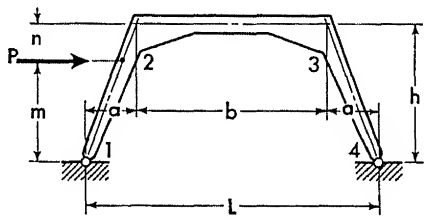
$$M_2 = \frac{Pbh}{2L} \qquad M_3 = -\frac{Pbh}{2L}$$

$$H_1 = -\frac{P}{2} \qquad H_4 = \frac{P}{2}$$

$$V_4 = \frac{Ph}{L} \qquad V_1 = -V_4$$

Apply Eqs. (16-1) through (16-3) to obtain the moment at any section of the frame members.

16-13. Horizontal Concentrated Load at Any Point of Inclined Column



Obtain value of load constant R_{21} from applicable Tables 1 to 8.

$$K = \frac{R_{21}}{A\phi}$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -P\left(Kh \mp \frac{mb}{2L}\right)$$

$$H_4 = P\left(K + \frac{m}{2h}\right) \qquad H_1 = -(P - H_4)$$

$$V_4 = \frac{Pm}{L} \qquad V_1 = -V_4$$

When $y_1 \leqq m$

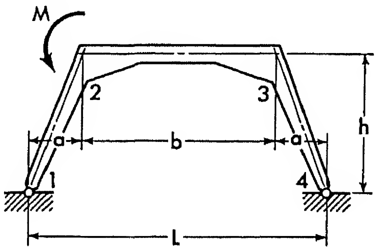
$$M_{y_1} = (M_2 + Pn)\frac{y_1}{h}$$

When $y_1 > m$

$$M_{y_1} = Pm\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (16-2) and (16-3) to obtain the moment at any section of frame members 2-3 and 3-4.

16-14. Moment Applied at Joint 2



$$K = \frac{\alpha_{23} + \beta_{23}}{A}$$

$$\left.\begin{matrix} M_{21} \\ M_{3} \end{matrix}\right\rangle = M\left(K \pm \frac{a}{L}\right) \qquad M_{23} = -(M - M_{21})$$

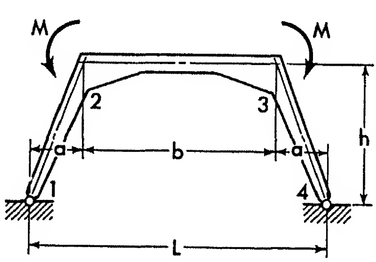
$$H_1 = H_4 = -\frac{MK}{h} \qquad V_1 = \frac{M}{L}$$

$$V_4 = -\frac{M}{L} \qquad M_{y_1} = M_{21}\frac{y_1}{h}$$

$$M_{x_2} = M_{23}\left(1 - \frac{x_2}{b}\right) + M_3\frac{x_2}{b}$$

Apply Eq. (16-3) to obtain the moment at any section of member 3-4.

16-15. Two Equal Moments Applied at Joints 2 and 3



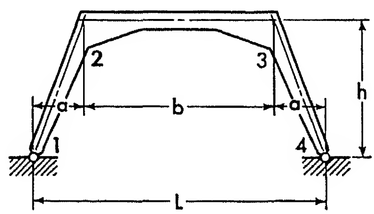
$$K = \frac{\alpha_{23} + \beta_{23}}{A}$$

$$M_{21} = M_{34} = 2MK \qquad M_{23} = M_{32} = -(M - M_{21})$$

$$H_1 = H_4 = -\frac{2MK}{h} \qquad V_1 = V_4 = 0$$

$$M_{y_1} = M_{21}\frac{y_1}{h} \qquad M_{x_2} = M_{23} \qquad M_{y_4} = M_{34}\frac{y_4}{h}$$

16-16. Effect of Temperature Rise. Range t° for entire frame.



$$H_1 = H_4 = \frac{12L\varepsilon t^\circ}{Ah^2q\phi} E(\min I_{1\text{-}2})$$

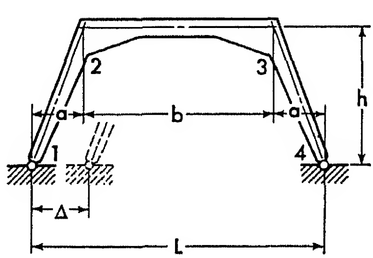
$$M_2 = M_3 = -H_4h$$

$$V_1 = V_4 = 0$$

Apply Eqs. (16-1) through (16-3) to obtain the moment at any section of the frame members.

Note: For temperature drop, introduce the value of t° with a negative sign.

16-17. Horizontal Displacement of One Support



$$H_1 = H_4 = \frac{12\Delta}{Ah^2q\phi} E(\min I_{1\text{-}2})$$

$$M_2 = M_3 = -H_4h \qquad V_1 = V_4 = 0$$

Apply Eqs. (16-1) through (16-3) to obtain the moment at any section of the frame members.

Note: If the direction of the frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.

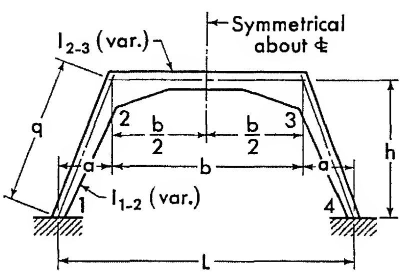
For Notations and Constants, see Arts. 16-1 and 16-2

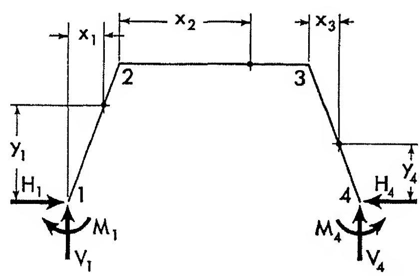
A close-up view of the three-mile-long concrete trestle which carries the track of the Missouri Pacific Railroad Company over the West Atchafalaya Floodway in Louisiana. This monumental structure stands over a dense forest and swamp area and was designed to withstand forest fire or lateral load created by a 25-foot-deep flow of water carrying forest debris. Lateral stability of the structure was obtained by inclining the columns outward and by providing heavy H-shaped concrete footings monolithically constructed with the columns. Designed by the Missouri Pacific Railroad Co. (Courtesy of the Portland Cement Association.)

SECTION 17

SYMMETRICAL TRAPEZOIDAL FRAMES WITH FIXED SUPPORTS

17-1. Notations, Coordinates, and Frame Constants





The sketch appearing on the left, above, explains notations for a representative trapezoidal frame with members of variable cross section.

The solutions of analysis given on the following pages are not limited to the shapes of the members shown, but are applicable to any shape, provided only that the frame is symmetrical.

The sketch appearing on the right, above, shows positive directions of the moments and the vertical and horizontal components of the frame reactions. It also defines the coordinates for any section of the frame. Coordinates to be considered only in the positive sense.

Frame Constants. Obtain numerical values of column and girder elastic parameters α_{12}, α_{21}, β_{12}, α_{23}, and β_{23} from applicable Charts 1 to 10 in the Appendix.

$$\phi = \frac{\min I_{1\text{-}2}}{\min I_{2\text{-}3}} \cdot \frac{b}{q}$$

$$\Theta_{12} = \alpha_{12} + \alpha_{21} + 2\beta_{12}$$

$$\Theta_{23} = 2(\alpha_{23} + \beta_{23}) \qquad A = \frac{\beta_{12}}{\alpha_{12}}$$

$$B = 2\Theta_{12} + \phi\Theta_{23}$$

$$D = \frac{4a}{b}\left[\alpha_{12}\left(1 + \frac{a}{b}\right) + \beta_{12}\right] + \Theta_{12} + \phi(\alpha_{23} - \beta_{23})$$

$$F = B - \frac{2(\alpha_{12} + \beta_{12})^2}{\alpha_{12}}$$

$$G = \frac{1}{D}\left[\alpha_{21} + \frac{L}{b}\beta_{12} + \phi(\alpha_{23} - \beta_{23})\right]$$

17-2. Equations of Frame Reactions and Moments. The equations for the redundant moments and the vertical and horizontal components of frame reactions are given on the following pages for a number of loading conditions. Corresponding expressions for the moments and forces at any section of a member with applied load are also provided.

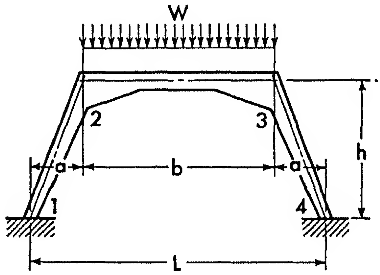
The equations for the moments of load-free members are listed below for reference.

$$M_{x_1} = M_1\left(1 - \frac{x_1}{a}\right) + M_2\frac{x_1}{a} \tag{17-1}$$

$$M_{x_2} = M_2\left(1 - \frac{x_2}{b}\right) + M_3\frac{x_2}{b} \tag{17-2}$$

$$M_{x_3} = M_3\left(1 - \frac{x_3}{a}\right) + M_4\frac{x_3}{a} \tag{17-3}$$

17-3. Vertical Uniform Load on Girder



W = Total load

Obtain value of load constant R_{23} from Chart 11 or 12.

$$K = \frac{2b\phi R_{23}}{F}$$

$$M_1 = M_4 = WAK \qquad M_2 = M_3 = -WK$$

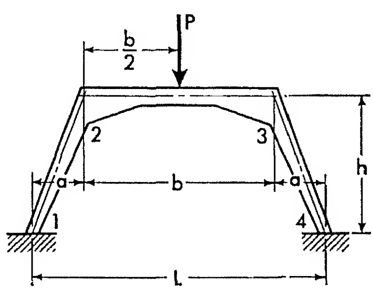
$$H_1 = H_4 = \frac{W}{h}\left[\frac{a}{2} + K(1 + A)\right]$$

$$V_1 = V_4 = \frac{W}{2}$$

$$M_{x_2} = \frac{Wx_2}{2}\left(1 - \frac{x_2}{b}\right) + M_2$$

Apply Eq. (17-1) to obtain the moment at any section of the left column. Moments at corresponding sections in the right half of the frame are identical to those in the left half.

17-4. Vertical Concentrated Load at Mid-point of Girder



Obtain value of load constant R_{23} from Table 1 or 3.

$$K = \frac{2b\phi R_{23}}{F}$$

$$M_1 = M_4 = PAK \qquad M_2 = M_3 = -PK$$

$$H_1 = H_4 = \frac{P}{h}\left[\frac{a}{2} + K(1 + A)\right]$$

$$V_1 = V_4 = \frac{P}{2}$$

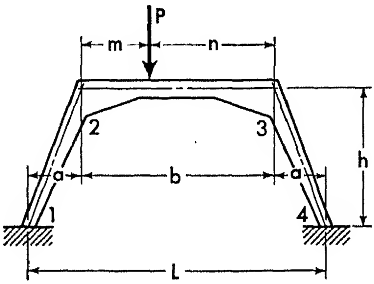
When $x_2 \leqq \frac{b}{2}$

$$M_{x_2} = \frac{Px_2}{2} + M_2$$

Apply Eq. (17-1) to obtain the moment at any section of the left column. Moments at corresponding sections in the right half of the frame are identical to those in the left half.

For Notations and Constants, see Arts. 17-1 and 17-2

17-5. Vertical Concentrated Load at Any Point of Girder



Obtain values of load constants R_{23} and R_{32} from applicable Tables 1 to 4.

$$J = \frac{b\phi(R_{23} + R_{32})}{F} \qquad K = \frac{b\phi(R_{23} - R_{32})}{2D}$$

$$S = \frac{a(1 - G)(b - 2m)}{2L} - K$$

$$T = \frac{Ga(b - 2m)}{2b} + \frac{KL}{b}$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = P(AJ \mp T)$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -P(J \mp S)$$

$$H_1 = H_4 = \frac{P}{h}\left[\frac{a}{2} + J(1 + A)\right]$$

$$\left.\begin{matrix} V_1 \\ V_4 \end{matrix}\right\rangle = \frac{P}{2} \pm \frac{P}{2L}(4T + n - m)$$

When $x_2 \leqq m$

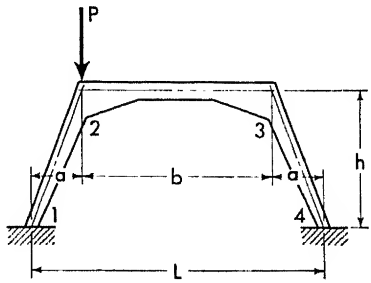
$$M_{x_2} = (Pn + M_3)\frac{x_2}{b} + M_2\left(1 - \frac{x_2}{b}\right)$$

When $x_2 > m$

$$M_{x_2} = (Pm + M_2)\left(1 - \frac{x_2}{b}\right) + M_3\frac{x_2}{b}$$

Apply Eqs. (17-1) and (17-3) to obtain the moment at any section of the frame columns.

17-6. Vertical Concentrated Load at Joint 2



$$M_1 = -\frac{PGa}{2} \qquad M_4 = \frac{PGa}{2}$$

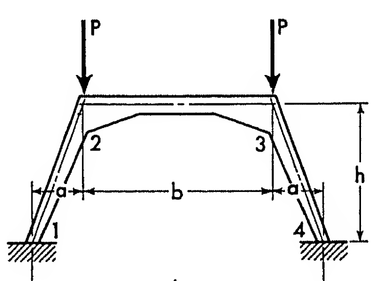
$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = \pm\frac{Pab(1-G)}{2L}$$

$$H_1 = H_4 = \frac{Pa}{2h}$$

$$V_4 = \frac{Pa(1-G)}{L} \qquad V_1 = P - V_4$$

Apply Eqs. (17-1) through (17-3) to obtain the moment at any section of the frame members.

17-7. Two Vertical Concentrated Loads at Joints 2 and 3

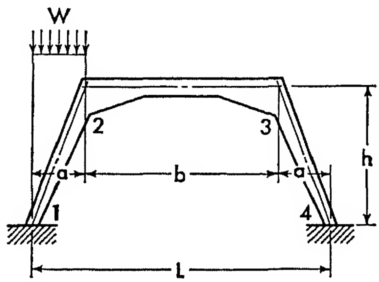


$$H_1 = H_4 = \frac{Pa}{h} \qquad V_1 = V_4 = P$$

There are no bending moments.

For Notations and Constants, see Arts. 17-1 and 17-2

17-8. Vertical Uniform Load on Inclined Column



W = Total load

Obtain values of load constants R_{12} and R_{21} from applicable Charts 11 to 16.

$$J = AR_{21} - \frac{R_{12}(\phi\Theta_{23} + 2\alpha_{21})}{2\alpha_{12}}$$

$$K = R_{21} - AR_{12}$$

$$N = \frac{R_{12} + R_{21}}{2D} + \frac{aR_{12}}{Db}$$

$$S = \frac{b(1 - G)}{4L} - N$$

$$T = \frac{G}{4} + \frac{LN}{b}$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = Wa\left(\frac{J}{F} \mp T\right)$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -Wa\left(\frac{K}{F} \mp S\right)$$

$$H_1 = H_4 = \frac{Wa}{h}\left(\frac{1}{4} + \frac{J + K}{F}\right)$$

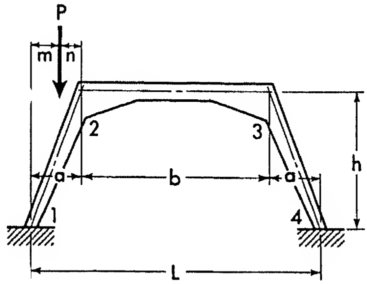
$$V_4 = \frac{Wa}{2L}(1 - 4T) \qquad V_1 = W - V_4$$

$$M_{x_1} = \left(\frac{Wx_1}{2} + M_1\right)\left(1 - \frac{x_1}{a}\right) + M_2\frac{x_1}{a}$$

Apply Eqs. (17-2) and (17-3) to obtain the moment at any section of frame members 2-3 and 3-4.

Members of Variable Section

17-9. Vertical Concentrated Load at Any Point of Inclined Column



Obtain values of load constants R_{12} and R_{21} from applicable Tables 1 to 8.

$$J = AR_{21} - \frac{R_{12}(\phi\Theta_{23} + 2\alpha_{21})}{2\alpha_{12}}$$

$$K = R_{21} - AR_{12}$$

$$N = \frac{R_{12} + R_{21}}{2D} + \frac{aR_{12}}{Db}$$

$$S = \frac{bm(1 - G)}{2La} - N$$

$$T = \frac{Gm}{2a} + \frac{LN}{b}$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = Pa\left(\frac{J}{F} \mp T\right)$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -Pa\left(\frac{K}{F} \mp S\right)$$

$$H_1 = H_4 = \frac{P}{h}\left[\frac{m}{2} + \frac{a(J + K)}{F}\right]$$

$$V_4 = \frac{P}{L}(m - 2Ta) \qquad V_1 = P - V_4$$

When $x_1 \leqq m$

$$M_{x_1} = (Pn + M_2)\frac{x_1}{a} + M_1\left(1 - \frac{x_1}{a}\right)$$

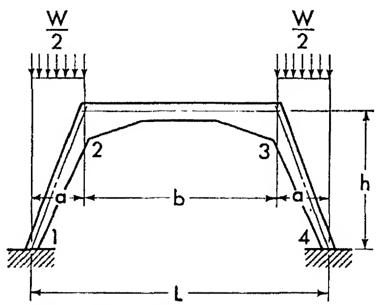
For Notations and Constants, see Arts. 17-1 and 17-2

When $x_1 > m$

$$M_{x_1} = (Pm + M_1)\left(1 - \frac{x_1}{a}\right) + M_2\frac{x_1}{a}$$

Apply Eqs. (17-2) and (17-3) to obtain the moment at any section of frame members 2-3 and 3-4.

17-10. Vertical Uniform Load over Both Inclined Columns



W = Total load

In view of the symmetry of the loading about the frame center line, the solution of the problem may be obtained by the use of the load constants of the left inclined column alone. Thus, obtaining values of the load constants R_{12} and R_{21} in the same way as for uniform load on member 1-2 only (Art. 17-8), and introducing them into the equations given below, the solution of the problem is readily obtained.

$$J = AR_{21} - \frac{R_{12}(\Theta_{23}\phi + 2\alpha_{21})}{2\alpha_{12}}$$

$$K = R_{21} - AR_{12}$$

$$M_1 = M_4 = \frac{WJa}{F} \qquad M_2 = M_3 = -\frac{WKa}{F}$$

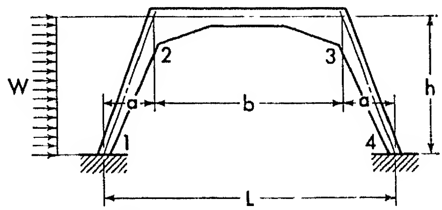
$$H_1 = H_4 = \frac{Wa}{h}\left(\frac{1}{4} + \frac{J+K}{F}\right) \qquad V_1 = V_4 = \frac{W}{2}$$

$$M_{x_1} = \left(\frac{Wx_1}{4} + M_1\right)\left(1 - \frac{x_1}{a}\right) + M_2\frac{x_1}{a}$$

$$M_{x_2} = M_2$$

Moments at corresponding sections in the right half of the frame are identical to those in the left half.

17-11. Horizontal Uniform Load on Inclined Column



W = Total load

Obtain values of load constants R_{12} and R_{21} from applicable Charts 11 to 16.

$$J = AR_{21} - \frac{R_{12}(\phi\Theta_{23} + 2\alpha_{21})}{2\alpha_{12}}$$

$$K = R_{21} - AR_{12}$$

$$N = \frac{R_{12} + R_{21}}{2D} + \frac{aR_{12}}{Db}$$

$$S = \frac{b(1 - G)}{4L} - N$$

$$T = \frac{G}{4} + \frac{LN}{b}$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = Wh\left(\frac{J}{F} \mp T\right)$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -Wh\left(\frac{K}{F} \mp S\right)$$

$$H_4 = \frac{W}{4}\left[1 + \frac{4(J + K)}{F}\right]$$

$$H_1 = -(W - H_4)$$

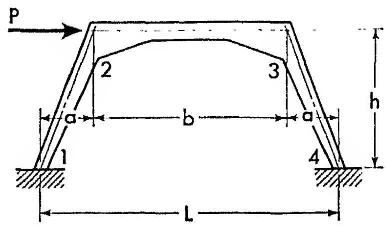
$$V_4 = \frac{Wh}{2L}(1 - 4T) \qquad V_1 = -V_4$$

$$M_{y_1} = \left(\frac{Wy_1}{2} + M_1\right)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (17-2) and (17-3) to obtain the moment at any section of frame members 2-3 and 3-4.

For Notations and Constants, see Arts. 17-1 and 17-2

17-12. Horizontal Concentrated Load at Joint 2



$$M_1 = -\frac{PGh}{2} \qquad M_4 = \frac{PGh}{2}$$

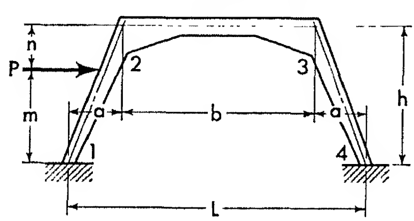
$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = \pm\frac{Pbh(1-G)}{2L}$$

$$H_1 = -\frac{P}{2} \qquad H_4 = \frac{P}{2}$$

$$V_4 = \frac{Ph}{L}(1-G) \qquad V_1 = -V_4$$

Apply Eqs. (17-1) through (17-3) to obtain the moment at any section of the frame members.

17-13. Horizontal Concentrated Load at Any Point of Inclined Column



Obtain values of load constants R_{12} and R_{21} from applicable Tables 1 to 8.

$$J = AR_{21} - \frac{R_{12}(\phi\Theta_{23} + 2\alpha_{21})}{2\alpha_{12}}$$

$$K = R_{21} - AR_{12}$$

$$N = \frac{R_{12} + R_{21}}{2D} + \frac{aR_{12}}{Db}$$

$$S = \frac{bm(1 - G)}{2Lh} - N$$

$$T = \frac{Gm}{2h} + \frac{LN}{b}$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = Ph\left(\frac{J}{F} \mp T\right)$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -Ph\left(\frac{K}{F} \mp S\right)$$

$$H_4 = \frac{P}{h}\left(\frac{m}{2} + \frac{J + K}{F}\right) \qquad H_1 = -(P - H_4)$$

$$V_4 = \frac{P}{L}(m - 2Th) \qquad V_1 = -V_4$$

When $y_1 \leqq m$

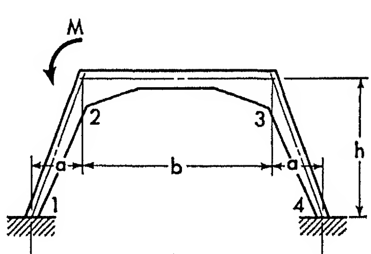
$$M_{y_1} = (Pn + M_2)\frac{y_1}{h} + M_1\left(1 - \frac{y_1}{h}\right)$$

When $y_1 > m$

$$M_{y_1} = (Pm + M_1)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (17-2) and (17-3) to obtain the moment at any section of frame members 2-3 and 3-4.

17-14. Moment Applied at Joint 2



$$J = \frac{\phi(\alpha_{23} + \beta_{23})}{F} \qquad K = \frac{\phi(\alpha_{23} - \beta_{23})}{2D}$$

For Notations and Constants, see Arts. 17-1 and 17-2

$$S = \frac{a(1 - G)}{L} + K$$

$$T = \frac{Ga - KL}{b}$$

$$\left.\begin{matrix} M_1 \\ M_4 \end{matrix}\right\rangle = -M(AJ \pm T)$$

$$\left.\begin{matrix} M_{21} \\ M_3 \end{matrix}\right\rangle = M(J \pm S) \qquad M_{23} = -(M - M_{21})$$

$$H_1 = H_4 = -\frac{MJ(1 + A)}{h}$$

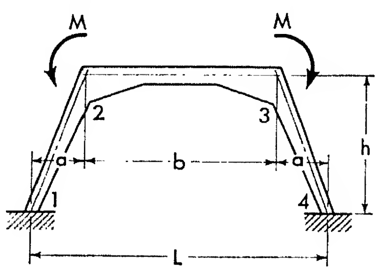
$$\left.\begin{matrix} V_1 \\ V_4 \end{matrix}\right\rangle = \pm \frac{M}{L}(1 + 2T)$$

$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) + M_{21}\frac{y_1}{h}$$

$$M_{x_2} = M_{23}\left(1 - \frac{x_2}{b}\right) + M_3\frac{x_2}{b}$$

$$M_{y_4} = M_4\left(1 - \frac{y_4}{h}\right) + M_3\frac{y_4}{h}$$

17-15. Two Equal Moments Applied at Joints 2 and 3



$$J = \frac{\phi(\alpha_{23} + \beta_{23})}{F}$$

$$M_1 = M_4 = -2MAJ$$

$$M_{21} = M_{34} = 2MJ$$

$$M_{23} = M_{32} = -(M - M_{21})$$

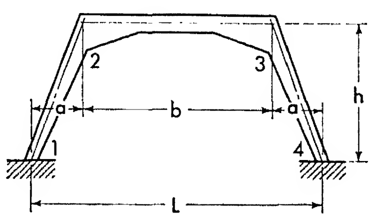
$$H_1 = H_4 = -\frac{2MJ(1 + A)}{h} \qquad V_1 = V_4 = 0$$

$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) + M_{21}\frac{y_1}{h}$$

$$M_{x_2} = M_{23}$$

$$M_{y_4} = M_4\left(1 - \frac{y_4}{h}\right) + M_{34}\frac{y_4}{h}$$

17-16. Effect of Temperature Rise. Range t° for the entire frame.



$$K = \frac{12L\varepsilon t^\circ}{Fhq} E(\min I_{1\text{-}2})$$

$$M_1 = M_4 = -K(1 + A) + \frac{BK}{2\alpha_{12}}$$

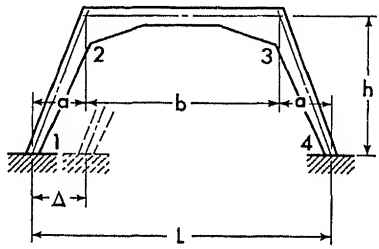
$$M_2 = M_3 = -K(1 + A)$$

$$H_1 = H_4 = \frac{BK}{2h\alpha_{12}} \qquad V_1 = V_4 = 0$$

Apply Eqs. (17-1) through (17-3) to obtain the moment at any section of the frame members.

Note: For temperature drop, introduce the value of t° with a negative sign.

17-17. Horizontal Displacement of One Support



$$K = \frac{12\Delta}{Fhq} E(\min I_{1\text{-}2})$$

$$M_1 = M_4 = -K(1 + A) + \frac{BK}{2\alpha_{12}}$$

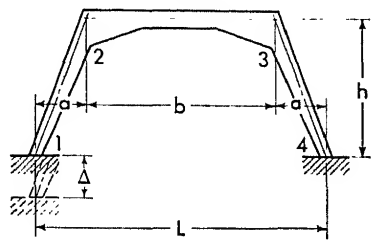
$$M_2 = M_3 = -K(1 + A)$$

$$H_1 = H_4 = \frac{BK}{2h\alpha_{12}} \qquad V_1 = V_4 = 0$$

Apply Eqs. (17-1) through (17-3) to obtain the moment at any section of the frame members.

Note: If the direction of the frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.

17-18. Vertical Settlement of One Support



$$K = \frac{12\Delta}{Db^2q} E(\min I_{1\text{-}2})$$

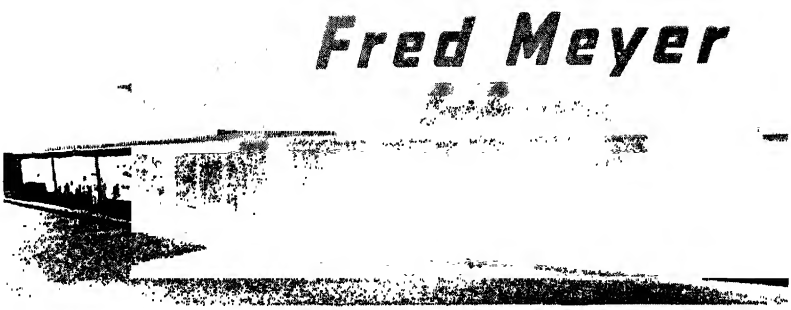
$$M_1 = -M_4 = KL$$

$$M_2 = -M_3 = Kb$$

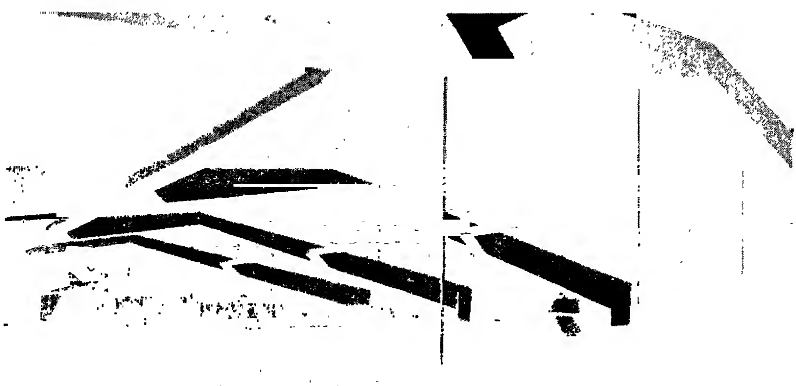
$$V_1 = -V_4 = -2K \qquad H_1 = H_4 = 0$$

Apply Eqs. (17-1) through (17-3) to obtain the moment at any section of the frame members.

Note: If the direction of the frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.



The Burlingame Shopping Center in Portland, Oregon, after dark. Elegantly shaped gable frames with members of variable cross section are attractively silhouetted in the background. Designed by Leslie E. Poole, consulting engineer, of Portland, Oregon. (Courtesy of Leslie E. Poole and the Photo-Art Commercial Studios, Portland, Ore.)

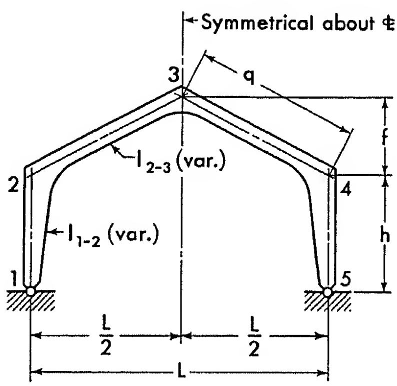


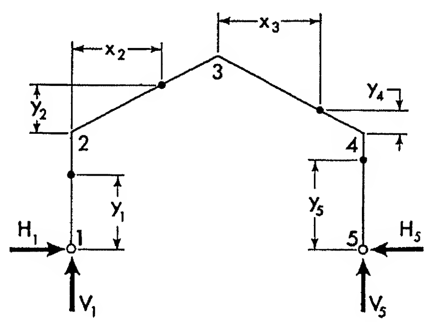
A roof-top view of the concrete gable girders of the Burlingame Shopping Center in Portland, Oregon. The frame girders of variable cross section are 130 feet long and support the building ceiling by means of steel hangers. Lightweight concrete was advantageously employed to reduce the dead load of the structure. (Courtesy of Leslie E. Poole, consulting engineer, Portland, Ore.)

SECTION 18

SYMMETRICAL GABLE FRAMES WITH HINGED SUPPORTS

18-1. Notations, Coordinates, and Frame Constants





The sketch appearing on the left, above, explains notations for a representative gable frame with members of variable cross section.

The solutions of analysis given on the following pages are not limited to the shapes of the members shown, but are applicable to any shape, provided only that the frame is symmetrical.

The sketch appearing on the right, above, shows positive directions of the vertical and horizontal components of the frame reactions. It also defines the coordinates for any section of the frame. Coordinates to be considered only in the positive sense.

General Frame Constants. Obtain numerical values of elastic parameters α_{21}, α_{23}, α_{32}, and β_{23} from applicable Charts 1 to 10 in the Appendix.

$$\phi = \frac{\min I_{1-2}}{\min I_{2-3}} \cdot \frac{q}{h} \qquad \psi = \frac{f}{h}$$

$$\Theta_{23} = \alpha_{23} + \alpha_{32} + 2\beta_{23}$$

$$A = \Theta_{23} + \psi^2\alpha_{32} + 2\psi(\alpha_{32} + \beta_{23}) + \frac{\alpha_{21}}{\phi} \qquad B = \alpha_{32}(1 + \psi) + \beta_{23}$$

Constant C. To be used only in cases of horizontal load on frame.

$$C = \alpha_{23} + \beta_{23}(1 + \psi) + \frac{\alpha_{21}}{\phi}$$

18-2. Equations of Frame Reactions and Moments. The equations for the vertical and the redundant horizontal components of frame reactions are given on the following pages for a number of loading conditions. Corresponding expressions for the moment and forces at any section of a member with applied load are also provided.

The equations for the moments of load-free members are listed below for reference.

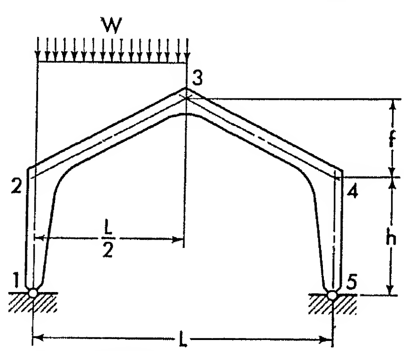
$$M_{y_1} = M_2 \frac{y_1}{h} \tag{18-1}$$

$$M_{x_2} = M_2 \left(1 - \frac{2x_2}{L}\right) + M_3 \frac{2x_2}{L} \tag{18-2}$$

$$M_{x_3} = M_3 \left(1 - \frac{2x_3}{L}\right) + M_4 \frac{2x_3}{L} \tag{18-3}$$

$$M_{y_5} = M_4 \frac{y_5}{h} \tag{18-4}$$

18-3. Vertical Uniform Load on Left Inclined Member



W = total load

Obtain values of load constants R_{23} and R_{32} from applicable Charts 11 to 16.

Members of Variable Section

$$K = R_{23} + R_{32}(1 + \psi)$$

$$H_1 = H_5 = \frac{WL}{8Ah}(B + 2K)$$

$$M_2 = M_4 = -H_5h$$

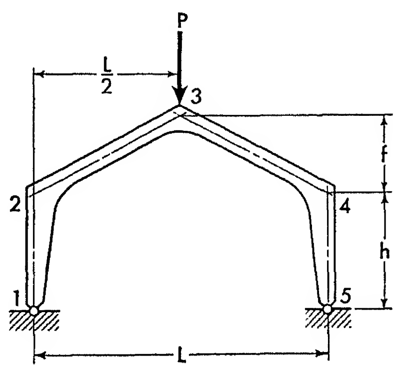
$$M_3 = \frac{WL}{8} - H_5h(1 + \psi)$$

$$V_1 = \frac{3}{4}W \qquad V_5 = \frac{W}{4}$$

$$M_{x_2} = \left(M_2 + \frac{Wx_2}{2}\right)\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (18-1), (18-3), and (18-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

18-4. Vertical Concentrated Load at Joint 3



$$H_1 = H_5 = \frac{PLB}{4Ah}$$

$$M_2 = M_4 = -H_5h$$

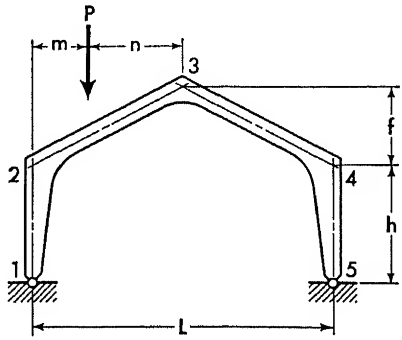
$$M_3 = \frac{PL}{4} - H_5h(1 + \psi)$$

$$V_1 = V_5 = \frac{P}{2}$$

Apply Eqs. (18-1) through (18-4) to obtain the moment at any section of the frame members.

For Notations and Constants, see Arts. 18-1 and 18-2

18-5. Vertical Concentrated Load at Any Point of Left Inclined Member



Obtain values of load constants R_{23} and R_{32} from applicable Tables 1 to 8.

$$K = R_{23} + R_{32}(1 + \psi)$$

$$H_1 = H_5 = \frac{P}{4Ah}(2Bm + KL) \qquad M_2 = M_4 = -H_5 h$$

$$M_3 = \frac{Pm}{2} - H_5 h(1 + \psi)$$

$$V_1 = P\left(1 - \frac{m}{L}\right) \qquad V_5 = \frac{Pm}{L}$$

When $x_2 \leqq m$

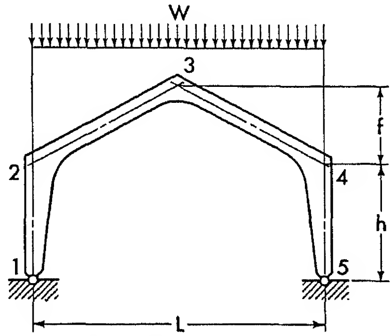
$$M_{x_2} = (M_3 + Pn)\frac{2x_2}{L} + M_2\left(1 - \frac{2x_2}{L}\right)$$

When $x_2 > m$, but $< \frac{L}{2}$

$$M_{x_2} = (M_2 + Pm)\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (18-1), (18-3), and (18-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

18-6. Vertical Uniform Load over Entire Girder



W = total load

In view of the symmetry of the loading about the frame center line, the solution of the problem may be obtained by the use of the load constants of the left inclined member alone. Thus, obtaining values of load constants R_{23} and R_{32} in the same way as for uniform load on member 2-3 only (Art. 18-3), and introducing them into the equations given below, the solution of the problem is readily obtained.

$$K = R_{23} + R_{32}(1 + \psi)$$

$$H_1 = H_5 = \frac{WL}{8Ah}(B + 2K) \qquad M_2 = M_4 = -H_5h$$

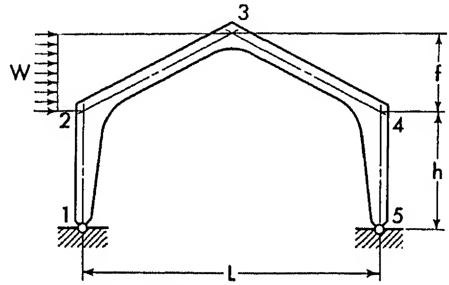
$$M_3 = \frac{WL}{8} - H_5h(1 + \psi) \qquad V_1 = V_5 = \frac{W}{2}$$

For any section of member 2-3

$$M_{x_2} = \left(\frac{Wx_2}{4} + M_2\right)\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eq. (18-1) to obtain the moment at any section of the left column. Moments at corresponding sections in the right half of the frame are identical to those in the left half.

18-7. Horizontal Uniform Load on Left Inclined Member



W = total load

Obtain values of load constants R_{23} and R_{32} from applicable Charts 11 to 16.

$$K = R_{23} + R_{32}(1 + \psi)$$

$$N = A + B + C + 2K\psi$$

$$H_5 = \frac{WN}{4A} \qquad H_1 = -(W - H_5)$$

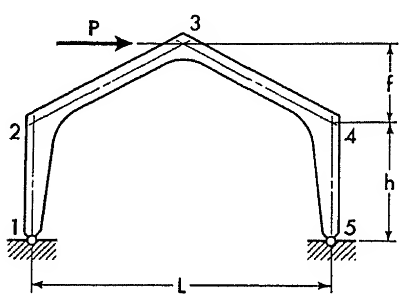
$$M_2 = h(W - H_5) \qquad M_4 = -H_5 h$$

$$M_3 = \frac{Wh}{4}(2 + \psi) - H_5 h(1 + \psi) \qquad V_5 = \frac{Wh}{2L}(2 + \psi)$$

$$V_1 = -V_5 \qquad M_{y_2} = \left(\frac{Wy_2}{2} + M_2\right)\left(1 - \frac{y_2}{f}\right) + M_3\frac{y_2}{f}$$

Apply Eqs. (18-1), (18-3), and (18-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

18-8. Horizontal Concentrated Load at Joint 3



$$H_1 = -\frac{P}{2} \qquad H_5 = \frac{P}{2}$$

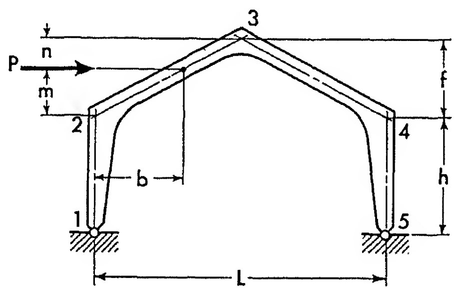
$$M_2 = \frac{Ph}{2} \qquad M_3 = 0$$

$$M_4 = -\frac{Ph}{2}$$

$$V_5 = \frac{Ph}{L}(1 + \psi) \qquad V_1 = -V_5$$

Apply Eqs. (18-1) through (18-4) to obtain the moment at any section of the frame members.

18-9. Horizontal Concentrated Load at Any Point of Left Inclined Member



Obtain values of load constants R_{23} and R_{32} from applicable Tables 1 to 8.

$$g = \frac{m}{f} \qquad K = R_{23} + R_{32}(1 + \psi)$$

$$N = B(1 + g\psi) + C + K\psi$$

$$H_5 = \frac{PN}{2A} \qquad H_1 = -(P - H_5)$$

$$M_2 = h(P - H_5)$$

$$M_3 = \frac{P}{2}(h + m) - H_5 h(1 + \psi)$$

$$M_4 = -H_5 h \qquad V_5 = \frac{P}{L}(h + m) \qquad V_1 = -V_5$$

When $x_2 \leqq b$

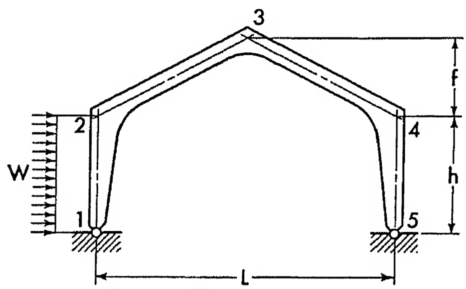
$$M_{x_2} = (Pn + M_3)\frac{2x_2}{L} + M_2\left(1 - \frac{2x_2}{L}\right)$$

When $x_2 > b$

$$M_{x_2} = (Pm + M_2)\left(1 - \frac{2x_2}{L}\right) + M_3 \frac{2x_2}{L}$$

Apply Eqs. (18-1), (18-3), and (18-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

18-10. Horizontal Uniform Load on Column



W = total load

Obtain value of load constant R_{21} from applicable Charts 11 to 16.

$$N = B + C + \frac{2R_{21}}{\phi}$$

$$H_5 = \frac{WN}{4A} \qquad H_1 = -(W - H_5)$$

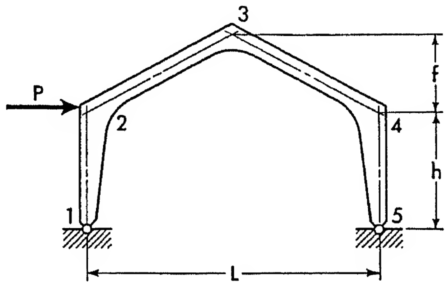
$$M_2 = h\left(\frac{W}{2} - H_5\right) \qquad M_3 = h\left[\frac{W}{4} - H_5(1 + \psi)\right]$$

$$M_4 = -H_5 h \qquad V_5 = \frac{Wh}{2L} \qquad V_1 = -V_5$$

$$M_{y_1} = \frac{Wy_1}{2}\left(1 - \frac{y_1}{h}\right) + M_2 \frac{y_1}{h}$$

Apply Eqs. (18-2) through (18-4) to obtain the moment at any section of frame members 2-3, 3-4, and 4-5.

18-11. Horizontal Concentrated Load at Joint 2



$$H_5 = \frac{P}{2A}(B + C) \qquad H_1 = -(P - H_5)$$

$$M_2 = h(P - H_5)$$

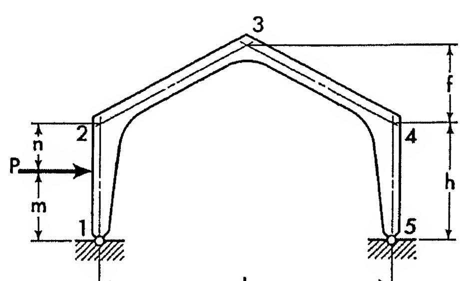
$$M_3 = h\left[\frac{P}{2} - H_5(1 + \psi)\right]$$

$$M_4 = -H_5 h$$

$$V_5 = \frac{Ph}{L} \qquad V_1 = -V_5$$

Apply Eqs. (18-1) through (18-4) to obtain the moment at any section of the frame members.

18-12. Horizontal Concentrated Load at Any Point of Column



Obtain value of load constant R_{21} from applicable Tables 1 to 8.

$$N = m(B + C) + \frac{hR_{21}}{\phi}$$

For Notations and Constants, see Arts. 18-1 and 18-2

$$H_5 = \frac{PN}{2Ah} \qquad H_1 = -(P - H_5)$$

$$M_2 = Pm - H_5h$$

$$M_3 = \frac{Pm}{2} - H_5h(1 + \psi) \qquad M_4 = -H_5h$$

$$V_5 = \frac{Pm}{L} \qquad V_1 = -V_5$$

When $y_1 \leqq m$

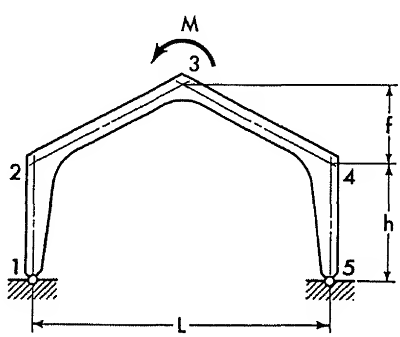
$$M_{y_1} = (Pn + M_2)\frac{y_1}{h}$$

When $y_1 > m$

$$M_{y_1} = Pm\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (18-2) through (18-4) to obtain the moment at any section of frame members 2-3, 3-4, and 4-5.

18-13. Moment Applied at Joint 3



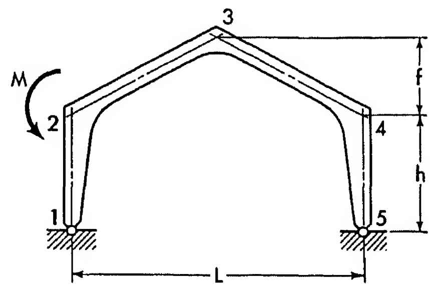
$$H_1 = H_5 = 0 \qquad M_2 = M_4 = 0$$

$$M_{32} = \frac{M}{2} \qquad M_{34} = -\frac{M}{2}$$

$$V_1 = \frac{M}{L} \qquad V_5 = -\frac{M}{L}$$

$$M_{x_2} = M_{32}\frac{2x_2}{L} \qquad M_{x_3} = M_{34}\left(1 - \frac{2x_3}{L}\right)$$

18-14. Moment Applied at Joint 2



$$K = \alpha_{23} + \beta_{23}(1 + \psi)$$

$$H_1 = H_5 = -\frac{M}{2Ah}(B + K)$$

$$M_{21} = M_4 = \frac{M}{2A}(B + K) \qquad M_{23} = -(M - M_{21})$$

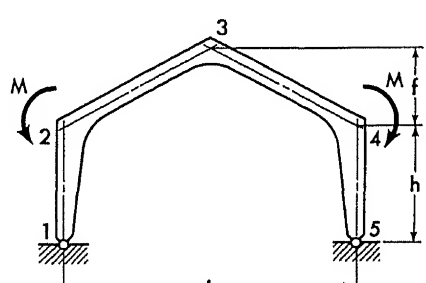
$$M_3 = -\frac{M}{2} - H_5 h(1 + \psi)$$

$$V_1 = \frac{M}{L} \qquad V_5 = -\frac{M}{L} \qquad M_{y_1} = M_{21}\frac{y_1}{h}$$

$$M_{x_2} = M_{23}\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (18-3) and (18-4) to obtain the moment at any section of frame members 3-4 and 4-5.

18-15. Two Equal Moments Applied at Joints 2 and 4



$$K = \alpha_{23} + \beta_{23}(1 + \psi)$$

$$H_1 = H_5 = -\frac{M}{Ah}(B + K)$$

$$M_{21} = M_{45} = \frac{M}{A}(B + K)$$

$$M_{23} = M_{43} = -M\left(1 - \frac{B + K}{A}\right)$$

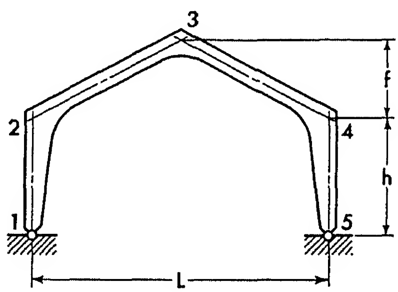
$$M_3 = -M\left[1 - \frac{(B + K)(1 + \psi)}{A}\right]$$

$$V_1 = V_5 = 0 \qquad M_{y_1} = M_{21}\frac{y_1}{h}$$

$$M_{x_2} = M_{23}\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Moments at corresponding sections in the right half of the frame are identical to those in the left half.

18-16. Effect of Temperature Rise. Range t° for the entire frame.



$$K = \frac{6L\varepsilon t^\circ}{Ah^2\phi}E(\min I_{1\text{-}2})$$

$$M_2 = M_4 = -K$$

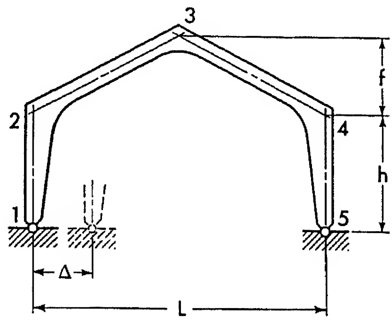
$$M_3 = M_2(1 + \psi)$$

$$H_1 = H_5 = \frac{K}{h} \qquad V_1 = V_5 = 0$$

Apply Eqs. (18-1) through (18-4) to obtain the moment at any section of the frame members.

Note: For temperature drop, introduce the value of t° with a negative sign.

18-17. Horizontal Displacement of One Support



$$K = \frac{6\Delta}{Ah^2\phi} E(\min I_{1\text{-}2})$$

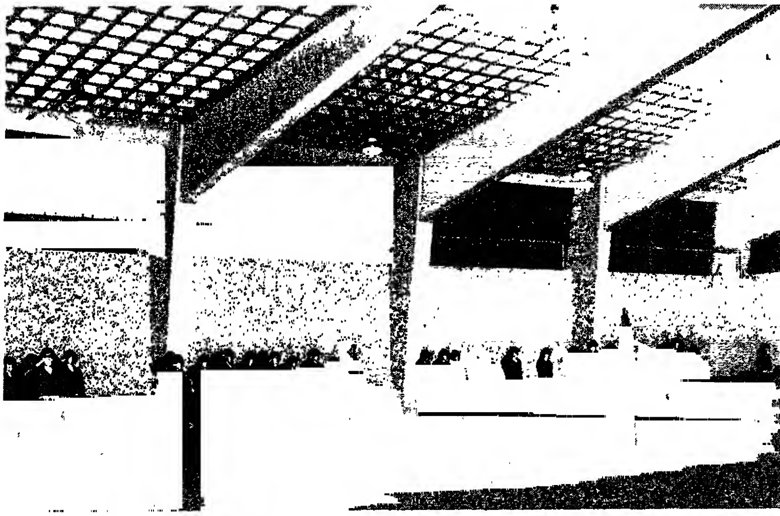
$$M_2 = M_4 = -K$$

$$M_3 = M_2(1 + \psi)$$

$$H_1 = H_5 = \frac{K}{h} \qquad V_1 = V_5 = 0$$

Apply Eqs. (18-1) through (18-4) to obtain the moment at any section of the frame members.

Note: If the direction of the frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.



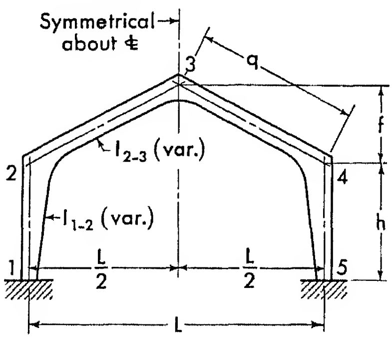
West Virginia State National Armory in Buckhannon, West Virginia. Neat lines of impressive gable frames spanning the 95-foot-wide armory and an abundance of light are the characteristic features of this outstanding structure. Note that the waffle ceiling effectively contributes to the over-all elegance of the building. (Courtesy of Baker and Coombs, Morgantown, W.Va., builders, and Williams Form Engineering Co., Grand Rapids, Mich., concrete accessories manufacturer.)

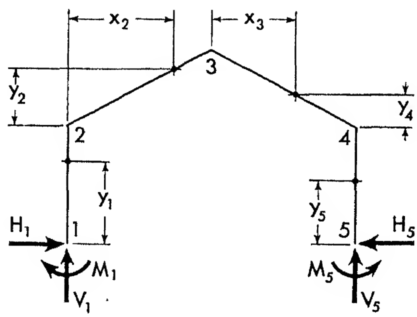
The Southern Cast Stone House exhibited in Knoxville, Tennessee. This modern reinforced concrete house was assembled from precast slabs, gable frames, wall panels, etc. Current construction techniques make full use of precast or prestressed structural members, inexpensively mass produced at the plant and assembled at the site with relative ease. Pleasant appearance is definitely exhibited by the house as a whole as well as by its individual components. (Courtesy of the Portland Cement Association.)

SECTION 19

SYMMETRICAL GABLE FRAMES WITH FIXED SUPPORTS

19-1. Notations, Coordinates, and Frame Constants





The sketch appearing on the left, above, explains notations for a representative gable frame with members of variable cross section.

The solutions of analysis given on the following pages are not limited to the shapes of the members shown, but are applicable to any shape, provided only that the frame is symmetrical.

The sketch appearing on the right, above, shows positive directions of the moments and the vertical and horizontal components of the frame reactions. It also defines the coordinates for any section of the frame. Coordinates to be considered only in the positive sense.

Frame Constants. Obtain numerical values of elastic parameters α_{12}, α_{21}, β_{12}, α_{23}, α_{32}, and β_{23} from applicable Charts 1 to 10 in the Appendix.

$$\phi = \frac{\min I_{1-2}}{\min I_{2-3}} \cdot \frac{q}{h} \qquad \psi = \frac{f}{h}$$

$$\Theta_{12} = \alpha_{12} + \alpha_{21} + 2\beta_{12} \qquad \Theta_{23} = \alpha_{23} + \alpha_{32} + 2\beta_{23}$$

$$A = \alpha_{12} + \beta_{12} - \phi\psi(\alpha_{32} + \beta_{23}) \qquad B = 2(\Theta_{12} + \phi\Theta_{23})$$

$$C = \alpha_{12} + \phi\psi^2\alpha_{32} \qquad D = 4(\Theta_{12} + \phi\alpha_{23})$$

$$F = B - 2AS \qquad G = \beta_{23} + (1 + S\psi)\,\alpha_{32}$$

$$N = S(\alpha_{32} + \beta_{23}) + \frac{B\psi\alpha_{32}}{2C} \qquad S = \frac{A}{C}$$

19-2. Equations of Frame Reactions and Moments. The equations for the redundant moments and the vertical and horizontal components of frame reactions are given on the following pages for a number of loading conditions. Corresponding expressions for the moment and forces at any section of a member with applied load are also provided.

The equations for the moments of load-free members are listed below for reference.

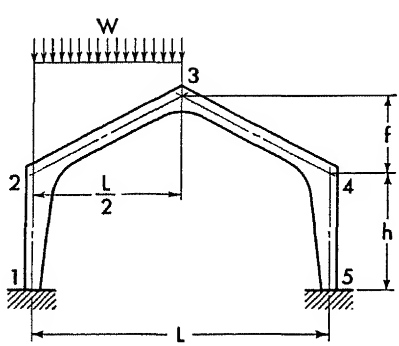
$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h} \tag{19-1}$$

$$M_{x_2} = M_2\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L} \tag{19-2}$$

$$M_{x_3} = M_3\left(1 - \frac{2x_3}{L}\right) + M_4\frac{2x_3}{L} \tag{19-3}$$

$$M_{y_5} = M_5\left(1 - \frac{y_5}{h}\right) + M_4\frac{y_5}{h} \tag{19-4}$$

19-3. Vertical Uniform Load on Left Inclined Member



W = Total load

Obtain values of load constants R_{23} and R_{32} from applicable Charts 11 to 16.

Members of Variable Section

$$J = R_{23} + R_{32}(1 + S\psi)$$

$$K = S(R_{23} + R_{32}) + \frac{B\psi R_{32}}{2C}$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = -WL\phi\left(\frac{G + 2J}{4F} \pm \frac{R_{23}}{D}\right)$$

$$H_1 = H_5 = \frac{WL\phi}{4Fh}(2K + N)$$

$$M_1 = M_2 + H_1h \qquad M_3 = \frac{M_2 + M_4}{2} + \frac{WL}{8} - H_1f$$

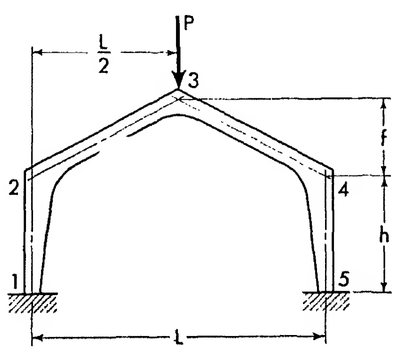
$$M_5 = M_4 + H_5h \qquad V_5 = \frac{W}{4D}(D - 8\phi R_{23})$$

$$V_1 = W - V_5$$

$$M_{x_2} = \left(\frac{Wx_2}{2} + M_2\right)\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (19-1), (19-3), and (19-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

19-4. Vertical Concentrated Load at Joint 3



$$M_2 = M_4 = -\frac{PLG\phi}{2F}$$

$$M_1 = M_5 = \frac{PL\phi}{2F}(N - G)$$

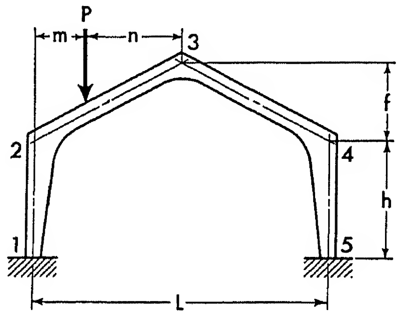
$$H_1 = H_5 = \frac{PLN\phi}{2Fh} \qquad V_1 = V_5 = \frac{P}{2}$$

For Notations and Constants, see Arts. 19-1 and 19-2

$$M_3 = -\frac{PL\phi}{2F}(G + N\psi) + \frac{PL}{4}$$

Apply Eqs. (19-1) through (19-4) to obtain the moment at any section of the frame members.

19-5. Vertical Concentrated Load at Any Point of Left Inclined Member



Obtain values of load constants R_{23} and R_{32} from applicable Tables 1 to 8.

$$J = R_{23} + R_{32}(1 + S\psi)$$

$$K = S(R_{23} + R_{32}) + \frac{B\psi R_{32}}{2C}$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = -\frac{P\phi}{2F}(JL + 2Gm) \mp \frac{PL\phi R_{23}}{D}$$

$$H_1 = H_5 = \frac{P\phi}{2Fh}(KL + 2Nm) \qquad M_1 = M_2 + H_1h$$

$$M_3 = \frac{M_2 + M_4}{2} + \frac{Pm}{2} - H_1f$$

$$M_5 = M_4 + H_5h \qquad V_5 = P\left(\frac{m}{L} - \frac{2\phi R_{23}}{D}\right)$$

$$V_1 = P - V_5$$

When $x_2 \leqq m$

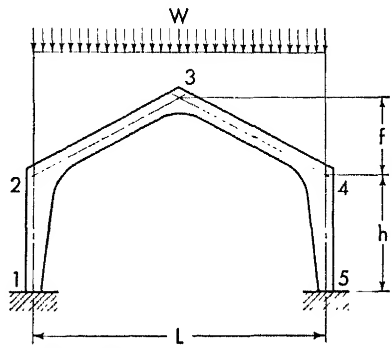
$$M_{x_2} = (Pn + M_3)\frac{2x_2}{L} + M_2\left(1 - \frac{2x_2}{L}\right)$$

When $x_2 > m$, but $\leq \frac{L}{2}$

$$M_{x_2} = (Pm + M_2)\left(1 - \frac{2x_2}{L}\right) + M_3\,\frac{2x_2}{L}$$

Apply Eqs. (19-1), (19-3), and (19-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

19-6. Vertical Uniform Load over Entire Girder



W = Total load

In view of the symmetry of the loading about the frame center line, the solution of the problem may be obtained by the use of the load constants of the left inclined member alone. Thus, obtaining values of load constants R_{23} and R_{32} in the same way as for uniform load on member 2-3 only (Art. 19-3), and introducing them into the equations given below, the solution of the problem is readily obtained.

$$J = R_{23} + R_{32}(1 + S\psi)$$

$$K = S(R_{23} + R_{32}) + \frac{B\psi R_{32}}{2C}$$

$$M_2 = M_4 = -\frac{WL\phi}{4F}(G + 2J)$$

$$H_1 = H_5 = \frac{WL\phi}{4Fh}(2K + N) \qquad M_1 = M_5 = M_2 + H_5 h$$

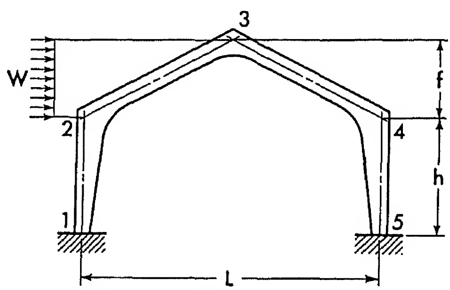
$$M_3 = M_2 + \frac{WL}{8} - H_5 f \qquad V_1 = V_5 = \frac{W}{2}$$

For any section of member 2-3

$$M_{x_2} = \left(\frac{Wx_2}{4} + M_2\right)\left(1 - \frac{2x_2}{L}\right) + M_3\,\frac{2x_2}{L}$$

Apply Eq. (19-1) to obtain the moment at any section of the left column. Moments at corresponding sections in the right half of the frame are identical to those in the left half.

19-7. Horizontal Uniform Load on Left Inclined Member



W = Total load

Obtain values of load constants R_{23} and R_{32} from applicable Charts 11 to 16.

$$J = R_{23} + R_{32}(1 + S\psi)$$

$$K = S(R_{23} + R_{32}) + \frac{B\psi R_{32}}{2C}$$

$$T = \alpha_{12} + \beta_{12} - \phi\psi R_{23}$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = -\frac{Wf\phi}{2F}(2J - G) \pm \frac{2WTh}{D}$$

$$H_5 = \frac{W}{2} - \frac{W\phi\psi}{2F}(N - 2K)$$

$$H_1 = -(W - H_5) \qquad M_1 = M_2 - h(W - H_5)$$

$$M_3 = \frac{M_2 + M_4}{2} - H_5 f + \frac{Wf}{4}$$

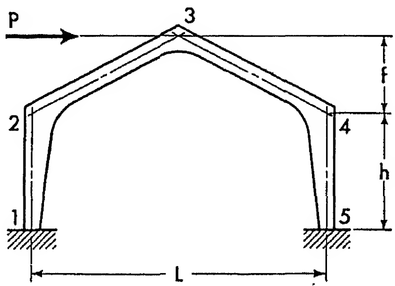
$$M_5 = M_4 + H_5 h \qquad V_5 = \frac{Wh}{2L}\left(\psi + \frac{8T}{D}\right)$$

$$V_1 = -V_5$$

$$M_{y_2} = \left(\frac{Wy_2}{2} + M_2\right)\left(1 - \frac{y_2}{f}\right) + M_3 \frac{y_2}{f}$$

Apply Eqs. (19-1), (19-3), and (19-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

19-8. Horizontal Concentrated Load at Joint 3



$$T = \alpha_{12} + \beta_{12}$$

$$M_2 = \frac{2PTh}{D} \qquad M_4 = -\frac{2PTh}{D}$$

$$H_1 = -\frac{P}{2} \qquad H_5 = \frac{P}{2}$$

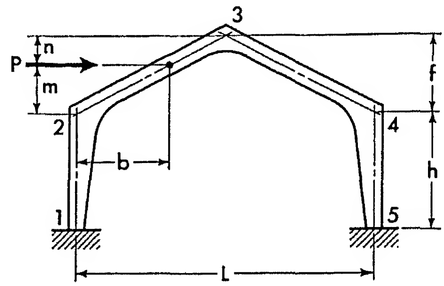
$$M_1 = M_2 - \frac{Ph}{2} \qquad M_3 = 0$$

$$M_5 = M_4 + \frac{Ph}{2}$$

$$V_5 = \frac{Ph}{L}\left(\psi + \frac{4T}{D}\right) \qquad V_1 = -V_5$$

Apply Eqs. (19-1) through (19-4) to obtain the moment at any section of the frame members.

19-9. Horizontal Concentrated Load at Any Point of Left Inclined Member



Obtain values of load constants R_{23} and R_{32} from applicable Tables 1 to 8.

$$J = R_{23} + R_{32}(1 + S\psi)$$

$$K = S(R_{23} + R_{32}) + \frac{B\psi R_{32}}{2C}$$

$$T = \alpha_{12} + \beta_{12} - \phi\psi R_{23}$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = -\frac{P}{F}(Jf - Gn) \pm \frac{2PTh}{D}$$

$$H_5 = \frac{P}{2} - \frac{P\phi}{Fh}(Nn - K)$$

$$H_1 = -(P - H_5)$$

$$M_1 = M_2 - h(P - H_5) \qquad M_3 = \frac{M_2 + M_4}{2} - H_5 f + \frac{Pm}{2}$$

$$M_5 = M_4 + H_5 h \qquad V_5 = \frac{P}{L}\left(m + \frac{4Th}{D}\right)$$

$$V_1 = -V_5$$

When $x_2 \leqq b$

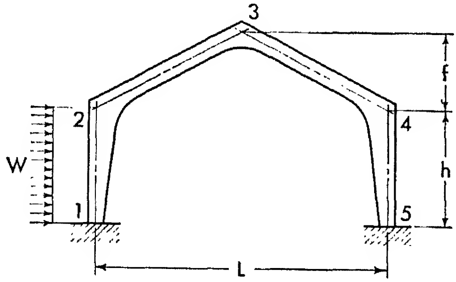
$$M_{x_2} = (Pn + M_3)\frac{2x_2}{L} + M_2\left(1 - \frac{2x_2}{L}\right)$$

When $x_2 > b$

$$M_{x_2} = (Pm + M_2)\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (19-1), (19-3), and (19-4) to obtain the moment at any section of frame members 1-2, 3-4, and 4-5.

19-10. Horizontal Uniform Load on Column



W = Total load

Obtain values of load constants R_{12} and R_{21} from applicable Charts 11 to 16.

$$J = \frac{BR_{12}}{2C} - S(R_{12} + R_{21})$$

$$K = R_{21} - R_{12}(S - 1)$$

$$T = \alpha_{12} + \beta_{12} - 2(R_{12} + R_{21})$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = \frac{Wh}{2F}(G\phi\psi - 2K) \pm \frac{WTh}{D}$$

$$H_5 = \frac{W}{4}\left[1 - \frac{2}{F}(N\phi\psi + 2J)\right]$$

$$H_1 = -(W - H_5) \qquad M_1 = M_2 + H_5h - \frac{Wh}{2}$$

$$M_3 = \frac{M_2 + M_4}{2} - H_5f \qquad M_5 = M_4 + H_5h$$

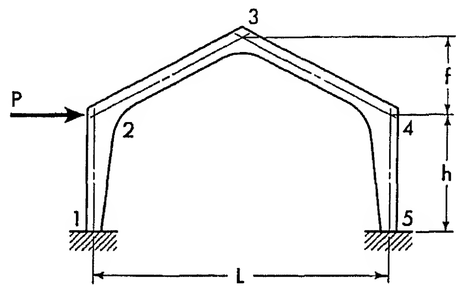
$$V_5 = \frac{2WTh}{DL} \qquad V_1 = -V_5$$

$$M_{y_1} = \left(\frac{Wy_1}{2} + M_1\right)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (19-2) through (19-4) to obtain the moment at any section of frame members 2-3, 3-4, and 4-5.

For Notations and Constants, see Arts. 19-1 and 19-2

19-11. Horizontal Concentrated Load at Joint 2



$$T = 2(\alpha_{12} + \beta_{12})$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = Ph\left(\frac{G\phi\psi}{F} \pm \frac{T}{D}\right)$$

$$H_5 = \frac{P}{2} - \frac{PN\phi\psi}{F} \qquad H_1 = -(P - H_5)$$

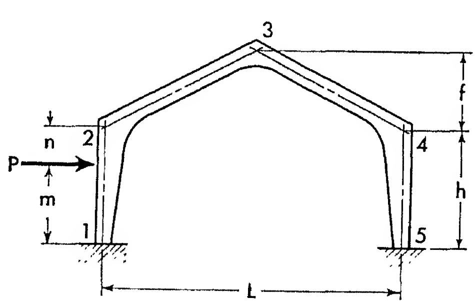
$$M_1 = M_2 - h(P - H_5) \qquad M_3 = \frac{M_2 + M_4}{2} - H_5 f$$

$$M_5 = M_4 + H_5 h$$

$$V_5 = \frac{2PTh}{DL} \qquad V_1 = -V_5$$

Apply Eqs. (19-1) through (19-4) to obtain the moment at any section of the frame members.

19-12. Horizontal Concentrated Load at Any Point of Column



Obtain values of load constants R_{12} and R_{21} from applicable Tables 1 to 8.

Members of Variable Section

$$J = \frac{BR_{12}}{2C} - S(R_{12} + R_{21})$$

$$K = R_{21} - R_{12}(S - 1)$$

$$T = m(\alpha_{12} + \beta_{12}) - h(R_{12} + R_{21})$$

$$\left.\begin{matrix} M_2 \\ M_4 \end{matrix}\right\rangle = \frac{P}{F}(Gm\phi\psi - Kh) \pm \frac{2PT}{D}$$

$$H_5 = \frac{P}{h}\left(\frac{m}{2} - \frac{Nm\phi\psi + Jh}{F}\right)$$

$$H_1 = -(P - H_5) \qquad M_1 = M_2 + H_5h - Pm$$

$$M_3 = \frac{M_2 + M_4}{2} - H_5f \qquad M_5 = M_4 + H_5h$$

$$V_5 = \frac{4PT}{DL} \qquad V_1 = -V_5$$

When $y_1 \leqq m$

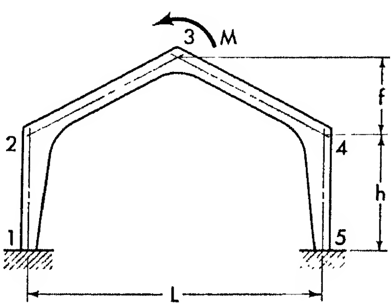
$$M_{y_1} = (Pn + M_2)\frac{y_1}{h} + M_1\left(1 - \frac{y_1}{h}\right)$$

When $y_1 > m$

$$M_{y_1} = (Pm + M_1)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (19-2) through (19-4) to obtain the moment at any section of frame members 2-3, 3-4, and 4-5.

19-13. Moment Applied at Joint 3



$$M_1 = M_2 = -\frac{2M\phi\beta_{23}}{D}$$

For Notations and Constants, see Arts. 19-1 and 19-2

$$M_4 = M_5 = \frac{2M\phi\beta_{23}}{D}$$

$$M_{32} = -M_{34} = \frac{M}{2} \qquad H_1 = H_5 = 0$$

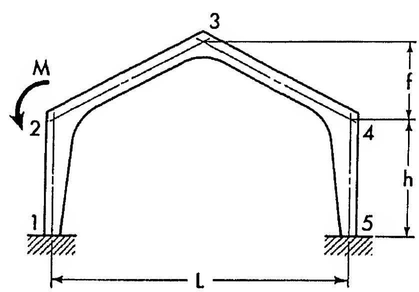
$$V_5 = -\frac{M}{L}\left(1 + \frac{4\phi\beta_{23}}{D}\right) \qquad V_1 = -V_5$$

$$M_{x_2} = M_2\left(1 - \frac{2x_2}{L}\right) + M_{32}\frac{2x_2}{L}$$

$$M_{x_3} = M_{34}\left(1 - \frac{2x_3}{L}\right) + M_4\frac{2x_3}{L}$$

Apply Eqs. (19-1) and (19-4) to obtain the moment at any section of frame members 1-2 and 4-5.

19-14. Moment Applied at Joint 2



$$T = \Theta_{23} + S\psi(\alpha_{32} + \beta_{23})$$

$$U = S\Theta_{23} + \frac{B\psi(\alpha_{32} + \beta_{23})}{2C}$$

$$\left.\begin{matrix} M_{21} \\ M_4 \end{matrix}\right\rangle = \frac{MT\phi}{F} \pm \frac{2M\phi\alpha_{23}}{D}$$

$$H_1 = H_5 = -\frac{MU\phi}{Fh} \qquad M_1 = M_{21} + H_5h$$

$$M_{23} = -(M - M_{21})$$

$$M_3 = \frac{MT\phi}{F} - \frac{M}{2} - H_5f \qquad M_5 = M_4 + H_5h$$

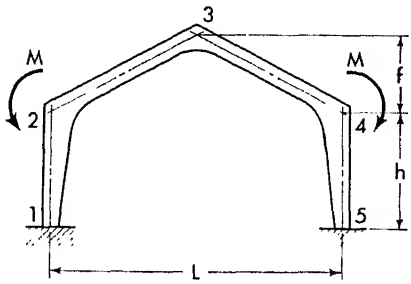
$$V_5 = -\frac{M}{L}\left(1 - \frac{4\phi\alpha_{23}}{D}\right) \qquad V_1 = -V_5$$

$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) + M_{21}\frac{y_1}{h}$$

$$M_{x_2} = M_{23}\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Apply Eqs. (19-3) and (19-4) to obtain the moment at any section of frame members 3-4 and 4-5.

19-15. Two Equal Moments Applied at Joints 2 and 4



$$T = \Theta_{23} + S\psi(\alpha_{32} + \beta_{23})$$

$$U = S\Theta_{23} + \frac{B\psi(\alpha_{32} + \beta_{23})}{2C}$$

$$M_{21} = M_{45} = \frac{2MT\phi}{F}$$

$$H_1 = H_5 = -\frac{2MU\phi}{Fh} \qquad V_1 = V_5 = 0$$

$$M_1 = M_5 = M_{21} + H_5 h \qquad M_{23} = M_{43} = -(M - M_{21})$$

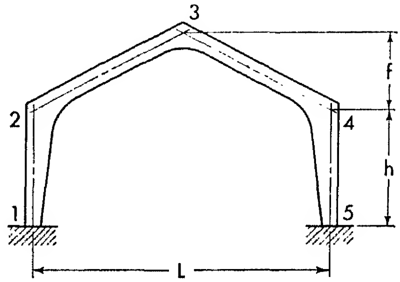
$$M_3 = \frac{2MT\phi}{F} - M - H_5 f$$

$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) + M_{21}\frac{y_1}{h}$$

$$M_{x_2} = M_{23}\left(1 - \frac{2x_2}{L}\right) + M_3\frac{2x_2}{L}$$

Moments at corresponding sections in the right half of the frame are identical to those in the left half.

19-16. Effect of Temperature Rise. Range $t°$ for the entire frame.



$$K = \frac{12L\varepsilon t°}{CFh^2} E(\min I_{1\text{-}2})$$

$$M_2 = M_4 = -AK$$

$$M_1 = M_5 = \frac{K}{2}(B - 2A)$$

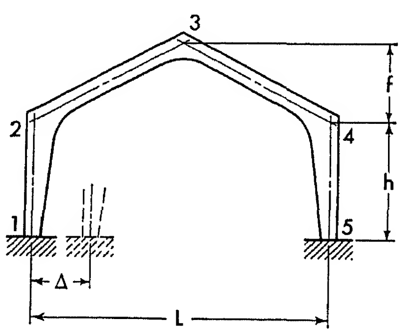
$$H_1 = H_5 = \frac{BK}{2h} \qquad V_1 = V_5 = 0$$

$$M_3 = M_1 - H_5(h + f)$$

Apply Eqs. (19-1) through (19-4) to obtain the moment at any section of the frame members.

Note: For temperature drop, introduce the value of $t°$ with a negative sign.

19-17. Horizontal Displacement of One Support



$$K = \frac{12\Delta}{CFh^2} E(\min I_{1\text{-}2})$$

$$M_2 = M_4 = -AK$$

$$M_1 = M_5 = \frac{K}{2}(B - 2A)$$

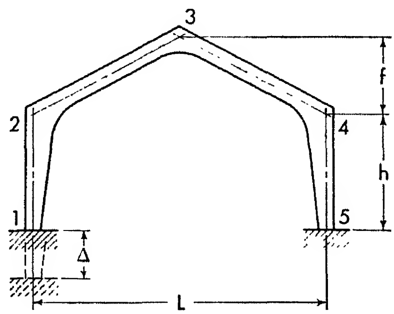
$$H_1 = H_5 = \frac{BK}{2h} \qquad V_1 = V_5 = 0$$

$$M_3 = M_1 - H_5(h + f)$$

Apply Eqs. (19-1) through (19-4) to obtain the moment at any section of the frame members.

Note: If the direction of the frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.

19-18. Vertical Settlement of One Support



$$K = \frac{48\Delta}{DLh} E(\min I_{1\text{-}2})$$

$$M_1 = M_2 = K \qquad M_3 = 0$$

$$M_4 = M_5 = -K$$

$$H_1 = H_5 = 0$$

$$V_5 = \frac{2K}{L} \qquad V_1 = -V_5$$

Apply Eqs. (19-1) through (19-4) to obtain the moment at any section of the frame.

Note: If the direction of the frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.

The Devils' Valley Bridge in Hermsdorf, Germany, carrying autobahn traffic between Jena and Gera is one of the biggest concrete arched structures in the world, spanning a ravine 452 feet wide. This bridge demonstrates the magnificence and aesthetic beauty inherent in arched structures. The economy of the application of arched structures to bridge construction may be realized from the fact that the thickness of the arch rib, spanning such a great distance, is only 4 feet 0 inch and 8 feet 6 inches at the crown and springing line, respectively. Designed and built by Grun and Bilfinger A.G. of Mannheim. (Courtesy of the Deutscher Beton-Verein E.V. of Wiesbaden.)

SECTION 20

INTRODUCTION TO ANALYSIS OF FRAMES WITH CURVED MEMBERS

20-1. General. Arches and arched frames with members of variable cross section are economical structures, but have not been widely used because certain associated factors outweigh their economic advantages. One of these factors is our inability to produce rapidly and accurately an analysis of arched structures of variable cross section. Existing analyses are involved, tedious, and time-consuming. To alleviate this difficulty, a special method, developed in this text,[1] permits the designer to make a precise analysis of arched structures in a short time, using only four or five significant figures in computations.

In the following sections, the condensed solutions of analysis of arched structures are given for many loading conditions. These solutions together with the auxiliary data provided in tabular form permit the engineer to obtain promptly the precise answers to the practical problems involved in the analysis of arches or arched structures.

20-2. Curvature of the Axes of Arched Members. Arched members may vary widely with respect to axis curvature and body shape. They may be segmental, parabolic, elliptical, multicentered, or catenarian in curvature, and of various shapes and rates of arch thickness variation. It is apparent, therefore, that a generalized solution of analysis for arched members is not practical because of its complexity. Conversely, if the solution of analysis encompasses members of only one particular curvature of axis, it may be presented in reasonably simple form, even though the shape variation of members is allowed. To determine the most common and usable curvature of the axes of arched members, a cursory investigation has been made and is herewith outlined.

[1] Based on the "Concept of Elastic Parameters," p. 221.

A review of technical literature pertaining to arched bridges built during the last two decades indicates a definite trend in bridge architecture toward the design of parabolic arched bridges with open spandrels. Bridges of this type are characterized by the fact that with their nearly uniform dead load per unit length, the arches work, predominantly, in pure compression. For such an application, parabolic arches are extremely efficient structural elements, leading to their popularity in bridge construction.

In another group of the arched structures such as employed in buildings of the industrial type, it may also be noticed that arched members generally carry uniformly distributed loads; hence, the parabolic arches are again very desirable structural elements.

These brief considerations illustrate that parabolic arches merit the particular attention of the structural designer. Consequently, considerable space in this text is devoted to symmetrical parabolically arched members of two basic classes. In all varieties, however, their axes conform to the quadratic parabolic curve, defined by Eq. (8-1).

20-3. Classification of Arched Members. As mentioned previously, two basic classes of arched members are considered in this text. Each class has its own advantages and the selection of the class shall be based on the economic and practical considerations.[1] The class of arched members which for the purpose of classification hereinafter is termed *prime arches* is most suitable for dams, arched frames, hangars, and other similar structures. The class of the arched members which for the purpose of classification is termed *quadratic arches* is generally considered more economical in bridge construction. The properties of these two classes will now be amplified.

A prime arch may be defined as an arch whose halves are simple prismoids of constant width, distorted to the arch curvature. Its section changes at a more or less uniform rate from the crown to the abutments, the moment of inertia of the section varying in accordance with the *lineal equation*

$$I = \frac{I_0}{\cos\varphi\left[1 - (1-k)\dfrac{2z}{L}\right]} \tag{20-1}$$

[1] The advantages of particular classes of arches with respect to construction features and preliminary design considerations are beyond the scope of this text. A comprehensive discussion may be found in C. B. McCullough and E. S. Thayer, *Elastic Arch Bridges*, chap. III and Appendix, John Wiley & Sons, Inc., New York, 1931.

where

I = moment of inertia of an arbitrary arch section about its neutral axis; the section defined by abscissa z, measured from the arch vertical center line

I_0 = moment of inertia of the arch crown section about its neutral axis

k = arch constant defined by the equation

$$k = \frac{I_0}{I_a \cos \varphi_a}$$

I_a = moment of inertia of arch section about its neutral axis, at the springing line

φ_a = angle of arch axis with the horizontal, at the springing line

φ = angle between the arch axis and the horizontal, at the section defined by the abscissa z

L = arch span

A quadratic arch [1] may be defined as an arch whose cross section is substantially constant in the central portion of the span, but varies rapidly in the proximity of the abutments. The moment of inertia of the arch section varies in accordance with the *quadratic equation*

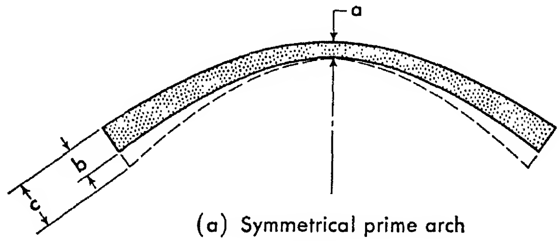
$$I = \frac{I_0}{\cos \varphi \left[1 - (1 - k)\frac{4z^2}{L^2}\right]} \tag{20-2}$$

The distinction between prime and quadratic arches can be better understood by referring to Fig. 20-1. In this figure, arches of both classes with respective congruent sections at the crowns and also at the springing lines are shown. The difference in the rate at which the depth of the members increases is evident in the illustration.

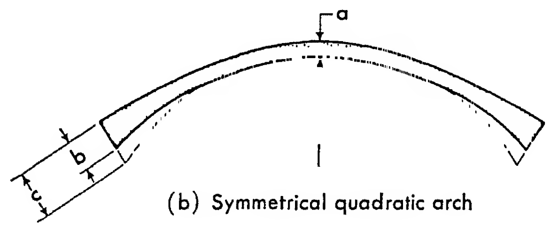
As explained above, the arches are classified as prime or quadratic, depending on the rate of thickness variation from crown to abutments. However, nothing limits their geometric dimensions and consequently they may be of any ratio of arch thickness at the springing line to that at the crown. The resulting broad range of arch shapes may be conveniently subdivided into the following three groups:

1. Tapered arches, whose thickness decreases toward the springing lines.
2. Arches of constant thickness.
3. Haunched arches, whose thickness increases toward the springing lines.

[1] Some authors use the term *slender* in defining this class of arch.



(a) Symmetrical prime arch



(b) Symmetrical quadratic arch

FIG. 20-1. Principal classes of arches

Tapered curved members are commonly used for two-hinged arches, while haunched curved members are used for hingeless arches or as girders for arched frames.

The above characteristics of arched members are given for general information only. In practical application, the engineer may select a suitable arched member directly from Table 11 or 12 in the Appendix. In these tables the relative thicknesses of arch sections are given at close intervals for arches of various shapes; the shapes are characterized by the k values of the arch.

A wide range of arches with k values from 0.02 to 5.00 is given in the tables. The upper limit of arch k value (5.00) represents a substantially tapered arch, while the lower limit (0.02) represents a severely haunched arch. Intermediate values represent arches with various degrees of tapering or haunching.

20-4. Correlation of Frame Axes. In the analysis of frames containing both arched and straight members with haunches, judgment should be exercised in the selection of a suitable system of frame axes. For the axis of a straight member with variable cross section a definition was already adopted in Section 13 of this text. As to the arched member, a common definition of its axis as being the line passing through the centroids of individual sections, while rigidly applicable to the frame consisting in its

entirety of an arched member, may be found by inspection to be unsuitable when taken in conjunction with an adjacent haunched straight member. Thus, in the case of a frame consisting of an arched girder and two columns haunched at their upper end, the arch axis may be more reasonably selected as passing through the centroid of the crown section but parallel to either the intrados or the extrados, depending on whether the column haunches are, respectively, interior or exterior.

It is apparent, of course, that the selection of axes in a complex frame of this kind will have some effect on the final outcome of the analysis. But it is believed that the results of the analysis will not be affected appreciably.

20-5. Frame Members. The procedure of reducing a frame to individual elementary members as described in Art. 13-3 for frames with straight members is also applicable to arched frames. Figure 20-2 illustrates the details of the rules to be followed in this procedure. After the shapes and dimensions of elementary members are established the elastic parameters may be determined for each member.

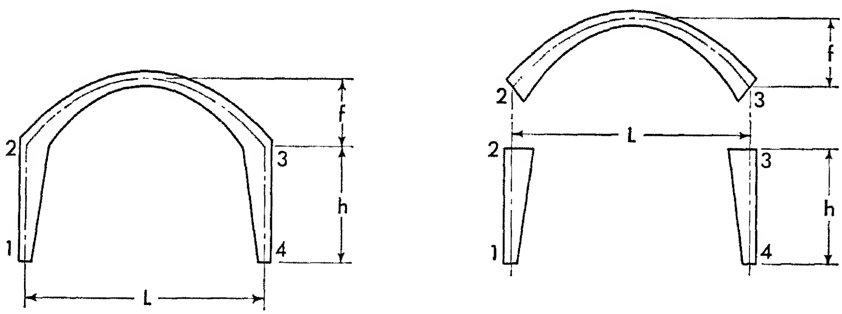


FIG. 20-2. Reduction of the frame to elementary members

20-6. Elastic Parameters of Arched Members. Any symmetrical arched member is characterized by four elastic parameters: α, β, γ, and δ. These parameters are functions of the arch shape and do not depend upon the dimensions of the member or the degree of fixity of its supports. Their numerical values may be precisely calculated by an analytical integration along the parabolic axis of the member. For convenience of application these quantities are listed as functions of the k value in Table 13 in the Appendix.

20-7. Load Constants of Arched Members. The moment area of any loaded curved member is characterized by three load constants S, T, and U. These

load constants, as explained previously, depend only upon the shape of the member and the manner of loading. Their numerical values may be precisely calculated by an analytical integration along the parabolic axis of the member for many loading conditions. They are tabulated and listed for a variety of arched members in Tables 15 through 20 in the Appendix.

The considered loading conditions cover the common vertical and horizontal loadings on the member as well as a number of special loadings used for calculation of maxima and minima of bending moments and forces in arched structures. For practical applications, the majority of loading conditions arising in the analysis of arched structures are included.

20-8. Assumptions. The condensed solutions of structural analysis given in Sections 21 through 24 have been derived without simplifications or approximations, using the theory of virtual work. Therefore, the solutions are applicable to frames and arches of any span, rise, or cross-sectional variation. These solutions are given as either method A alone or as methods A and B, for reasons similar to those given in Art. 8-4.

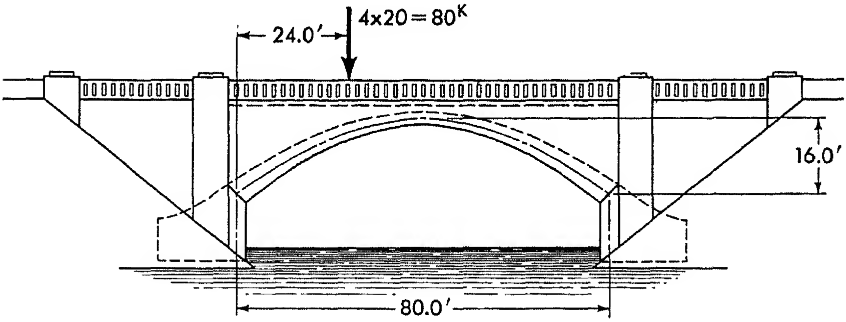
For convenience of performing calculations by method B, the numerical values of the elastic arch constant τ, representing the effect of axial deformation, have been computed for various arches and are listed in Table 14 in the Appendix. These values have been calculated by an integration along the parabolic arch axis with due consideration for arch thickness variation.

20-9. Condensed Solutions of Structural Analysis. After the elastic parameters and the load constants for the members of a frame are determined, the redundant quantities of the frame may be readily obtained using equations of the condensed solutions of analysis. In order to facilitate determination of bending moments and shearing and axial forces at various intermediate sections of the structure, expressions for them are also provided.

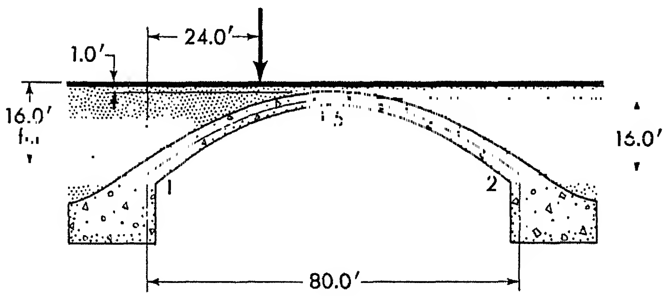
ILLUSTRATIVE EXAMPLES

Example 20-1. In this example, an earth-filled arch of variable cross section is analyzed. The use of various auxiliary data is illustrated and steps for practical analysis are outlined.

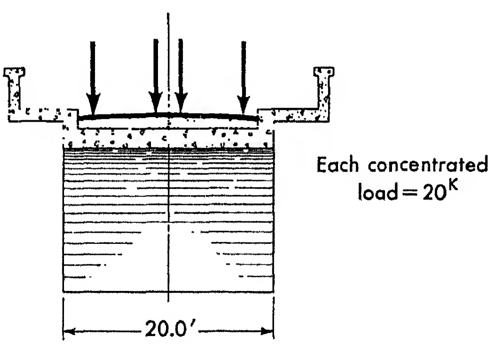
A hingeless, parabolic earth-filled arch of 80-ft span carries a dead load and is subjected to the action of four 20-kip concentrated live loads acting on a transverse section, 24 ft from the left arch support, as illustrated in Fig. 20-3. The rise and width of the arch are 16 and 20 ft, re-



(a) Elevation



(b) Longitudinal section



(c) Transverse section at crown

FIG. 20-3. Earth-filled arch bridge with applied vertical load

spectively. The thickness of the earth fill varies from 1 ft at the crown to 16 ft at the abutments.

The unit weight of the concrete arch is taken as 150 lb/cu ft and the unit weight of the fill as 120 lb/cu ft. For the sake of simplicity, it is assumed that the weight of the spandrel walls and road pavement is included in the weight of the fill. Further, because of the fill, the live loads may be considered uniformly distributed across the entire width of the arch. As a result, a concentrated load of 4 kip/ft of arch width is assumed for analysis. For convenience, all calculations of this example are carried out in the kip-foot dimensional system for longitudinal strip of arch 1 ft wide.

The first step of the analysis is the selection of the barrel shape and its dimensions. A prime arch has been arbitrarily selected for this example. In order to arrive at the dimensions of the barrel various empirical formulas [1] may be utilized. Assume that computed barrel thickness at the crown and at the springing lines is 1 ft 4½ in. and 3 ft 6¼ in., respectively. The ratio of barrel thickness at the springing to that at the crown may then be computed and the k value obtained from Table 11 in the Appendix for the corresponding rise to span ratio. Thus,

$$\frac{d_{spr.}}{d_{crown}} = \frac{3.52}{1.375} = 2.56 \qquad \text{and} \qquad \frac{f}{L} = \frac{16}{80} = 0.2$$

for which the k value of arch is found to be 0.08.

Note that Tables 9 through 20, in the Appendix, provide considerable amount of information for the analysis and design of arches. Tables 9 and 10 list coordinates of arch axes and their respective angles of inclination. Tables 11 and 12 give variation in thickness for arch sections at 10 intervals, so that the dimensions of the barrel or arch rib may be readily established.[2] The remaining tables provide various constants for arch analysis.

When the shape of the arch has been established and its k value found, the redundant quantities of the arch may be readily calculated by

[1] See, for example, H. Sutherland and R. O. Reese, *Introduction to Reinforced Concrete*, John Wiley & Sons, Inc., New York, 1946, p. 442.

[2] For example, using numerical values given in Table 11, the arch barrel under consideration would have the following thicknesses:

Section 1:	$d_r \times d_{crown} = 2.56 \times 1.375$ ft $= 3.52$ ft
Section 1.1:	$d_r \times d_{crown} = 1.65 \times 1.375$ ft $= 2.27$ ft
Section 1.2:	$d_r \times d_{crown} = 1.35 \times 1.375$ ft $= 1.86$ ft, etc.

applying the equations of the condensed solutions of analysis given in Section 22. Since these solutions are for the individual loading conditions, the complex loading should be resolved into a number of simple loadings. When solutions for individual loadings are found, the final results may be readily obtained by superposition.

The earth-fill load of variable intensity, acting on the arch, may be represented by a vertical uniform load of 120 lb/ft over the entire span and a vertical complementary parabolic load with a maximum intensity of 1,800 lb/ft at the abutments, as shown in Fig. 20-4.

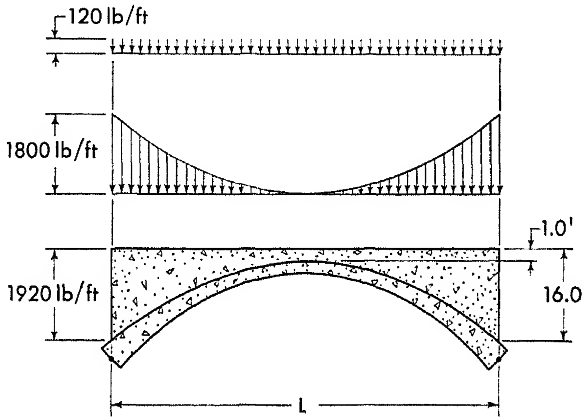


FIG. 20-4. Substitution of earth-fill load

The uniform load, thus, is $120 \times 80 = 9{,}600$ lb, and the complementary parabolic load is

$$\frac{pL}{3} = \frac{1{,}800 \times 80}{3} = 48{,}000 \text{ lb}$$

The spanwise distribution of the barrel dead load is quite complex because of the variable thickness of the barrel. However, for practical purposes, this load may be resolved into uniform and complementary parabolic loads using the same approach as above for the earth-fill load.

The first step of computation consists of the determination of the weight of the barrel. For this purpose the barrel may be considered as consisting of two symmetric curved prismoids spanning the distances between the crown and springing lines. Then the volume V of the barrel may be found by the use of the following equation:

$$V = \frac{s}{6}(w_1 + 4w + w_{1.5})$$

where

s = length of parabolic arch axis

w_1 = cross-sectional area of barrel at springing line

w = cross-sectional area of barrel at quarter point of arch parabolic axis

$w_{1.5}$ = cross-sectional area of barrel at crown

To facilitate this calculation, the numerical constants given in Table 9 may be advantageously utilized. Thus, the length of the barrel axis for the given rise to span ratio of the arch in this example is 1.098L, or $1.098 \times 80 = 87.84$ ft. Similarly, the abscissa of the quarter point of the parabolic axis is found to be 0.234L (measured from the left end of the arch); the cross-sectional area of the barrel at this section may be determined, with the aid of Table 11 in the Appendix. Entering Table 11 with a k value of 0.08, the relative thickness of the barrel at this section of the span with respect to the thickness at the crown section is found to be 1.30. The actual thickness, therefore, is

$$d = d_r \times d_{crown} = 1.30 \times 1.375 \text{ ft} = 1.79 \text{ ft}$$

The cross-sectional area of the barrel at the springing line, quarter point of the length, and at the crown may now be determined and the weight of the barrel calculated.

The areas are

$$w_1 = 3.52 \times 1 = 3.52 \text{ sq ft}$$

$$w = 1.79 \times 1 = 1.79 \text{ sq ft}$$

$$w_{1.5} = 1.38 \times 1 = 1.38 \text{ sq ft}$$

and the weight of the barrel is

$$V_w = \left[\frac{87.84}{6}(3.52 + 4 \times 1.79 + 1.38)\right] 150 = 26{,}500 \text{ lb}$$

This load of variable intensity may be resolved into uniform and complementary parabolic loads, as explained above. The intensity of the uniform load should be taken as corresponding to the weight of the barrel per unit of horizontal projection at the crown, expressed in proper dimensional units. The weight of foot of barrel at crown is

$$1.38 \times 150 = 207 \text{ lb/ft}$$

Therefore the intensity of the uniform load is 207 lb/ft. The uniformly

distributed load on the arch (one part of barrel weight) is $207 \times 80 =$ 16,500 lb. The complementary parabolic load (another part of barrel weight) is determined as the difference between the total weight of the barrel and the assumed uniformly distributed load. That is,

$$26{,}500 - 16{,}560 = 9{,}940 \text{ lb}$$

In order to reduce numerical computations, it is advisable to combine all uniform loads together into one group and all complementary parabolic loads into another; the group loadings may then be applied to the arch. For the considered case, the combined uniform load is

$$9{,}600 + 16{,}560 = 26{,}160 \text{ lb}$$

and the combined complementary load is

$$48{,}000 + 9{,}940 = 57{,}940 \text{ lb}$$

After the loadings have been grouped, the redundant moments and forces may be determined for separate loading conditions by the use of the condensed solutions of arch analysis given in Section 22.

Arch Elastic Parameters and Constants. The numerical values of the elastic parameters α, β, γ, and δ of the selected arch now may be obtained from Table 13 in the Appendix. Entering the table with a k value of 0.08, the following numerical values of the parameters are found:

$$\alpha = 1.930 \qquad \beta = 1.310$$

$$\gamma = 4.56 \qquad \delta = 0.68$$

Inserting the values into the equations given in Art. 22-1, the constants of the arch Θ, D, J, and F are obtained; they are

$$\Theta = 2(\alpha + \beta) = 2(1.93 + 1.31) = 6.48$$

$$D = 2(\alpha - \beta) = 2(1.93 - 1.31) = 1.24$$

$$J = 1 + \frac{\delta}{\gamma} = 1 + \frac{0.68}{4.56} = 1.149$$

$$F = \Theta - \gamma J^2 = 6.48 - 4.56 \times 1.149^2 = 0.46$$

Loading Condition 1. Uniform Load over the Arch. The condensed solution of arch analysis for the uniformly loaded arch is given in Art. 22-6. The calculations begin with the selection of numerical values of load constants. Entering Table 15 in the Appendix, with the arch k value of 0.08, the following numerical values of load constants are found:

$$S = 0.57 \quad \text{and} \quad T = 0.3275$$

Applying the equations given in Art. 22-6, and using, as stated previously, the kip-foot dimensional system, the term K and the redundants are

$$K = \frac{S\Theta}{\gamma} = \frac{0.57 \times 6.48}{4.56} = 0.81$$

Moments M_1 and M_2 are both zero.

$$H_1 = H_2 = \frac{WL}{Ff} (K - 2JT)$$

$$= \frac{26.16 \times 80}{0.46 \times 16} (0.81 - 2 \times 1.149 \times 0.3275) = 16.32 \text{ kip}$$

$$V_1 = V_2 = \frac{W}{2} = \frac{26.16}{2} = 13.08 \text{ kip}$$

Loading Condition 2. Complementary Parabolic Load over the Arch. Following the procedure described in condition 1, the numerical values of the load constants for the complementary parabolic load on the arch are found from Table 15, in the Appendix, as

$$S = 0.3134 \qquad \text{and} \qquad T = 0.185$$

Applying equations of the condensed solution of arch analysis given in Art. 22-8, the term K and the arch redundants are

$$K = \frac{S\Theta}{\gamma} = \frac{0.3134 \times 6.48}{4.56} = 0.4454$$

$$M_1 = M_2 = \frac{WL}{F} (JS - 2T)$$

$$= \frac{57.94 \times 80}{0.46} (1.149 \times 0.3134 - 2 \times 0.185) = -99.76 \text{ ft-kip}$$

$$H_1 = H_2 = \frac{WL}{Ff} (K - 2JT)$$

$$= \frac{57.94 \times 80}{0.46 \times 16} (0.4454 - 2 \times 1.149 \times 0.185) = 12.78 \text{ kip}$$

$$V_1 = V_2 = \frac{W}{2} = \frac{57.94}{2} = 28.97 \text{ kip}$$

Hence, the moment at an arbitrary arch section 1.3, $x = 24$ ft, is

$$M_{1.3} = M_1 + \frac{WL}{16}\left[1 - \frac{(L - 2x)^4}{L^4}\right] - H_1 y$$

$$= (-99.76) + \frac{57.94 \times 80}{16}\left[1 - \frac{(80 - 2 \times 24)^4}{80^4}\right]$$

$$- 12.78(0.84 \times 16) = 10.76 \text{ ft-kip}$$

Loading Condition 3. Concentrated Load on the Arch. The numerical values of the load constants for the arch with concentrated load applied at a distance of 0.3L from the left support are found from Table 16, in the Appendix, as

$$S = 0.7224 \qquad T = 0.4557 \qquad U = 0.3738$$

Applying equations of the condensed solution given in Art. 22-10, the term K and the arch redundants are

$$K = \frac{S\Theta}{\gamma} = \frac{0.7224 \times 6.48}{4.56} = 1.027$$

$$\left.\begin{matrix} M_1 \\ M_2 \end{matrix}\right\rangle = \frac{PL}{F}(JS - T - U) \mp \frac{PL(T - U)}{D}$$

$$= \frac{4 \times 80}{0.46}(1.149 \times 0.7224 - 0.4557 - 0.3738)$$

$$\mp \frac{4 \times 80(0.4557 - 0.3738)}{1.24} = \begin{matrix} -20.81 \text{ ft-kip} \\ +21.51 \text{ ft-kip} \end{matrix}$$

$$H_1 = H_2 = \frac{PL}{Ff}[K - J(T + U)]$$

$$= \frac{4 \times 80}{0.46 \times 16}[1.027 - 1.149(0.4557 + 0.3738)] = 3.22 \text{ kip}$$

$$V_1 = P\left[1 - \frac{m}{L} + \frac{2(T - U)}{D}\right]$$

$$= 4\left[1 - \frac{24}{80} + \frac{2(0.4557 - 0.3738)}{1.24}\right] = 3.33 \text{ kip}$$

$$V_2 = P - V_1 = 4 - 3.33 = 0.67 \text{ kip}$$

The moment at an arbitrary arch section 1.3 may be obtained by the use of the equation

$$M_x = M_1 + V_1x - H_1y$$

which gives

$$M_{1.3} = -20.81 + 3.33 \times 24 - 3.22(0.84 \times 16) = 15.83 \text{ ft-kip}$$

Final Moments and Forces. Superposing the values of the moment and forces determined for individual loading conditions 1, 2, and 3, the magnitudes of redundant quantities due to entire load on the arch are obtained. Thus

$$M_1 = 0 + (-99.76) + (-20.81) = -120.57 \text{ ft-kip}$$

$$M_2 = 0 + (-99.76) + 21.51 = -78.25 \text{ ft-kip}$$

$$H_1 = H_2 = 16.32 + 12.78 + 3.22 = 32.32 \text{ kip}$$

$$V_1 = 13.08 + 28.97 + 3.33 = 45.38 \text{ kip}$$

$$V_2 = 13.08 + 28.97 + 0.67 = 42.72 \text{ kip}$$

and the moment at an arbitrary arch section 1.3 is

$$M_{1.3} = 0 + 10.76 + 15.83 = 26.59 \text{ ft-kip}$$

The axial and shearing forces due to individual loadings on the arch may be found in the manner illustrated in Art. 8-5. Applying the method of superposition, the magnitudes of axial and shearing forces for the entire load on the arch may be obtained.

Solution by Method B. The same problem may be solved more precisely by method B, which includes correction for the effect of axial deformation. As an illustration, the analysis is presented below for condition 3, where the arch is loaded with a single concentrated load of 4 kip.

Noting that the value of τ, for the arch under consideration, from Table 14, in the Appendix, is 1.80, the arch redundants may be obtained by the use of equations given in Art. 22-10. Introducing numerical values into the equations, the following values of the arch constant G and the redundant forces and moments are obtained:

$$G = \tau \frac{d^2_{crown}}{f^2} = 1.8 \frac{1.375^2}{16^2} = 0.0133$$

$$H_1 = H_2 = \frac{PL[K - J(T + U)]}{Ff(1 + G)}$$

$$= \frac{4 \times 80[1.027 - 1.149(0.4557 + 0.3778)]}{0.46 \times 16(1 + 0.0133)} = 3.17 \text{ kip}$$

$$\left.\begin{matrix} M_1 \\ M_2 \end{matrix}\right\rangle = \frac{1}{\Theta}[H_1 f(\gamma + \delta) - PL(T + U)] \mp \frac{PL(T - U)}{D}$$

$$= \frac{1}{6.48}[3.17 \times 16(4.56 + 0.68) - 4 \times 80(0.4557 + 0.3738)]$$

$$\mp \frac{4 \times 80(0.4557 - 0.3738)}{1.24} = \begin{matrix} -21.09 \text{ ft-kip} \\ +21.19 \text{ ft-kip} \end{matrix}$$

The remaining forces and moments may be found in accordance with instructions given in Art. 22-10.

Example 20-2. An analysis of the butterfly frame of the passengers' shelter is illustrated in this example. The butterfly frame consists of a two-hinged arched frame with extended canopies from joints 2 and 3, as shown in Fig. 20-5. The frame is 80 ft end to end, and the spacing between columns is 48 ft. The column width at the base is 1.75 ft and at the junction with the girder is 3.5 ft. The girder is a quadratic parabolic arch with depths of 1 ft and 3.25 ft, at the crown and at the springing line, respectively. The width of each member of the frame normal to the plane of the drawing is 1 ft.

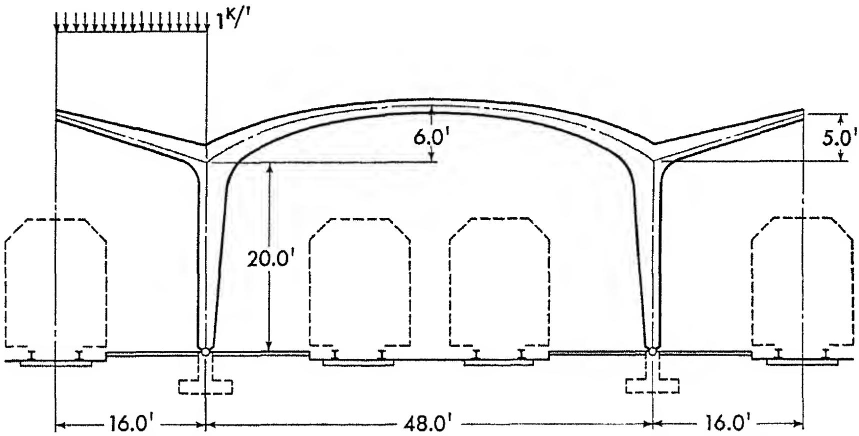


FIG. 20-5. Arched passenger shelter with applied vertical uniform load on the left canopy

To illustrate the application of the analysis to more involved loadings, the case of a live load acting only on the left canopy is considered. For the load of 1 kip/ft of canopy length, the bending moments in principal sections of the frame are to be determined.

The effect of the loaded cantilever on the structure may be represented by a moment acting in a counterclockwise direction on joint 2 and an axial vertical force acting concentrically on the left column, as shown in Fig. 20-6. The magnitude of the vertical force acting on the left column is 16 kip, and the magnitude of the applied moment on joint 2 is $1 \times 16 \times 8 = 128$ ft-kip. The axial force produces only compression in the left column, while the applied moment produces flexural, shearing, and axial effects over the entire arched frame.

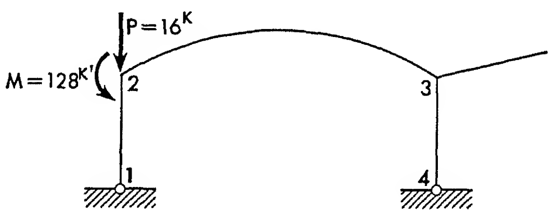


FIG. 20-6. Substitution of the canopy load

The vertical compressive force on the left column may be regarded as one load, and the applied moment at joint 2 as another. The vertical compressive force causes a direct axial compression in the column and this evident effect does not require any further comments. The effect of the applied moment on the arched frame, however, is quite complex, and a complete analysis is given below.

The procedure of assigning numerals to the joints and isolating members of the frame for the purpose of selecting elastic parameters and load constants has been previously described and will not be repeated.

The ratio of the arch thickness at the springing line to that at the crown of arched member 2-3 is $3.25/1 = 3.25$. For this ratio and for $f/L = 0.125$, the arch k value is obtained from Table 12, in the Appendix, as 0.04. Entering Table 13 with this k value, the following numerical values of the elastic parameters are obtained:

$\alpha_{23} = 2.464 \qquad \beta_{23} = 1.616$

$\gamma_{23} = 5.522 \qquad \text{and} \qquad \delta_{23} = 0.9417$

Observing that the geometric constants of member 1-2 are

$$v = \frac{l_h}{l} = \frac{20}{20} = 1$$

and

$$t = \left(\frac{\min d}{\max d}\right)^3 = \left(\frac{1.75}{3.50}\right)^3 = 0.125$$

the value of the elastic parameter α_{21} is obtained from Chart 6 as 0.81.

Using the equations of the condensed solution of analysis given in Art. 23-1, and noting that the reference moments of inertia of members 1-2 and 2-3 are

$$\min I_{1\text{-}2} = \frac{1.75^3 \times 1}{12} = 0.447 \text{ ft}^4$$

and

$$I_{2.5} = \frac{1^3 \times 1}{12} = 0.0833 \text{ ft}^4$$

the frame constants ϕ, ψ, Θ_{23}, and A may be determined; they are

$$\phi = \frac{\min I_{1\text{-}2}}{I_{2.5}} \cdot \frac{L}{h} = \frac{0.447}{0.0833} \cdot \frac{48}{20} = 12.88$$

$$\psi = \frac{f}{h} = \frac{6}{20} = 0.3$$

$$\Theta_{23} = 2(\alpha_{23} + \beta_{23}) = 2(2.464 + 1.616) = 8.16$$

$$\begin{aligned} A &= 2\alpha_{21} + \phi[\Theta_{23} + \psi^2\gamma_{23} + 2\psi(\gamma_{23} + \delta_{23})] \\ &= 2 \times 0.81 + 12.88[8.16 + 0.3^2 \times 5.522 \\ &\qquad + 2 \times 0.3(5.522 + 0.9417)] = 163.1 \end{aligned}$$

Proceeding further and using the equations given in Art. 23-17, the term B and redundant reaction H are

$$\begin{aligned} B &= \Theta_{23} + \psi(\gamma_{23} + \delta_{23}) \\ &= 8.16 + 0.3(5.522 + 0.9417) = 10.1 \end{aligned}$$

$$H_1 = H_4 = -\frac{MB\phi}{2Ah} = -\frac{128 \times 10.1 \times 12.88}{2 \times 163.1 \times 20} = -2.55 \text{ kip}$$

Likewise, the vertical reactions of the frame are

$$V_1 = \frac{M}{L} = \frac{128}{48} = 2.67 \text{ kip}$$

(This does not include the effect of the 16-kip vertical force acting on column 1-2.)

$$V_4 = -\frac{M}{L} = -\frac{128}{48} = -2.67 \text{ kip}$$

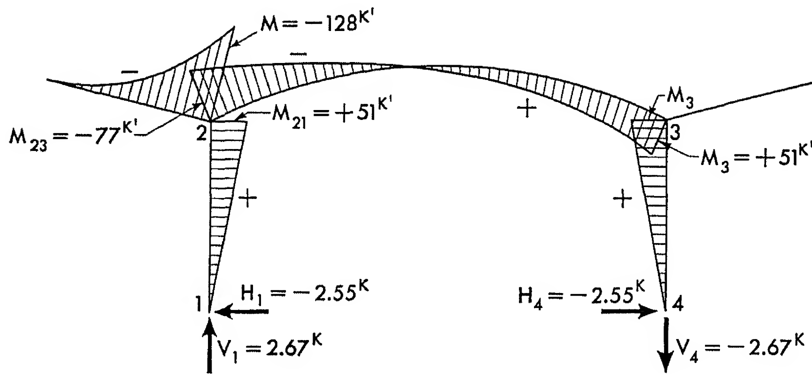


FIG. 20-7. Bending moment diagram and frame reactions

And finally, the bending moments at the frame joints and the arch crown are

$$M_{21} = -H_1 h = -(-2.55)20 = 51.0 \text{ ft-kip}$$

$$M_{23} = -(M - M_{21}) = -(128 - 51) = -77.0 \text{ ft-kip}$$

$$M_3 = M_{21} = 51.0 \text{ ft-kip}$$

and

$$M_{2.5} = M_{23}\left(1 - \frac{x}{L}\right) + M_3 \frac{x}{L} - H_4 y_2$$

$$= (-77)\left(1 - \frac{24}{48}\right) + 51 \cdot \frac{24}{48} - (-2.55)6 = 2.3 \text{ ft-kip}$$

The complete bending moment diagram together with frame reactions is shown in Fig. 20-7.

Example 20-3. In the construction of arched buildings such as hangars, auditoriums, warehouses, and similar structures the parabolic arches of rolled steel sections are very often utilized. Steel members being

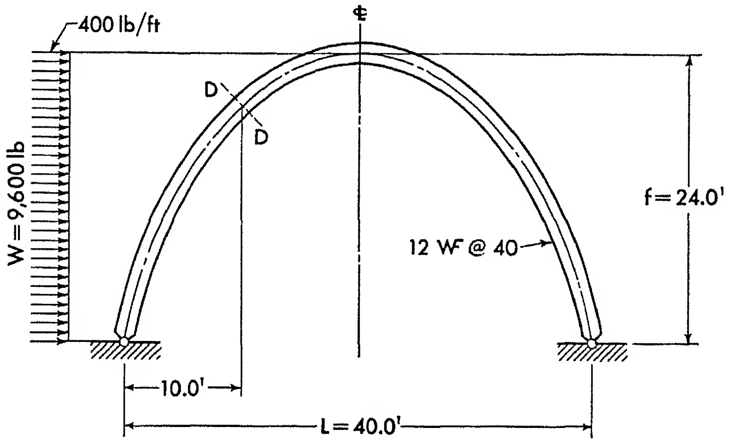


FIG. 20-8. Arch rib of the quonset with applied horizontal uniform load

bent to curvature retain complete uniformity of cross section along the entire length of the arch. The geometry of such arches is not easily expressed in mathematical terms, thereby presenting considerable difficulty in performing a rigorous arch analysis. However, by the use of the condensed solutions of analysis given in this text and with aid of the data listed in the Appendix, the solution may be readily obtained.

To illustrate the analysis of an arch of uniform cross section, a rib of a parabolic quonset having a 24-ft rise and 40-ft span, as shown in Fig. 20-8, is selected. The rib is made of 12 WF 40 lb section and is bent to the parabolic curvature. Assuming a wind load of 400 lb/ft intensity on the vertical projection of the rib, find the numerical values of redundant reactions.

For an arch of uniform cross section and a rise to span ratio of 0.6, the arch k value is found from Table 12, in the Appendix, as 2.80. (Small deviations of the arch cross-sectional dimensions from the values listed in the table have an insignificant effect on the final results.)

Entering Tables 13 and 20, in the Appendix, with the known k value, the numerical values of the elastic parameters and the load constants are found to be

$$\gamma = 8.046 \qquad \text{and} \qquad S = 2.377$$

Then, the term K and the vertical and horizontal components of the arch reactions are obtained by the use of the equations given in Art. 21-11; they are

$$K = \frac{S}{\gamma} = \frac{2.377}{8.046} = 0.2954$$

$$H_2 = WK = 9.6 \times 0.2954 = 2.836 \text{ kip}$$

$$H_1 = -(W - H_2) = -(9.6 - 2.836) = -6.764 \text{ kip}$$

$$V_2 = \frac{Wf}{2L} = \frac{9.6 \times 24}{2 \times 40} = 2.88 \text{ kip}$$

$$V_1 = -2.88 \text{ kip}$$

The magnitude of the bending moment at the arbitrary section D-D of the arch ($x = 10$ ft from the left support) may be determined by the use of the equation

$$M_x = -\frac{Wf}{2}\left(\frac{x}{L} + \frac{y^2}{f^2}\right) - H_1 y$$

and upon substitution of numerical values, the moment M_D is

$$M_D = -\frac{9.6 \times 24}{2}\left(\frac{10}{40} + \frac{18^2}{24^2}\right) - (-6.764)18 = 28.15 \text{ ft-kip}$$

Example 20-4. The effect of temperature change on the arched hangar shown in Fig. 20-9 is investigated in this example. The arched roof of the hangar rises from the concrete shop buildings which serve as abutments for the arch. The parabolic arch is built monolithically with the buildings, and for the purposes of this analysis it is assumed fully fixed at its springing lines. The parabolic arch is of the prime type with variation of barrel thickness from 2 ft 0 in. at the crown to 2 ft 11¼ in. at its springings. Assuming the arch width as 1 ft 0 in., find the horizontal thrust of the arch for a 50°F temperature rise.

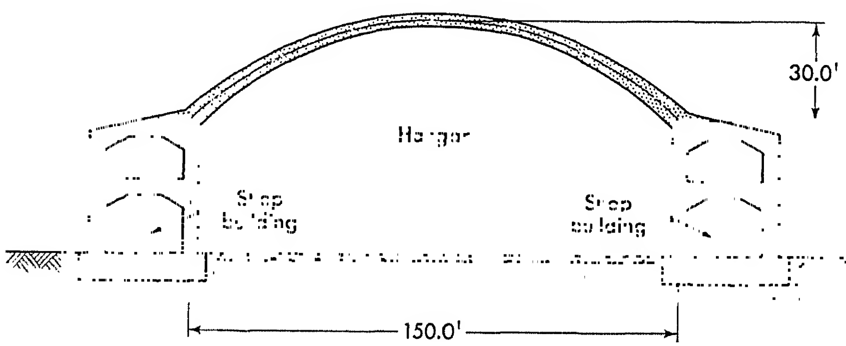


FIG. 20-9. Cross section of the arched hangar

Observing that the ratio of arch thickness at springing to that at crown is

$$\frac{2.95}{2.0} = 1.475$$

and the arch rise to span ratio is 0.2, the arch k value is found from Table 11, in the Appendix, as 0.4. Entering Table 13 with this arch k value, the following numerical values of the arch elastic parameters are obtained:

$$\alpha = 2.65 \qquad \beta = 1.55 \qquad \gamma = 5.20 \qquad \delta = 1.00$$

Substituting numerical values into equations given in Art. 22-1, the arch constants are

$$\Theta = 2(\alpha + \beta) = 2(2.65 + 1.55) = 8.40$$

$$J = 1 + \frac{\delta}{\gamma} = 1 + \frac{1}{5.2} = 1.192$$

$$F = \Theta - \gamma J^2 = 8.40 - 5.2 \times 1.192^2 = 1.01$$

$$C = \frac{\Theta}{\gamma} = \frac{8.4}{5.2} = 1.615$$

The equation of the horizontal thrust is given in Art. 22-14 as

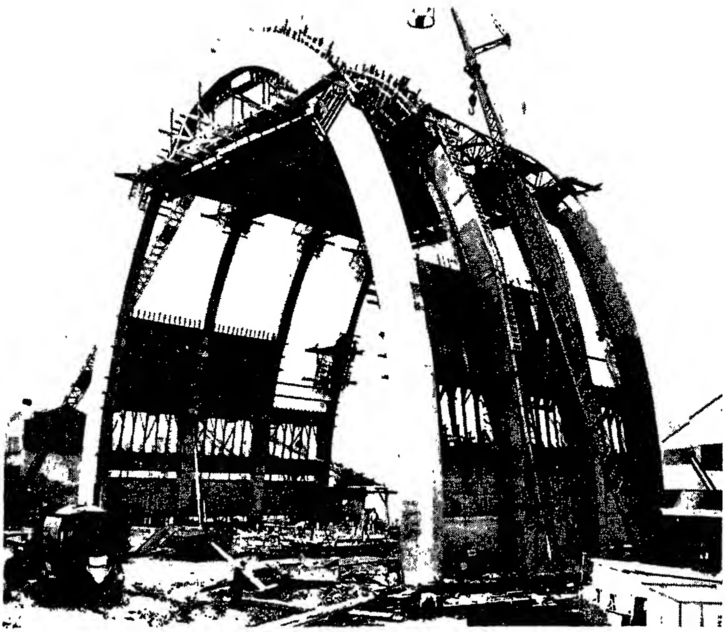
$$H_1 = H_2 = \frac{12C\varepsilon t^\circ}{Ff^2} EI_{1.5}$$

Assuming the modulus of elasticity of concrete as 432,000 kip/ft², the coefficient of thermal expansion of concrete as 0.000006 per degree Fahrenheit, and noting that the moment of inertia of the crown section is

$$I_{1.5} = \frac{2^3 \times 1}{12} = 0.667 \text{ ft}^4$$

the numerical value of the thrust then is

$$H_1 = H_2 = \frac{12 \times 1.615 \times 0.000006 \times 50}{1.01 \times 30^2} (432{,}000 \times 0.667) = 1.84 \text{ kip}$$

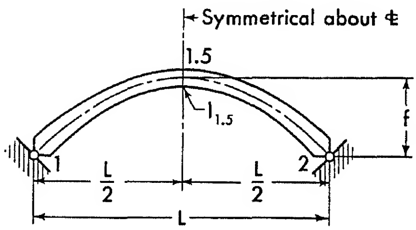


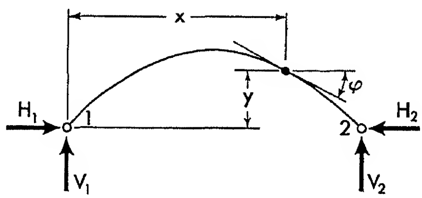
Construction view of the framework of the asphalt-mixing plant of the City of New York. This is an interesting example of a structure designed to meet the specific functional requirement of enclosing the tanks and machinery of the mixing plant. These unusual arches are of 90-foot span and 80-foot rise and boast the unique application of structural rolled steel for reinforcement of arches and for support of concrete forms. This idea eliminated the necessity of providing false-works for the construction of the arches. Designed by E. J. Kahn and R. A. Jacobs, architect-engineers of New York, New York. (Courtesy of the Portland Cement Association.)

SECTION 21

SYMMETRICAL PARABOLIC TWO-HINGED ARCHES

21-1. Notations, Coordinates, and Arch Constant





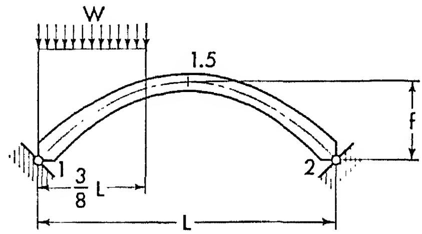
The sketch appearing on the left, above, explains notations for a representative arch with variable cross section. The arch axis is a symmetrical parabola conforming to Eq. (8-1).
The solutions of analysis given on the following pages are not limited to the shape of the arch shown, but are applicable to any symmetrical parabolic arch.

The sketch appearing on the right, above, shows positive directions of the vertical and horizontal components of the arch reactions. It also defines the angle of inclination and coordinates at any section of the arch. Angles of inclination and coordinates are to be considered only in the positive sense.

Arch Constant. Obtain value of the arch parameter γ from Table 13 in the Appendix.

21-2. Equations of Forces and Moments. The equations for the vertical and the redundant horizontal components of the arch reactions are given on the following pages for a number of loading conditions. Corresponding expressions for the moment and the axial and shearing forces at any section of the arch are also provided.

21-3. Vertical Uniform Load over Three-eighths of Span



W = total load

Obtain value of load constant S from Table 15 or 18.

$$K = \frac{S}{\gamma} \qquad H_1 = H_2 = \frac{WLK}{f}$$

$$V_1 = \frac{13}{16} W \qquad V_2 = \frac{3W}{16}$$

When $x \leqq \frac{3}{8} L$

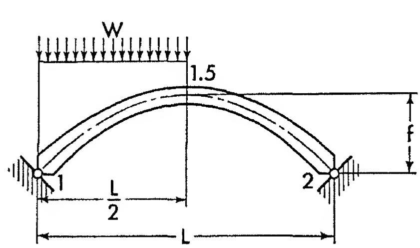
$$M_x = Wx\left(\frac{13}{16} - \frac{4x}{3L}\right) - H_1 y$$

When $x > \frac{3}{8} L$

$$M_x = \frac{3W}{16}(L - x) - H_1 y$$

Apply Eqs. (9-1) through (9-3) to determine the axial and shearing forces in the arch.

21-4. Vertical Uniform Load over Left Half of Span



W = total load

Obtain value of load constant S from Table 15 or 18.

$$K = \frac{S}{\gamma} \qquad H_1 = H_2 = \frac{WLK}{f}$$

$$V_1 = \frac{3}{4}W \qquad V_2 = \frac{W}{4}$$

When $x \leqq \frac{L}{2}$

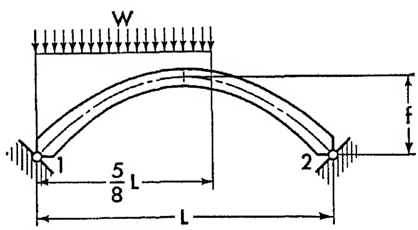
$$M_x = Wx\left(\frac{3}{4} - \frac{x}{L}\right) - H_1y$$

When $x > \frac{L}{2}$

$$M_x = \frac{W}{4}(L - x) - H_1y$$

Apply Eqs. (9-4) and (9-5) to determine the axial and shearing forces in the arch.

21-5. Vertical Uniform Load over Five-eighths of Span



Obtain value of load constant S from Table 15 or 18.

$$K = \frac{S}{\gamma} \qquad H_1 = H_2 = \frac{WLK}{f}$$

$$V_1 = \frac{11}{16}W \qquad V_2 = \frac{5W}{16}$$

When $x \leqq \frac{5}{8}L$

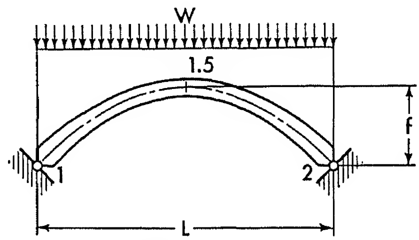
$$M_x = Wx\left(\frac{11}{16} - \frac{4x}{5L}\right) - H_1y$$

When $x > \frac{5}{8} L$

$$M_x = \frac{5W}{16}(L - x) - H_1 y$$

Apply Eqs. (9-6) through (9-8) to determine the axial and shearing forces in the arch.

21-6. Vertical Uniform Load over Entire Span



W = total load

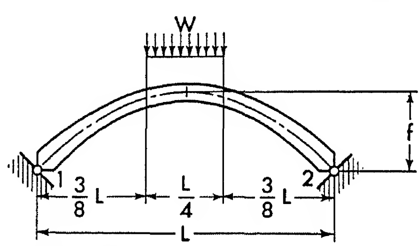
Obtain value of load constant S from Table 15 or 18.

$$K = \frac{S}{\gamma} \qquad H_1 = H_2 = \frac{WLK}{f}$$

$$V_1 = V_2 = \frac{W}{2}$$

M and Q are zero at any section of the arch. Apply Eq. (9-9) to determine the axial force in the arch.

21-7. Vertical Uniform Load over Center Quarter of Span



W = total load

Obtain value of load constant S from Table 15 or 18.

$$K = \frac{S}{\gamma} \qquad H_1 = H_2 = \frac{WLK}{f}$$

$$V_1 = V_2 = \frac{W}{2}$$

When $x \leqq \frac{3}{8}L$

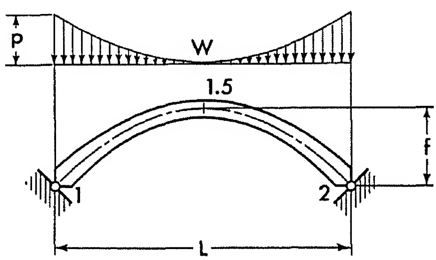
$$M_x = \frac{Wx}{2} - H_1 y$$

When $x > \frac{3}{8}L$, but $\leqq \frac{L}{2}$

$$M_x = \frac{Wx}{2} - \frac{2W}{L}\left(x - \frac{3L}{8}\right)^2 - H_1 y$$

Apply Eqs. (9-10) and (9-11) to determine the axial and shearing forces in the left half of the arch. Moments and axial forces at corresponding sections in the right half of the arch are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

21-8. Vertical Complementary Parabolic Load over Entire Arch



Total load $W = \frac{pL}{3}$

Obtain value of load constant S from Table 15 or 18.

$$K = \frac{S}{\gamma} \qquad H_1 = H_2 = \frac{WLK}{f}$$

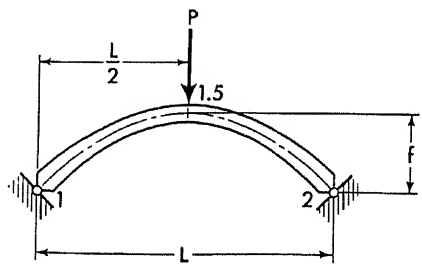
$$V_1 = V_2 = \frac{W}{2}$$

When $x \leqq \frac{L}{2}$

$$M_x = \frac{WL}{16}\left[1 - \left(\frac{L - 2x}{L}\right)^4\right] - H_1 y$$

Apply Eqs. (9-12) to determine the axial and shearing forces in the left half of the arch. Moments and axial forces at corresponding sections in the right half of the arch are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

21-9. Vertical Concentrated Load at Crown



Obtain value of load constant S from Table 16 or 19.

$$K = \frac{S}{\gamma} \qquad H_1 = H_2 = \frac{PLK}{f}$$

$$V_1 = V_2 = \frac{P}{2}$$

When $x \leqq \frac{L}{2}$

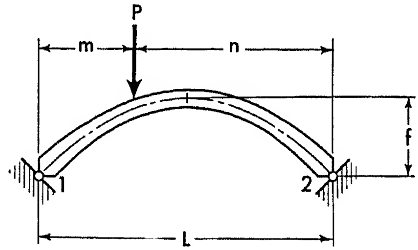
$$M_x = \frac{Px}{2} - H_1 y$$

When $x > \frac{L}{2}$

$$M_x = \frac{P}{2}(L - x) - H_1 y$$

Apply Eqs. (9-13) through (9-15) to determine the axial and shearing forces in the arch.

21-10. Vertical Concentrated Load on Arch



Obtain value of load constant S from Table 16 or 19.

$$K = \frac{S}{\gamma} \qquad H_1 = H_2 = \frac{PLK}{f}$$

$$V_1 = P\left(1 - \frac{m}{L}\right) \qquad V_2 = \frac{Pm}{L}$$

When $x \leqq m$

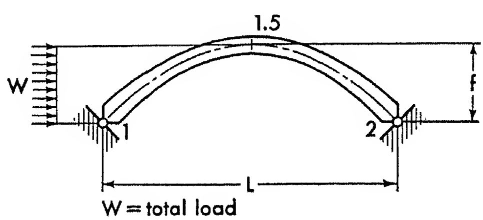
$$M_x = Px\left(1 - \frac{m}{L}\right) - H_1 y$$

When $x > m$

$$M_x = Pm\left(1 - \frac{x}{L}\right) - H_1 y$$

Apply Eqs. (9-16) through (9-19) to determine the axial and shearing forces in the arch.

21-11. Horizontal Uniform Load on Left Half of Arch



Obtain value of load constant S from Table 17 or 20.

$$K = \frac{S}{\gamma} \qquad H_2 = WK \qquad H_1 = -(W - H_2)$$

For Notations and Constants, see Arts. 21-1 and 21-2

$$V_2 = \frac{Wf}{2L} \qquad V_1 = -V_2$$

When $x \leqq \frac{L}{2}$

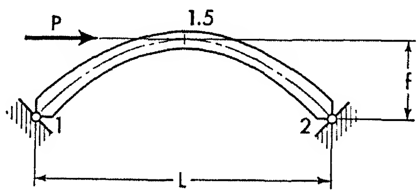
$$M_x = -\frac{Wf}{2}\left[\frac{x}{L} + \left(\frac{y}{f}\right)^2\right] - H_1 y$$

When $x > \frac{L}{2}$

$$M_x = \frac{Wf}{2}\left(1 - \frac{x}{L}\right) - H_2 y$$

Apply Eqs. (9-20) and (9-21) to determine the axial and shearing forces in the arch.

21-12. Horizontal Concentrated Load at Crown



$$H_1 = -\frac{P}{2} \qquad H_2 = \frac{P}{2} \qquad V_1 = -\frac{Pf}{L}$$

$$V_2 = \frac{Pf}{L}$$

When $x \leqq \frac{L}{2}$

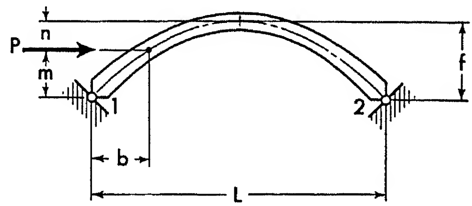
$$M_x = Pf\left(\frac{y}{2f} - \frac{x}{L}\right)$$

When $x > \frac{L}{2}$

$$M_x = \frac{Pf}{L}(L - x) - \frac{Py}{2}$$

Apply Eqs. (9-22) through (9-24) to determine the axial and shearing forces in the arch.

21-13. Horizontal Concentrated Load at Any Point of Left Half of Arch



Obtain value of load constant S from Table 17 or 20.

$$K = \frac{S}{\gamma} \qquad H_2 = PK \qquad H_1 = -(P - H_2)$$

$$V_2 = \frac{Pm}{L} \qquad V_1 = -V_2$$

When $x \leqq b$

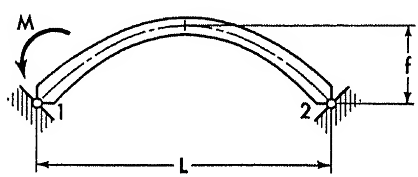
$$M_x = -\frac{Pmx}{L} - H_1 y$$

When $x > b$

$$M_x = Pm\left(1 - \frac{x}{L}\right) - H_2 y$$

Apply Eqs. (9-25) through (9-27) to determine the axial and shearing forces in the arch.

21-14. Moment Applied at Left End of Arch



Obtain value of arch parameter δ from Table 13.

$$J = 1 + \frac{\delta}{\gamma} \qquad H_1 = H_2 = -\frac{MJ}{2f}$$

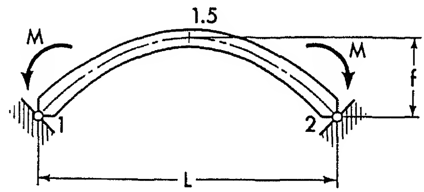
$$V_1 = \frac{M}{L} \qquad V_2 = -V_1$$

For Notations and Constants, see Arts. 21-1 and 21-2

$$M_x = -M\left(1 - \frac{x}{L}\right) - H_1 y$$

Apply Eqs. (9-28) and (9-29) to determine the axial and shearing forces in the arch.

21-15. Equal Moments Applied at Both Ends of Arch



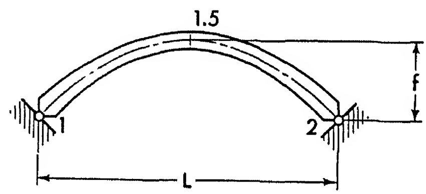
Obtain value of arch parameter δ from Table 13.

$$J = 1 + \frac{\delta}{\gamma} \qquad H_1 = H_2 = -\frac{MJ}{f}$$

$$V_1 = V_2 = 0 \qquad M_x = -M - H_1 y$$

Apply Eqs. (9-30) and (9-31) to determine the axial and shearing forces in the arch.

21-16. Effect of Temperature Rise. Range $t°$ for entire arch.



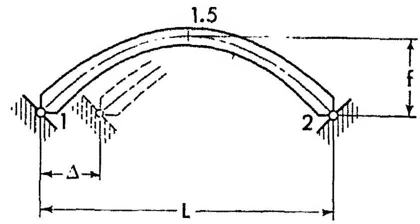
$$H_1 = H_2 = \frac{12\varepsilon t°}{f^2 \gamma} EI_{1.5}$$

$$V_1 = V_2 = 0 \qquad M_y = -H_1 y$$

Apply Eqs. (9-30) and (9-31) to determine the axial and shearing forces in the arch.

Note: For temperature drop, introduce the value of $t°$ with a negative sign.

21-17. Horizontal Displacement of One Support

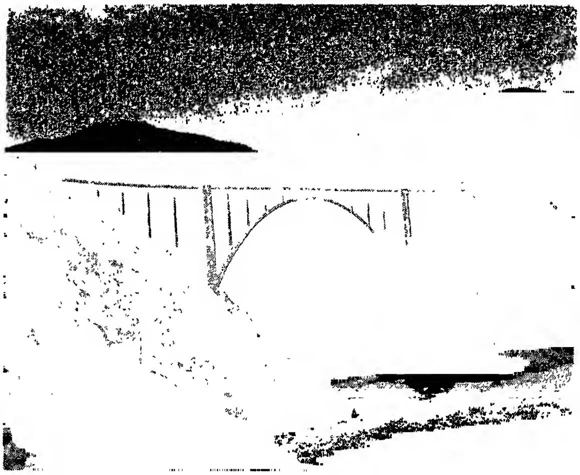


$$H_1 = H_2 = \frac{12\Delta}{Lf^2\gamma} EI_{1.5}$$

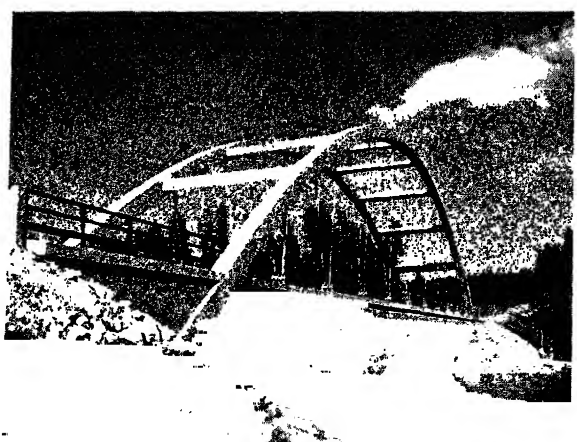
$$V_1 = V_2 = 0 \qquad M_x = -H_1 y$$

Apply Eqs. (9-30) and (9-31) to determine the axial and shearing forces in the arch.

Note: If the direction of arch displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.



A magnificent arched bridge over the Devil's Gorge on the Monterey Peninsula in California. Spanning a ravine 320 feet wide, 240 feet above the stream bed, the arch abutments appear to be glued to the rocky cliffs. This is an example of an arched bridge with the spandrel columns supporting a two-lane highway deck. Designed as a hingeless arch, the bridge exhibits neatness, excellent proportioning of the members, harmony, and expressiveness. Designed by the California State Highway Commission. (Courtesy of the Portland Cement Association.)

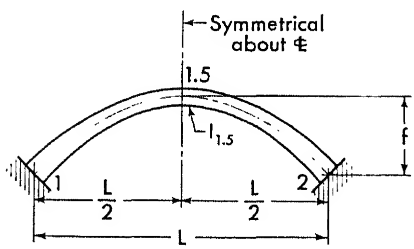


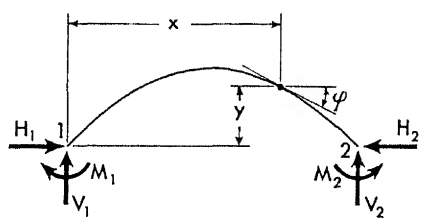
The Djupsund Bridge over a deep strait near Slåttö, Sweden. This arched bridge illustrates the use of an arch in a different functional capacity from that of the bridge over the Devil's Gorge. The highway deck of this bridge is suspended from the arches by steel round rods. The economical aspects of this arched structure may be appreciated by noting the dimensions of the arch. The effective span is 230 feet, while the arch section at the crown is only 2 feet 6 inches wide by 2 feet 1 inch thick. Designed by R. Lyckeberg, civil engineer of Stockholm, Sweden. (Courtesy of Christiani & Nielsen of Stockholm, Sweden, bridge builders.)

SECTION 22

SYMMETRICAL PARABOLIC HINGELESS ARCHES

22-1. Notations, Coordinates, and Arch Constants





The sketch appearing on the left, above, explains notations for a representative arch with variable cross section. The arch axis is a symmetrical parabola conforming to Eq. (8-1).

The solutions of analysis given on the following pages are not limited to the shape of the arch shown, but are applicable to any symmetrical parabolic arch.

The sketch appearing on the right, above, shows positive directions of the moments and the vertical and horizontal components of the arch reactions. It also defines the angle of inclination and coordinates at any section of the arch. Angles of inclination and coordinates are to be considered only in the positive sense.

General Arch Constants. Obtain values of arch parameters α, β, γ, and δ from Table 13 in the Appendix.

$$\Theta = 2(\alpha + \beta) \qquad D = 2(\alpha - \beta)$$

$$F = \Theta - \gamma J^2 \qquad J = 1 + \frac{\delta}{\gamma}$$

Arch Constant G, *for the Analyses by Method B*

$$G = \frac{d^2_{1.5}\tau}{f^2}$$

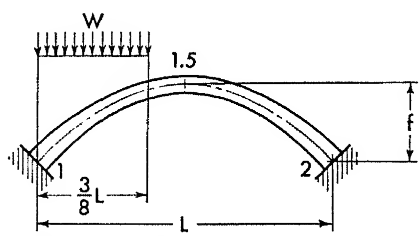
where

$d_{1.5}$ = arch thickness at the crown (in proper dimensional units)

τ = numerical constant from Table 14, in the Appendix

22-2. Equations of Moments and Forces. The equations for the redundant moments and the vertical and horizontal components of the arch reactions are given on the following pages for a number of loading conditions. Corresponding expressions for the moment and the axial and shearing forces at any section of the arch are also provided. In addition, rigorous solution of arch analysis, defined in Art. 20-8 as method B, are also included for many loading conditions.

22-3. Vertical Uniform Load over Three-eighths of Span



W = Total load

Obtain values of load constants S, T, and U from Table 15 or 18.

$$K = \frac{S\Theta}{\gamma}$$

METHOD A

$$\left.\begin{matrix} M_1 \\ M_2 \end{matrix}\right\rangle = \frac{WL}{F}(JS - T - U) \mp \frac{WL}{D}(T - U)$$

$$H_1 = H_2 = \frac{WL}{Ff}[K - J(T + U)]$$

$$V_1 = W\left[\frac{13}{16} + \frac{2(T-U)}{D}\right] \qquad V_2 = W - V_1$$

$$M_{1.5} = \frac{M_1 + M_2}{2} + \frac{3WL}{32} - H_2 f \tag{22-1}$$

When $x \leqq \frac{3}{8}L$

$$M_x = M_1 + V_1 x - \frac{4Wx^2}{3L} - H_1 y \tag{22-2}$$

When $x > \frac{3}{8}L$

$$M_x = M_2 + V_2(L - x) - H_1 y \tag{22-3}$$

Apply Eqs. (10-3) through (10-5) to determine the axial and shearing forces in the arch.

METHOD B

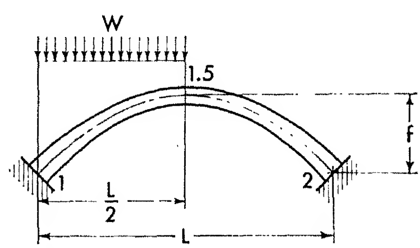
$$H_1 = H_2 = \frac{WL[K - J(T+U)]}{Ff(1+G)}$$

$$\left.\begin{matrix} M_1 \\ M_2 \end{matrix}\right\rangle = \frac{1}{\Theta}[JH_1 f\gamma - WL(T+U)] \mp \frac{WL(T-U)}{D}$$

$$V_1 = W\left[\frac{13}{16} + \frac{2(T-U)}{D}\right] \qquad V_2 = W - V_1$$

Apply Eqs. (22-1) through (22-3) to obtain the moment at any section of the arch, and use Eqs. (10-3) through (10-5) to determine the axial and shearing forces in the arch.

22-4. Vertical Uniform Load over Left Half of Span



W = Total load

Obtain values of load constants S, T, and U from Table 15 or 18.

For Notations and Constants, see Arts. 22-1 and 22-2

$$K = \frac{S\Theta}{\gamma}$$

METHOD A

$$\left.\begin{matrix} M_1 \\ M_2 \end{matrix}\right\rangle = \frac{WL}{F}(JS - T - U) \mp \frac{WL}{D}(T - U)$$

$$H_1 = H_2 = \frac{WL}{Ff}[K - J(T + U)]$$

$$V_1 = W\left[\frac{3}{4} + \frac{2(T - U)}{D}\right] \qquad V_2 = W - V_1$$

$$M_{1.5} = \frac{M_1 + M_2}{2} + \frac{WL}{8} - H_1 f \tag{22-4}$$

When $x \leqq \frac{L}{2}$

$$M_x = M_1 + V_1 x - \frac{Wx^2}{L} - H_1 y \tag{22-5}$$

When $x > \frac{L}{2}$

$$M_x = M_2 + V_2(L - x) - H_1 y \tag{22-6}$$

Apply Eqs. (10-7) and (10-9) to determine the axial and shearing forces in the arch.

METHOD B

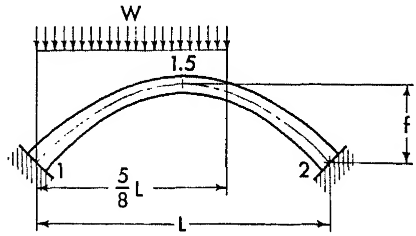
$$H_1 = H_2 = \frac{WL[K - J(T + U)]}{Ff(1 + G)}$$

$$\left.\begin{matrix} M_1 \\ M_2 \end{matrix}\right\rangle = \frac{1}{\Theta}[H_1 f(\gamma + \delta) - WL(T + U)] \mp \frac{WL}{D}(T - U)$$

$$V_1 = W\left[\frac{3}{4} + \frac{2(T - U)}{D}\right] \qquad V_2 = W - V_1$$

Apply Eqs. (22-4) through (22-6) to obtain the moment at any section of the arch, and use Eqs. (10-7) and (10-9) to determine the axial and shearing forces in the arch.

22-5. Vertical Uniform Load over Five-eighths of Span



W = Total load

Obtain values of load constants S, T, and U from Table 15 or 18.

$$K = \frac{S\Theta}{\gamma}$$

METHOD A

$$\left.\begin{matrix} M_1 \\ M_2 \end{matrix}\right\rangle = \frac{WL}{F}(JS - T - U) \mp \frac{WL}{D}(T - U)$$

$$H_1 = H_2 = \frac{WL}{Ff}[K - J(T + U)]$$

$$V_1 = W\left[\frac{11}{16} + \frac{2(T - U)}{D}\right] \qquad V_2 = W - V_1$$

$$M_{1.5} = \frac{M_1 + M_2}{2} + \frac{23WL}{160} - H_1 f \tag{22-7}$$

When $x \leqq \frac{5}{8}L$

$$M_x = M_1 + V_1 x - \frac{4Wx^2}{5L} - H_1 y \tag{22-8}$$

When $x > \frac{5}{8}L$

$$M_x = M_2 + V_2(L - x) - H_1 y \tag{22-9}$$

Apply Eqs. (10-12) through (10-14) to determine the axial and shearing forces in the arch.

For Notations and Constants, see Arts. 22-1 and 22-2

METHOD B

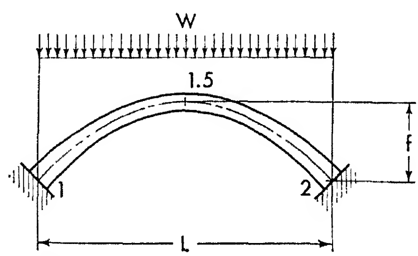
$$H_1 = H_2 = \frac{WL[K - J(T + U)]}{Ff(1 + G)}$$

$$\left.\begin{matrix} M_1 \\ M_2 \end{matrix}\right\rangle = \frac{1}{\Theta}[H_1 Jf\gamma - WL(T + U)] \mp \frac{WL}{D}(T - U)$$

$$V_1 = W\left[\frac{11}{16} + \frac{2(T - U)}{D}\right] \qquad V_2 = W - V_1$$

Apply Eqs. (22-7) through (22-9) to obtain the moment at any section of the arch, and use Eqs. (10-12) through (10-14) to determine the axial and shearing forces in the arch.

22-6. Vertical Uniform Load over Entire Span



W = Total load

Obtain values of load constants S and T from Table 15 or 18.

$$K = \frac{S\Theta}{\gamma}$$

METHOD A

M and Q are zero at any section of the arch.

$$H_1 = H_2 = \frac{WL}{Ff}(K - 2JT) \qquad V_1 = V_2 = \frac{W}{2}$$

Apply Eqs. (10-15) to determine the axial force in the arch.

METHOD B

$$H_1 = H_2 = \frac{WL(K - 2JT)}{Ff(1 + G)}$$

$$M_1 = M_2 = \frac{1}{\Theta}[H_1 f(\gamma + \delta) - 2WLT]$$

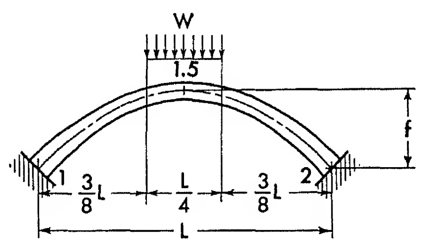
$$V_1 = V_2 = \frac{W}{2}$$

$$M_{1.5} = M_1 + \frac{WL}{8} - H_1 f$$

$$M_x = M_1 + \frac{Wx}{2}\left(1 - \frac{x}{L}\right) - H_1 y$$

Apply Eqs. (10-15) and (10-16) to determine the axial and shearing forces in the arch.

22-7. Vertical Uniform Load over Center Quarter of Span



W = Total load

Obtain values of load constants S and T from Table 15 or 18.

$$K = \frac{S\Theta}{\gamma}$$

METHOD A

$$M_1 = M_2 = \frac{WL}{F}(JS - 2T)$$

$$H_1 = H_2 = \frac{WL}{Ff}(K - 2JT) \qquad V_1 = V_2 = \frac{W}{2}$$

$$M_{1.5} = M_1 + \frac{7WL}{32} - H_1 f$$

Apply Eqs. (10-17) and (10-18) to determine the moment and the axial and shearing forces at any section of the left half of the arch. Moments and axial forces at corresponding sections in the right half of the arch are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

For Notations and Constants, see Arts. 22-1 and 22-2

METHOD B

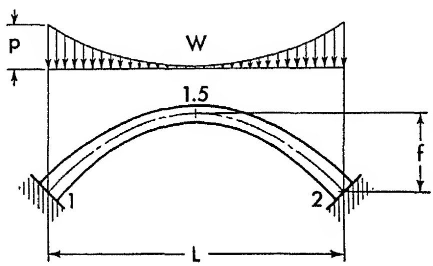
$$H_1 = H_2 = \frac{WL(K - 2JT)}{Ff(1 + G)}$$

$$M_1 = M_2 = \frac{1}{\Theta}[H_1 f(\gamma + \delta) - 2WLT]$$

$$V_1 = V_2 = \frac{W}{2}$$

Moment and axial and shearing forces, at any section of the arch, may be determined by the use of Eqs. (10-17) and (10-18) and the application of the symmetry relation described above.

22-8. Vertical Complementary Parabolic Load over Entire Arch



Total load $W = \frac{pL}{3}$

Obtain values of load constants S and T from Table 15 or 18.

$$K = \frac{S\Theta}{\gamma}$$

METHOD A

$$M_1 = M_2 = \frac{WL}{F}(JS - 2T)$$

$$H_1 = H_2 = \frac{WL}{Ff}(K - 2JT) \qquad V_1 = V_2 = \frac{W}{2}$$

Apply Eqs. (10-19) and (10-20) to determine the moment and the axial and shearing forces at any section of the left half of the arch. Moments and axial forces at corresponding sections in the right half of the arch are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

METHOD B

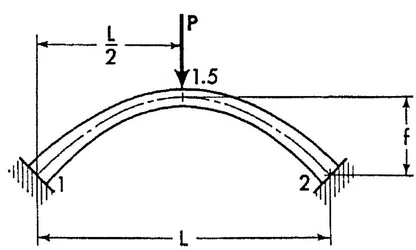
$$H_1 = H_2 = \frac{WL(K - 2JT)}{Ff(1 + G)}$$

$$M_1 = M_2 = \frac{1}{\Theta}[H_1 f(\gamma + \delta) - 2WLT]$$

$$V_1 = V_2 = \frac{W}{2}$$

Moment and axial and shearing forces, at any section of the arch, may be determined by the use of Eqs. (10-19) and (10-20) and the application of the symmetry relation described above.

22-9. Vertical Concentrated Load at Crown



Obtain values of load constants S and T from Table 16 or 19.

$$K = \frac{S\Theta}{\gamma}$$

METHOD A

$$M_1 = M_2 = \frac{PL}{F}(JS - 2T)$$

$$H_1 = H_2 = \frac{PL}{Ff}(K - 2JT) \qquad V_1 = V_2 = \frac{P}{2}$$

$$M_{1.5} = M_1 + \frac{PL}{4} - H_1 f \tag{22-10}$$

When $x \leqq \frac{L}{2}$

$$M_x = M_1 + \frac{Px}{2} - H_1 y \tag{22-11}$$

For Notations and Constants, see Arts. 22-1 and 22-2

Apply Eqs. (10-21) to determine the axial and shearing forces in the left half of the arch. Moments and axial forces at corresponding sections in the right half of the arch are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

METHOD B

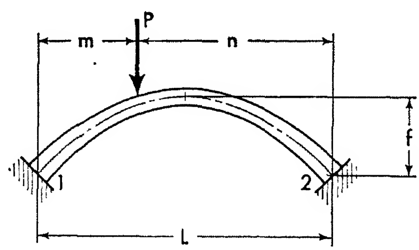
$$H_1 = H_2 = \frac{PL(K - 2JT)}{Ff(1 + G)}$$

$$M_1 = M_2 = \frac{1}{\Theta}[H_1 f(\gamma + \delta) - 2PLT]$$

$$V_1 = V_2 = \frac{P}{2}$$

Moment and axial and shearing forces, at any section of the arch, may be determined by the use of Eqs. (22-10), (22-11), and (10-21) and the application of the symmetry relation described above.

22-10. Vertical Concentrated Load on Arch



Obtain values of load constants S, T, and U from Table 16 or 19.

$$K = \frac{S\Theta}{\gamma}$$

METHOD A

$$\left.\begin{matrix} M_1 \\ M_2 \end{matrix}\right\rangle = \frac{PL}{F}(JS - T - U) \mp \frac{PL}{D}(T - U)$$

$$H_1 = H_2 = \frac{PL}{Ff}[K - J(T + U)]$$

$$V_1 = P\left[1 - \frac{m}{L} + \frac{2(T-U)}{D}\right] \qquad V_2 = P - V_1$$

When $x \leqq m$

$$M_x = M_1 + V_1x - H_1y \tag{22-12}$$

When $x > m$

$$M_x = M_2 + V_2(L - x) - H_2y \tag{22-13}$$

Apply Eqs. (10-29) through (10-32) to determine the axial and shearing forces in the arch.

METHOD B

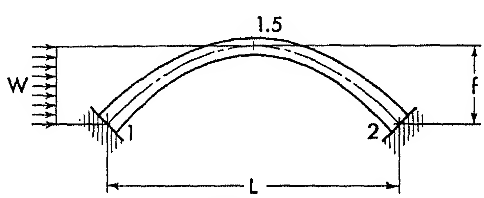
$$H_1 = H_2 = \frac{PL[K - J(T+U)]}{Ff(1+G)}$$

$$\left.\begin{matrix} M_1 \\ M_2 \end{matrix}\right\rangle = \frac{1}{\Theta}[H_1f(\gamma+\delta) - PL(T+U)] \mp \frac{PL}{D}(T-U)$$

$$V_1 = P\left[1 - \frac{m}{L} + \frac{2(T-U)}{D}\right] \qquad V_2 = P - V_1$$

Apply Eqs. (22-12) and (22-13) to obtain the moment at any section of the arch, and use Eqs. (10-29) through (10-32) to determine the axial and shearing forces in the arch.

22-11. Horizontal Uniform Load on Left Half of Arch



W=Total load

Obtain values of load constants S, T, and U from Table 17 or 20.

$$K = \frac{S\Theta}{\gamma} \qquad \left.\begin{matrix} M_1 \\ M_2 \end{matrix}\right\rangle = \frac{Wf}{F}(JS - T - U) \mp \frac{Wf}{D}(T-U)$$

$$H_2 = \frac{W}{F}[K - J(T + U)] \qquad H_1 = -(W - H_2)$$

$$V_2 = \frac{Wf}{2L}\left[1 - \frac{4(T - U)}{D}\right] \qquad V_1 = -V_2$$

When $x \leqq \frac{L}{2}$

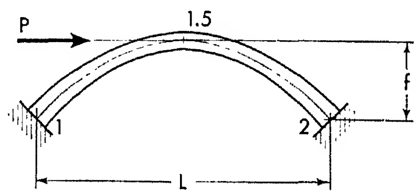
$$M_x = M_1 + V_1 x - \frac{Wy^2}{2f} - H_1 y$$

When $x > \frac{L}{2}$

$$M_x = M_2 + V_2(L - x) - H_2 y$$

Apply Eqs. (10-33) and (10-34) to determine the axial and shearing forces in the arch.

22-12. Horizontal Concentrated Load at Crown



Obtain values of load constants T and U from Table 17 or 20.

$$\left.\begin{matrix} M_1 \\ M_2 \end{matrix}\right\rangle = \mp \frac{Pf}{D}(T - U) \qquad H_1 = -\frac{P}{2}$$

$$H_2 = \frac{P}{2} \qquad V_2 = \frac{Pf}{L}\left[1 - \frac{2(T - U)}{D}\right]$$

$$V_1 = -V_2$$

When $x \leqq \frac{L}{2}$

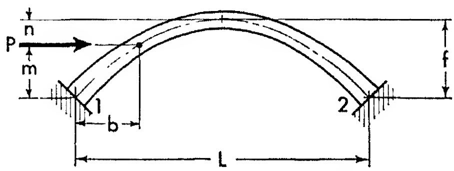
$$M_x = M_1 + V_1 x + \frac{Py}{2}$$

When $x > \frac{L}{2}$

$$M_x = M_2 + V_2(L - x) - \frac{Py}{2}$$

Apply Eqs. (10-35) through (10-37) to determine the shearing and axial forces in the arch.

22-13. Horizontal Concentrated Load at Any Point of Left Half of Arch



Obtain values of load constants S, T, and U from Table 17 or 20.

$$K = \frac{S\Theta}{\gamma}$$

$$\left.\begin{matrix} M_1 \\ M_2 \end{matrix}\right\rangle = \frac{Pf}{F}(JS - T - U) \mp \frac{Pf}{D}(T - U)$$

$$H_2 = \frac{P}{F}[K - J(T + U)] \qquad H_1 = -(P - H_2)$$

$$V_2 = \frac{Pf}{L}\left[\frac{m}{f} - \frac{2(T - U)}{D}\right] \qquad V_1 = -V_2$$

When $x \leqq b$

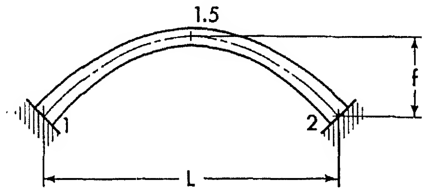
$$M_x = M_1 + V_1x - H_1y$$

When $x > b$

$$M_x = M_2 + V_2(L - x) - H_2y$$

Apply Eqs. (10-38) through (10-40) to determine the axial and shearing forces in the arch.

22-14. Effect of Temperature Rise. Range t° for entire arch.



$$C = \frac{\Theta}{\gamma} \qquad K = \frac{12\varepsilon t^\circ}{Ff} EI_{1.5}$$

$$M_1 = M_2 = JK \qquad H_1 = H_2 = \frac{CK}{f}$$

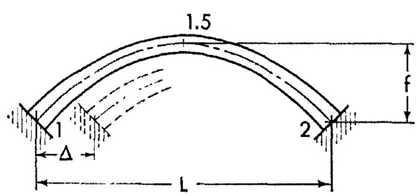
$$V_1 = V_2 = 0 \qquad M_{1.5} = M_1 - H_1 f$$

$$M_x = M_1 - H_1 y$$

Apply Eqs. (10-41) and (10-42) to determine the axial and shearing forces in the arch.

Note: For temperature drop, introduce the value of t° with a negative sign.

22-15. Horizontal Displacement of One Support



$$C = \frac{\Theta}{\gamma} \qquad K = \frac{12\Delta}{FLf} EI_{1.5}$$

$$M_1 = M_2 = JK \qquad H_1 = H_2 = \frac{CK}{f}$$

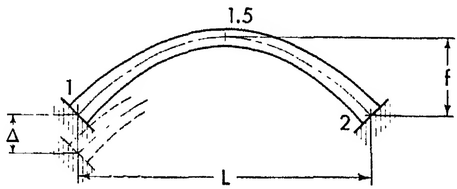
$$V_1 = V_2 = 0 \qquad M_{1.5} = M_1 - H_1 f$$

$$M_x = M_1 - H_1 y$$

Apply Eqs. (10-41) and (10-42) to determine the axial and shearing forces in the arch.

Note: If the direction of the arch displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.

22-16. Vertical Settlement of One Support



$$K = \frac{24\Delta}{DL^2} EI_{1.5}$$

$$M_1 = K \qquad M_2 = -K$$

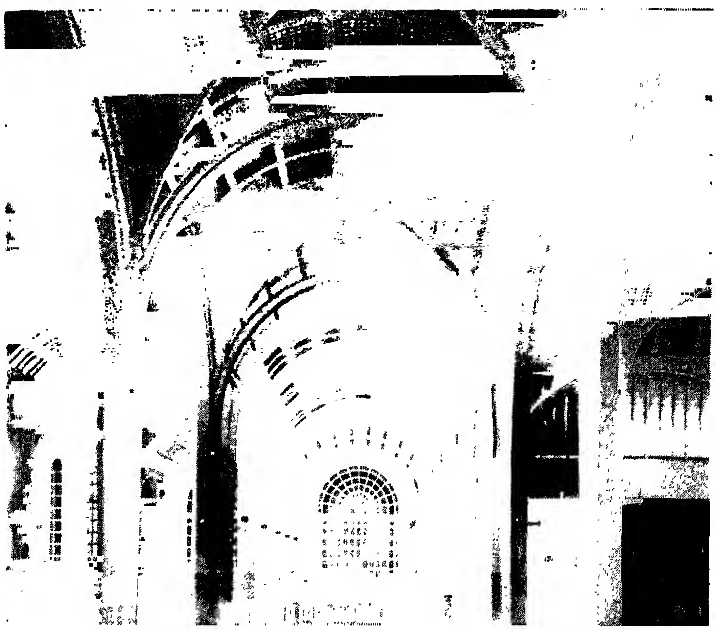
$$H_1 = H_2 = 0 \qquad V_1 = -\frac{2K}{L}$$

$$V_2 = \frac{2K}{L} \qquad M_{1.5} = 0$$

$$M_x = M_1 + V_1 x$$

Apply Eqs. (10-43) and (10-44) to determine the shearing and axial forces in the arch.

Note: If the direction of the arch displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.

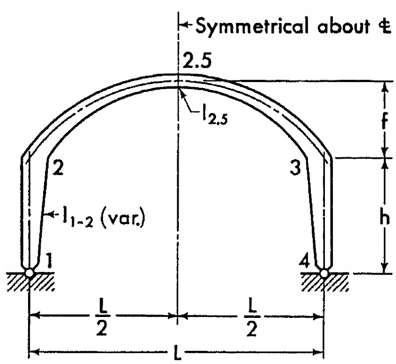


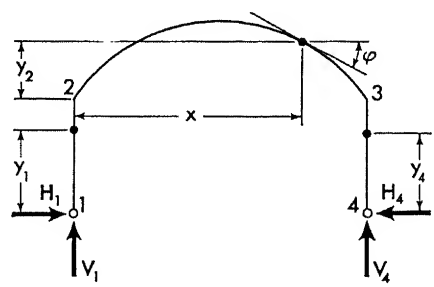
The main arcade of the Mercado de Abasto Proveedor Shopping Center in Buenos Aires, Argentina, illustrates an imaginative design of one of the contemporary forms of reinforced concrete construction. It consists of a series of interlocking frames of 80-foot span arranged in a crisscrossed pattern, with molded translucent glass blocks between them. The elegance and neatness of the building and the harmony of architectural design are apparent from the illustration. Delpini, Sulcic and Bes of Buenos Aires are engineers. (Courtesy of J. L. Delpini, engineer.)

SECTION 23

SYMMETRICAL PARABOLIC FRAMES WITH HINGED SUPPORTS

23-1. Notations, Coordinates, and Frame Constants





The sketch appearing on the left, above, explains notations for a representative arched frame with members of variable cross section. The axis of the arched girder is a symmetrical parabola conforming to Eq. (8-1).

The solutions of analysis given on the following pages are not limited to the shapes of the members shown, but are applicable to any shape, provided only that the frame is symmetrical and the axis of the girder is a parabola.

The sketch appearing on the right, above, shows positive directions of the vertical and horizontal components of the frame reactions. It also defines the angle of inclination and the coordinates at any section of the frame. Angles of inclination and coordinates are to be considered only in the positive sense.

Frame Constants. Obtain numerical value of the column parameter

α_{21} from applicable Charts 1 to 10 in the Appendix, and values of girder parameters α_{23}, β_{23}, γ_{23}, and δ_{23} from Table 13 in the Appendix.

$$\phi = \frac{\min I_{1\text{-}2}}{I_{2.5}} \cdot \frac{L}{h} \qquad \psi = \frac{f}{h}$$

$$\Theta_{23} = 2(\alpha_{23} + \beta_{23})$$

$$A = 2\alpha_{21} + \phi[\Theta_{23} + \psi^2\gamma_{23} + 2\psi(\gamma_{23} + \delta_{23})]$$

23-2. Equations of Frame Reactions and Moments. The equations for the vertical and the redundant horizontal components of frame reactions are given on the following pages for a number of loading conditions. Corresponding expressions for the moment and forces at any section of a member with applied load are also provided.

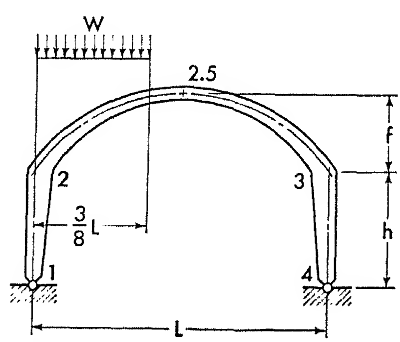
The equations for the moments and forces of load-free members are listed below for reference.

$$M_{y_1} = M_2 \frac{y_1}{h} \tag{23-1}$$

$$M_x = M_2\left(1 - \frac{x}{L}\right) + M_3 \frac{x}{L} - H_4 y_2 \tag{23-2}$$

$$M_{y_4} = M_3 \frac{y_4}{h} \tag{23-3}$$

23-3. Vertical Uniform Load over Three-eighths of Span



W = total load

Obtain values of load constants S, T, and U from Table 15 or 18.

$$K = S\psi + T + U$$

$$H_1 = H_4 = \frac{WLK\phi}{Ah} \qquad M_2 = M_3 = -H_1h$$

$$V_1 = \frac{13}{16}W \qquad V_4 = \frac{3W}{16}$$

When $x \leqq \frac{3}{8}L$

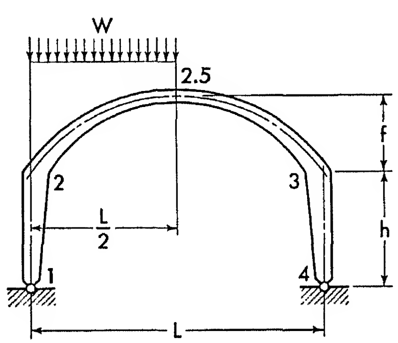
$$M_x = Wx\left(\frac{13}{16} - \frac{4x}{3L}\right) - H_1(h + y_2)$$

When $x > \frac{3}{8}L$

$$M_x = \frac{3W}{16}(L - x) - H_1(h + y_2)$$

Apply Eqs. (23-1) and (23-3) to obtain the moment at any section of the frame columns, and use Eqs. (11-6) through (11-8) to determine the axial and shearing forces in the arched girder.

23-4. Vertical Uniform Load over Left Half of Span



W = total load

Obtain values of load constants S, T, and U from Table 15 or 18.

$$K = S\psi + T + U$$

$$H_1 = H_4 = \frac{WLK\phi}{Ah} \qquad M_2 = M_3 = -H_1h$$

$$V_1 = \frac{3}{4}W \qquad V_4 = \frac{W}{4}$$

For Notations and Constants, see Arts. 23-1 and 23-2

When $x \leqq \frac{L}{2}$

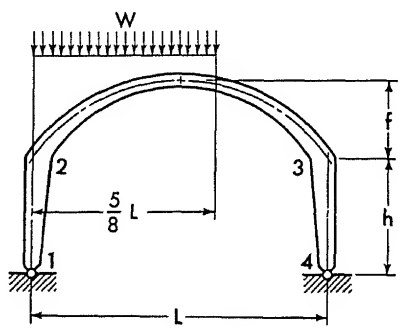
$$M_x = Wx\left(\frac{3}{4} - \frac{x}{L}\right) - H_1(h + y_2)$$

When $x > \frac{L}{2}$

$$M_x = \frac{W}{4}(L - x) - H_1(h + y_2)$$

Apply Eqs. (23-1) and (23-3) to obtain the moment at any section of the frame columns and use Eqs. (11-9) and (11-10) to determine the axial and shearing forces in the arched girder.

23-5. Vertical Uniform Load over Five-eighths of Span



W = total load

Obtain values of load constants S, T, and U from Table 15 or 18.

$$K = S\psi' + T + U$$

$$H_1 = H_4 = \frac{WLK\phi}{Ah} \qquad M_2 = M_3 = -H_1h$$

$$V_1 = \frac{11}{16}W \qquad V_4 = \frac{5W}{16}$$

When $x \leqq \frac{5}{8}L$

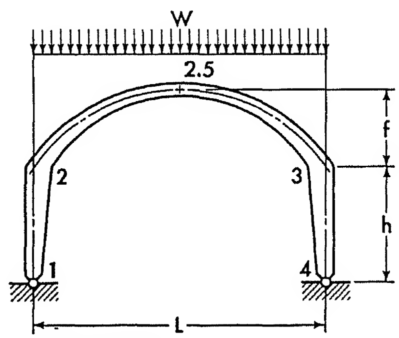
$$M_x = Wx\left(\frac{11}{16} - \frac{4x}{5L}\right) - H_1(h + y_2)$$

When $x > \frac{5}{8} L$

$$M_x = \frac{5W}{16}(L - x) - H_1(h + y_2)$$

Apply Eqs. (23-1) and (23-3) to obtain the moment at any section of the frame columns, and use Eqs. (11-11) through (11-13) to determine the axial and shearing forces in the arched girder.

23-6. Vertical Uniform Load over Entire Span



W = total load

Obtain values of load constants S and T from Table 15 or 18.

$$K = S\psi + 2T$$

$$H_1 = H_4 = \frac{WLK\phi}{Ah} \qquad M_2 = M_3 = -H_1 h$$

$$V_1 = V_4 = \frac{W}{2}$$

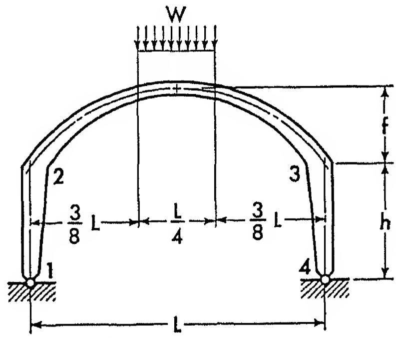
$$M_x = \frac{Wx}{2}\left(1 - \frac{x}{L}\right) - H_1(h + y_2)$$

$$M_{2.5} = \frac{WL}{8} - H_1(h + f)$$

Apply Eqs. (23-1) and (23-3) to obtain the moment at any section of the frame columns, and use Eqs. (11-14) and (11-15) to determine the axial and shearing forces in the arched girder.

For Notations and Constants, see Arts. 23-1 and 23-2

23-7. Vertical Uniform Load over Center Quarter of Span



W = total load

Obtain values of load constants S and T from Table 15 or 18.

$$K = S\psi + 2T$$

$$H_1 = H_4 = \frac{WLK\phi}{Ah} \qquad M_2 = M_3 = -H_1h$$

$$V_1 = V_4 = \frac{W}{2}$$

When $x \leqq \frac{3}{8}L$

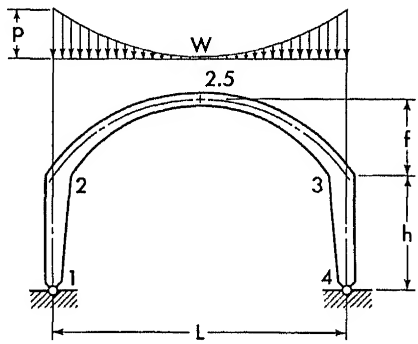
$$M_x = \frac{Wx}{2} - H_1(h + y_2)$$

When $x > \frac{3}{8}L$, but $\leqq \frac{L}{2}$

$$M_x = \frac{Wx}{2} - \frac{2W}{L}\left(x - \frac{3L}{8}\right)^2 - H_1(h + y_2)$$

Apply Eq. (23-1) to obtain the moment at any section of the left column, and use Eqs. (11-16) and (11-17) to determine the axial and shearing forces in the left half of the arched girder. Moments and axial forces at corresponding sections in the right half of the frame are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

23-8. Vertical Complementary Parabolic Load over Entire Span



Total load $W = \frac{pL}{3}$

Obtain values of load constants S and T from Table 15 or 18.

$$K = S\psi + 2T$$

$$H_1 = H_4 = \frac{WLK\phi}{Ah} \qquad M_2 = M_3 = -H_1h$$

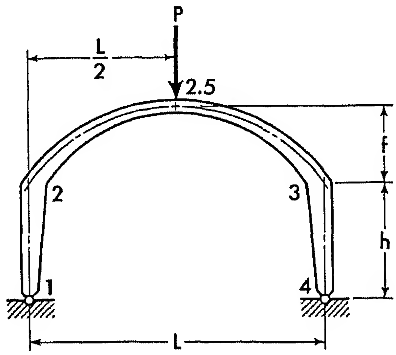
$$V_1 = V_4 = \frac{W}{2}$$

When $x \leqq \frac{L}{2}$

$$M_x = \frac{WL}{16}\left[1 - \left(\frac{L-2x}{L}\right)^4\right] - H_1(h + y_2)$$

Apply Eq. (23-1) to obtain the moment at any section of the left column, and use Eqs. (11-18) to determine the axial and shearing forces in the left half of the arched girder. Moments and axial forces at corresponding sections in the right half of the frame are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

23-9. Vertical Concentrated Load at Mid-point of Girder



Obtain values of load constants S and T from Table 16 or 19.

$$K = S\psi + 2T$$

$$H_1 = H_4 = \frac{PLK\phi}{Ah} \qquad M_2 = M_3 = -H_1h$$

$$V_1 = V_4 = \frac{P}{2} \qquad M_{2.5} = \frac{PL}{4} - H_1(h + f)$$

When $x \leqq \frac{L}{2}$

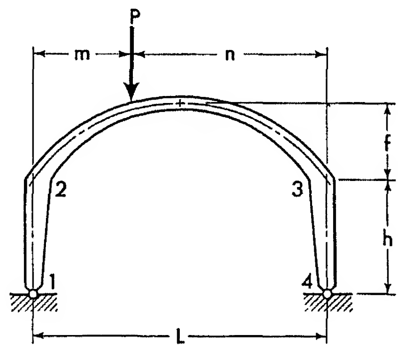
$$M_x = \frac{Px}{2} - H_1(h + y_2)$$

When $x > \frac{L}{2}$

$$M_x = \frac{P}{2}(L - x) - H_1(h + y_2)$$

Apply Eqs. (23-1) and (23-3) to obtain the moment at any section of the frame columns, and use Eqs. (11-19) through (11-21) to determine the axial and shearing forces in the arched girder.

23-10. Vertical Concentrated Load on Arched Girder



Obtain values of load constants S, T, and U from Table 16 or 19.

$$K = S\psi + T + U$$

$$H_1 = H_4 = \frac{PLK\phi}{Ah} \qquad M_2 = M_3 = -H_1h$$

$$V_1 = P\left(1 - \frac{m}{L}\right) \qquad V_4 = \frac{Pm}{L}$$

When $x \leqq m$

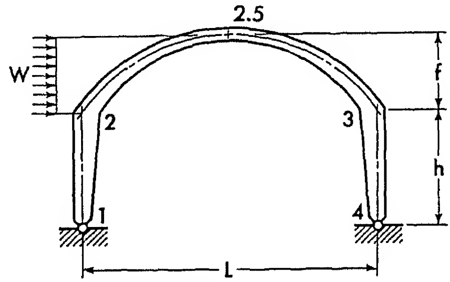
$$M_x = \frac{Pnx}{L} - H_1(h + y_2)$$

When $x > m$

$$M_x = Pm\left(1 - \frac{x}{L}\right) - H_1(h + y_2)$$

Apply Eqs. (23-1) and (23-3) to obtain the moment at any section of the frame columns, and use Eqs. (11-22) through (11-25) to determine the axial and shearing forces in the arched girder.

23-11. Horizontal Uniform Load on Left Half of Arched Girder



W = total load

Obtain values of load constants S, T, and U from Table 17 or 20.

$$C = \phi[\Theta_{23} + \psi(\gamma_{23} + \delta_{23})] + 2\alpha_{21}$$

$$K = S\psi + T + U$$

$$H_4 = \frac{W}{2A}(2K\phi\psi + C) \qquad H_1 = -(W - H_4)$$

$$M_2 = -H_1 h \qquad M_3 = -H_4 h$$

$$V_1 = -\frac{W}{2L}(2h + f) \qquad V_4 = -V_1$$

When $x \leqq \frac{L}{2}$

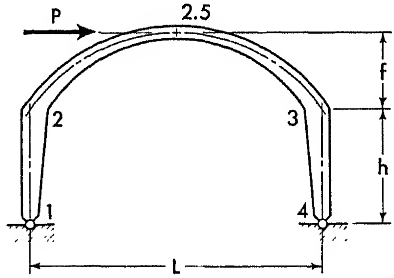
$$M_x = V_1 x - \frac{W y_2^2}{2f} - H_1(h + y_2)$$

When $x > \frac{L}{2}$

$$M_x = V_4(L - x) - H_4(h + y_2)$$

Apply Eqs. (23-1) and (23-3) to obtain the moment at any section of the frame columns, and use Eqs. (11-26) and (11-27) to determine the axial and shearing forces in the arched girder.

23-12. Horizontal Concentrated Load at Crown of Arched Girder



$$M_2 = \frac{Ph}{2} \qquad M_3 = -\frac{Ph}{2} \qquad M_{2.5} = 0$$

$$H_1 = -\frac{P}{2} \qquad H_4 = \frac{P}{2} \qquad V_4 = \frac{P(h+f)}{L} \qquad V_1 = -V_4$$

When $x \leqq \frac{L}{2}$

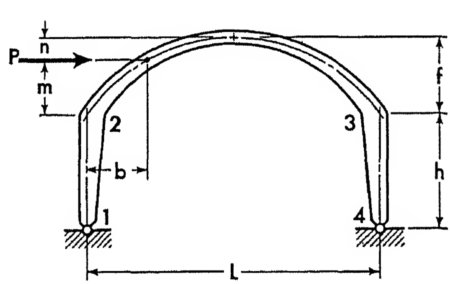
$$M_x = \frac{P}{2}(h + y_2) - V_4 x$$

When $x > \frac{L}{2}$

$$M_x = V_4(L - x) - \frac{P}{2}(h + y_2)$$

Apply Eqs. (23-1) and (23-3) to obtain the moment at any section of the frame columns, and use Eqs. (11-28) through (11-30) to determine the axial and shearing forces in the arched girder.

23-13. Horizontal Concentrated Load at Any Point of Left Half of Arched Girder



Obtain values of load constants S, T, and U from Table 17 or 20.

$$C = \phi[\Theta_{23} + \psi(\gamma_{23} + \delta_{23})] + 2\alpha_{21}$$

$$K = S\psi + T + U$$

$$H_4 = \frac{P}{2A}(2K\phi\psi + C) \qquad H_1 = -(P - H_4)$$

$$M_2 = -H_1 h \qquad M_3 = -H_4 h$$

$$V_4 = \frac{P(h+m)}{L} \qquad V_1 = -V_4$$

When $x \leqq b$

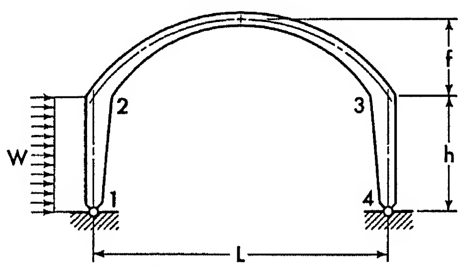
$$M_x = -\frac{Px(h+m)}{L} - H_1(h + y_2)$$

When $x > b$

$$M_x = P(h+m)\left(1 - \frac{x}{L}\right) - H_4(h + y_2)$$

Apply Eqs. (23-1) and (23-3) to obtain the moment at any section of the frame columns, and use Eqs. (11-31) through (11-33) to determine the axial and shearing forces in the arched girder.

23-14. Horizontal Uniform Load on Column



W = total load

Obtain value of load constant R_{21} from applicable Charts 11 to 16.

$$C = \phi[\Theta_{23} + \psi(\gamma_{23} + \delta_{23})] + 2\alpha_{21}$$

$$H_4 = \frac{W}{A}\left(R_{21} + \frac{C}{4}\right) \qquad H_1 = -(W - H_4)$$

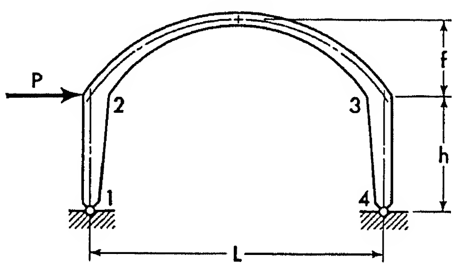
$$M_2 = \frac{Wh}{2} - H_4 h \qquad M_3 = -H_4 h$$

$$V_4 = \frac{Wh}{2L} \qquad V_1 = -V_4$$

$$M_{y_1} = \frac{Wy_1}{2}\left(1 - \frac{y_1}{h}\right) + M_2 \frac{y_1}{h}$$

Apply Eqs. (23-2) and (23-3) to obtain the moment at any section of frame members 2-3 and 3-4, and use Eqs. (11-4) and (11-5) to determine the axial and shearing forces in the arched girder.

23-15. Horizontal Concentrated Load at Joint 2



$$C = \phi[\Theta_{23} + \psi(\gamma_{23} + \delta_{23})] + 2\alpha_{21}$$

$$H_4 = \frac{PC}{2A} \qquad H_1 = -(P - H_4)$$

$$M_2 = -H_1 h \qquad M_3 = -H_4 h$$

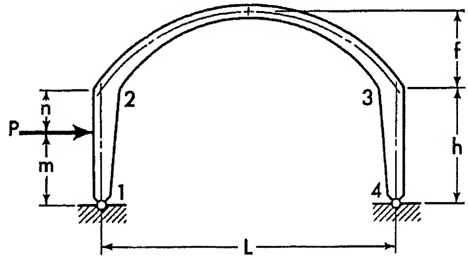
$$V_4 = \frac{Ph}{L}$$

$$V_1 = -V_4$$

$$M_{y_1} = M_2 \frac{y_1}{h}$$

Apply Eqs. (23-2) and (23-3) to obtain the moment at any section of frame members 2-3 and 3-4, and use Eqs. (11-4) and (11-5) to determine the axial and shearing forces in the arched girder.

23-16. Horizontal Concentrated Load at Any Point of Column



Obtain value of load constant R_{21} from applicable Tables 1 to 8.

$$C = \phi[\Theta_{23} + \psi(\gamma_{23} + \delta_{23})] + 2\alpha_{21}$$

$$H_4 = \frac{P}{A}\left(R_{21} + \frac{Cm}{2h}\right)$$

$$H_1 = -(P - H_4)$$

$$M_2 = Pm - H_4h \qquad M_3 = -H_4h$$

$$V_4 = \frac{Pm}{L} \qquad V_1 = -V_4$$

When $y_1 \leqq m$

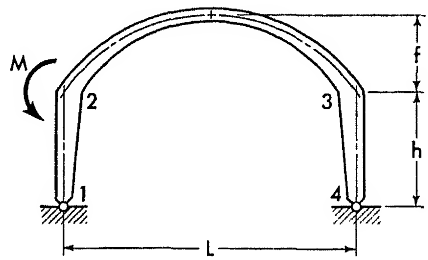
$$M_{y_1} = (Pn + M_2)\frac{y_1}{h}$$

When $y_1 > m$

$$M_{y_1} = Pm\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (23-2) and (23-3) to obtain the moment at any section of frame members 2-3 and 3-4, and use Eqs. (11-4) and (11-5) to determine the axial and shearing forces in the arched girder.

23-17. Moment Applied at Joint 2



$$B = \Theta_{23} + \psi(\gamma_{23} + \delta_{23})$$

$$H_1 = H_4 = -\frac{MB\phi}{2Ah}$$

$$M_{21} = M_3 = -H_1h \qquad M_{23} = -(M - M_{21})$$

$$V_1 = \frac{M}{L} \qquad V_4 = -\frac{M}{L}$$

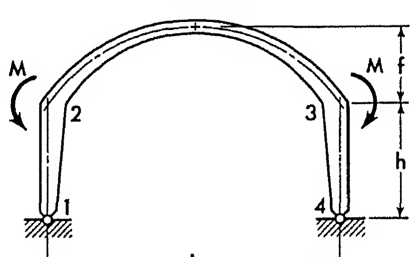
$$M_{y_1} = M_{21}\frac{y_1}{h}$$

$$M_x = M_{23}\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L} - H_4y_2$$

$$M_{y_4} = M_3\frac{y_4}{h}$$

Apply Eqs. (11-4) and (11-5) to determine the axial and shearing forces in the arched girder.

23-18. Two Equal Moments Applied at Joints 2 and 3



$$B = \Theta_{23} + \psi(\gamma_{23} + \delta_{23})$$

$$H_1 = H_4 = -\frac{MB\phi}{Ah}$$

$$M_{21} = M_{34} = -H_1h$$

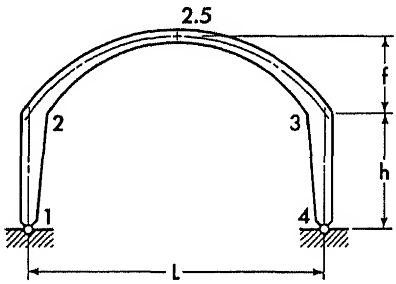
$$M_{23} = M_{32} = -(M - M_{21}) \qquad V_1 = V_4 = 0$$

$$M_{y_1} = M_{21}\frac{y_1}{h} \qquad M_x = M_{23} - H_1y_2$$

$$M_{y_4} = M_{34}\frac{y_4}{h}$$

Apply Eqs. (11-4) and (11-5) to determine the axial and shearing forces in the arched girder.

23-19. Effect of Temperature Rise. Range t° for entire frame.



$$H_1 = H_4 = \frac{12L\varepsilon t^\circ}{Ah^3}E(\min I_{1\text{-}2})$$

$$M_2 = M_3 = -H_1h \qquad V_1 = V_4 = 0$$

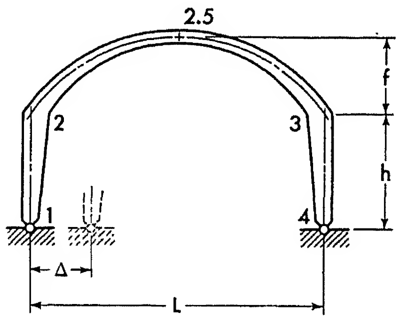
$$M_{y_1} = M_2\frac{y_1}{h} \qquad M_x = M_2 - H_1y_2$$

$$M_{y_4} = M_3\frac{y_4}{h}$$

Apply Eqs. (11-4) and (11-5) to determine the axial and shearing forces in the arched girder.

Note: For temperature drop, introduce the value of t° with a negative sign.

23-20. Horizontal Displacement of One Support



$$H_1 = H_4 = \frac{12\Delta}{Ah^3} E(\min I_{1\text{-}2})$$

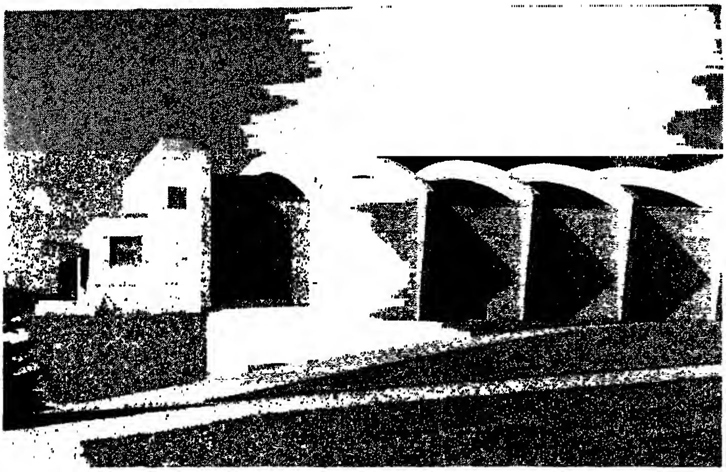
$$M_2 = M_3 = -H_1 h \qquad V_1 = V_4 = 0$$

$$M_{y_1} = M_2 \frac{y_1}{h} \qquad M_x = M_2 - H_1 y_2$$

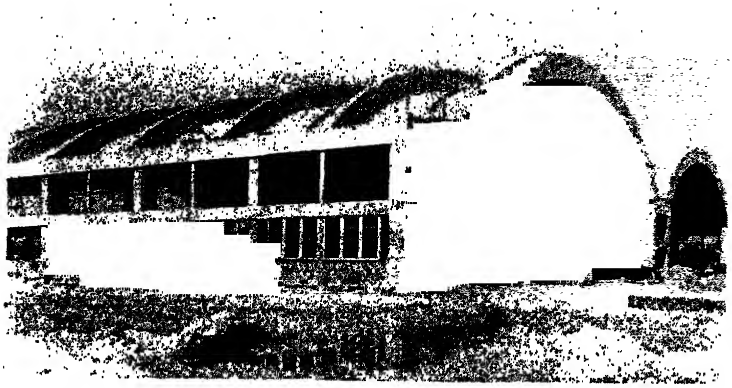
$$M_{y_4} = M_3 \frac{y_4}{h}$$

Apply Eqs. (11-4) and (11-5) to determine the axial and shearing forces in the arched girder.

Note: If the direction of the frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.



Recreation Hall of the Theodore Roosevelt Junior High School in Williamsport, Pennsylvania. Arched rigid frames of variable cross section are effectively utilized as the principal framework of the building. Note that the curved girders and slanted columns are extremely attractive even without any architectural treatment. Designed by D. H. Grootenboer, architect of Williamsport, Pennsylvania, and A. W. Lookup Co., engineers of Philadelphia, Pennsylvania. (Courtesy of the Portland Cement Association.)

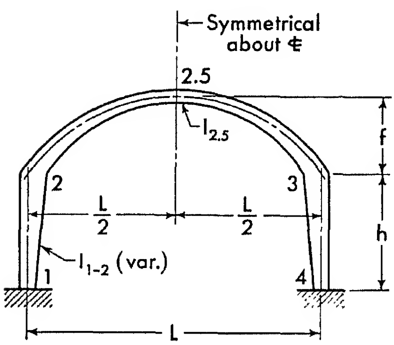


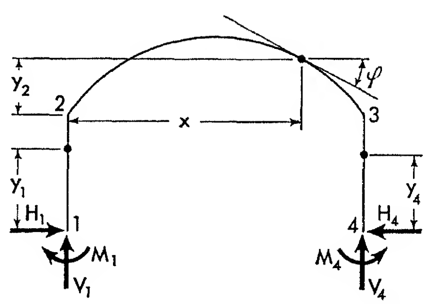
The reinforced concrete Seaplane Hangar at the Patuxent River Naval Air Station. The photograph shows one of the common types of hangar construction, which permits effective transmittal of the arch thrust to the foundations through the two-story lean-tos flanking each of the hangar bays. The lean-to bents may be treated as columns of the hangar's frames, and the arched members spanning the distance between the lean-tos may be considered as girders. Great economy is obtained in the construction of this seaplane hangar through the placement of the slab reinforcement in the directions of the principal tensile stresses. Designed by Roberts and Schaefer Company, engineers, New York, New York. (Courtesy of Roberts and Schaefer Co.)

SECTION 24

SYMMETRICAL PARABOLIC FRAMES WITH FIXED SUPPORTS

24-1. Notations, Coordinates, and Frame Constants





The sketch appearing on the left, above, explains notations for a representative arched frame with members of variable cross section. The axis of the arched girder is a symmetrical parabola conforming to Eq. (8-1).
The solutions of analysis given on the following pages are not limited to the shapes of the members shown, but are applicable to any shape, provided only that the frame is symmetrical and the axis of the girder is a parabola.

The sketch appearing on the right, above, shows positive directions of the moments and the vertical and horizontal components of the frame reactions. It also defines the angle of inclination and the coordinates at any section of the frame. Angles of inclination and coordinates are to be considered only in the positive sense.

Frame Constants. Obtain numerical values of column parameters α_{12}, α_{21}, and β_{12} from applicable Charts 1 to 10 in the Appendix, and values of girder parameters α_{23}, β_{23}, γ_{23}, and δ_{23} from Table 13 in the Appendix.

$$\phi = \frac{\min I_{1\text{-}2}}{I_{2.5}} \cdot \frac{L}{h} \qquad \psi = \frac{f}{h}$$

$$\Theta_{12} = \alpha_{12} + \alpha_{21} + 2\beta_{12}$$

$$\Theta_{23} = 2(\alpha_{23} + \beta_{23})$$

$$A = 2(\alpha_{12} + \beta_{12}) - \phi\psi(\gamma_{23} + \delta_{23})$$

$$B = 2\Theta_{12} + \phi\Theta_{23} \qquad C = 2\alpha_{12} + \phi\psi^2\gamma_{23}$$

$$D = \frac{B}{2} - 2\phi\beta_{23} \qquad F = B - AJ \qquad J = \frac{A}{C}$$

24-2. Equations of Frame Reactions and Moments. The equations for the redundant moments and the vertical and horizontal components of frame reactions are given on the following pages for a number of loading conditions. Corresponding expressions for the moment and forces at any section of a member with applied load are also provided.

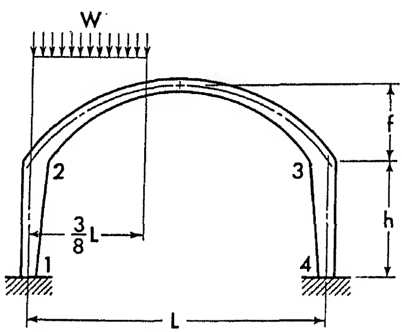
The equations for the moments and forces of load-free members are listed below for reference.

$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h} \tag{24-1}$$

$$M_x = M_2\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L} - H_4y_2 \tag{24-2}$$

$$M_{y_4} = M_3\frac{y_4}{h} + M_4\left(1 - \frac{y_4}{h}\right) \tag{24-3}$$

24-3. Vertical Uniform Load over Three-eighths of Span



W = Total load

Obtain values of load constants S, T, and U from Table 15 or 18.

Members of Variable Section

$$G = 1 + \frac{JS\psi}{U} \qquad K = \frac{BS\psi}{C}$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -\frac{WL\phi}{F}(T + GU) \mp \frac{WL\phi}{2D}(T - U)$$

$$H_1 = H_4 = \frac{WL\phi}{Fh}[K + J(T + U)]$$

$$V_1 = W\left[\frac{13}{16} + \frac{\phi(T - U)}{D}\right] \qquad V_4 = W - V_1$$

$$M_1 = M_2 + H_1h \qquad M_4 = M_3 + H_4h$$

When $x \leqq \frac{3}{8}L$

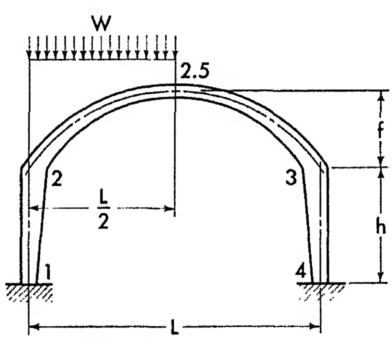
$$M_x = Wx\left(\frac{13}{16} - \frac{4x}{3L}\right) + M_2\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L} - H_4y_2$$

When $x > \frac{3}{8}L$

$$M_x = \frac{3W}{16}(L - x) + M_2\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L} - H_4y_2$$

Apply Eqs. (24-1) and (24-3) to obtain the moment at any section of the frame columns, and use Eqs. (12-6) through (12-8) to determine the axial and shearing forces in the arched girder.

24-4. Vertical Uniform Load over Left Half of Span



W = Total load

Obtain values of load constants S, T, and U from Table 15 or 18.

For Notations and Constants, see Arts. 24-1 and 24-2

$$G = 1 + \frac{JS\psi}{U} \qquad K = \frac{BS\psi}{C}$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\} = -\frac{WL\phi}{F}(T + GU) \mp \frac{WL\phi}{2D}(T - U)$$

$$H_1 = H_4 = \frac{WL\phi}{Fh}[K + J(T + U)]$$

$$M_1 = M_2 + H_1h \qquad M_4 = M_3 + H_4h$$

$$V_1 = W\left[\frac{3}{4} + \frac{\phi(T - U)}{D}\right] \qquad V_4 = W - V_1$$

When $x \leqq \frac{L}{2}$

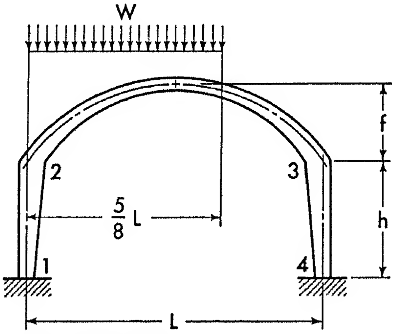
$$M_x = Wx\left(\frac{3}{4} - \frac{x}{L}\right) + M_2\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L} - H_4y_2$$

When $x > \frac{L}{2}$

$$M_x = \frac{W}{4}(L - x) + M_2\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L} - H_4y_2$$

Apply Eqs. (24-1) and (24-3) to obtain the moment at any section of the frame columns, and use Eqs. (12-9) and (12-10) to determine the axial and shearing forces in the arched girder.

24-5. Vertical Uniform Load over Five-eighths of Span



W = Total load

Obtain values of load constants S, T, and U from Table 15 or 18.

Members of Variable Section

$$G = 1 + \frac{JS\psi}{U} \qquad K = \frac{BS\psi}{C}$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -\frac{WL\phi}{F}(T + GU) \mp \frac{WL\phi}{2D}(T - U)$$

$$H_1 = H_4 = \frac{WL\phi}{Fh}[K + J(T + U)]$$

$$M_1 = M_2 + H_1 h \qquad M_4 = M_3 + H_4 h$$

$$V_1 = W\left[\frac{11}{16} + \frac{\phi(T - U)}{D}\right] \qquad V_4 = W - V_1$$

When $x \leqq \frac{5}{8}L$

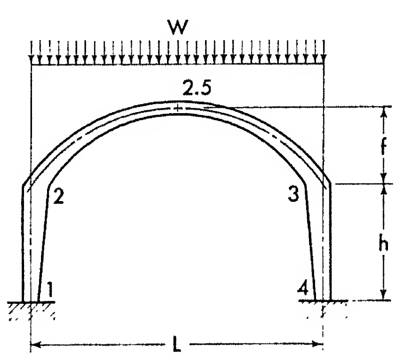
$$M_x = Wx\left(\frac{11}{16} - \frac{4x}{5L}\right) + M_2\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L} - H_4 y_2$$

When $x > \frac{5}{8}L$

$$M_x = \frac{5W}{16}(L - x) + M_2\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L} - H_4 y_2$$

Apply Eqs. (24-1) and (24-3) to obtain the moment at any section of the frame columns, and use Eqs. (12-11) through (12-13) to determine the axial and shearing forces in the arched girder.

24-6. Vertical Uniform Load over Entire Span



W = Total load

Obtain values of load constants S and T from Table 15 or 18.

For Notations and Constants, see Arts. 24-1 and 24-2

$$G = 1 + \frac{JS\psi}{T} \qquad K = \frac{BS\psi}{C}$$

$$M_2 = M_3 = -\frac{WLT\phi}{F}(1 + G)$$

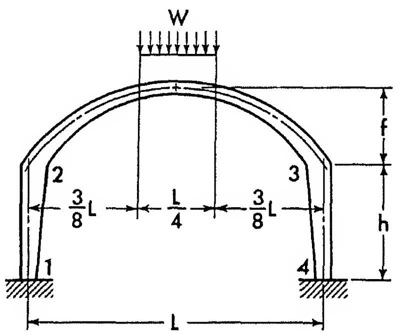
$$H_1 = H_4 = \frac{WL\phi}{Fh}(K + 2JT) \qquad M_1 = M_4 = M_2 + H_1h$$

$$V_1 = V_4 = \frac{W}{2} \qquad M_{2.5} = \frac{WL}{8} + M_2 - H_1f$$

$$M_x = \frac{Wx}{2}\left(1 - \frac{x}{L}\right) + M_2 - H_1y_2$$

Apply Eqs. (24-1) and (24-3) to obtain the moment at any section of the frame columns, and use Eqs. (12-14) and (12-15) to determine the axial and shearing forces in the arched girder.

24-7. Vertical Uniform Load over Center Quarter of Span



W = Total load

Obtain values of load constants S and T from Table 15 or 18.

$$G = 1 + \frac{JS\psi}{T} \qquad K = \frac{BS\psi}{C}$$

$$M_2 = M_3 = -\frac{WLT\phi}{F}(1 + G)$$

$$H_1 = H_4 = \frac{WL\phi}{Fh}(K + 2JT)$$

$$M_1 = M_4 = M_2 + H_1h \qquad V_1 = V_4 = \frac{W}{2}$$

When $x \leqq \frac{3}{8}L$

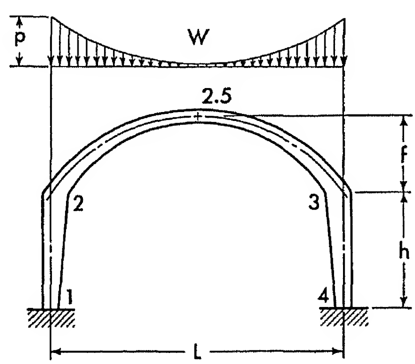
$$M_x = \frac{Wx}{2} + M_2 - H_1 y_2$$

When $x > \frac{3}{8}L$, but $\leqq \frac{L}{2}$

$$M_x = \frac{Wx}{2} - \frac{2W}{L}\left(x - \frac{3L}{8}\right)^2 + M_2 - H_1 y_2$$

Apply Eq. (24-1) to obtain the moment at any section of the left column, and use Eqs. (12-16) and (12-17) to determine the axial and shearing forces in the left half of the arched girder. Moments and axial forces at corresponding sections in the right half of the frame are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

24-8. Vertical Complementary Parabolic Load over Entire Span



Total load $W = \frac{pL}{3}$

Obtain values of load constants S and T from Table 15 or 18.

$$G = 1 + \frac{JS\psi}{T} \qquad K = \frac{BS\psi}{C}$$

$$M_2 = M_3 = -\frac{WLT\phi}{F}(1 + G)$$

$$H_1 = H_4 = \frac{WL\phi}{Fh}(K + 2JT)$$

For Notations and Constants, see Arts. 24-1 and 24-2

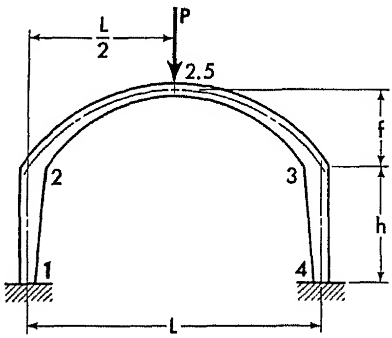
$$M_1 = M_4 = M_2 + H_1 h \qquad V_1 = V_4 = \frac{W}{2}$$

When $x \leqq \frac{L}{2}$

$$M_x = M_2 + \frac{WL}{16}\left[1 - \left(\frac{L - 2x}{L}\right)^4\right] - H_1 y_2$$

Apply Eq. (24-1) to obtain the moment at any section of the left column, and use Eqs. (12-18) to determine the axial and shearing forces in the left half of the arched girder. Moments and axial forces at corresponding sections in the right half of the frame are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

24-9. Vertical Concentrated Load at Mid-point of Girder



Obtain values of load constants S and T from Table 16 or 19.

$$G = 1 + \frac{JS\psi}{T} \qquad K = \frac{BS\psi}{C}$$

$$M_2 = M_3 = -\frac{PLT\phi}{F}(1 + G)$$

$$H_1 = H_4 = \frac{PL\phi}{Fh}(K + 2JT) \qquad M_1 = M_4 = M_2 + H_1 h$$

$$V_1 = V_4 = \frac{P}{2}$$

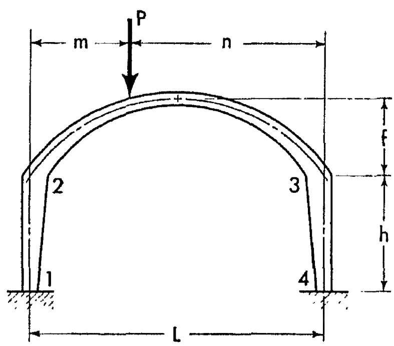
$$M_{2.5} = M_2 + \frac{PL}{4} - H_1 f$$

When $x \leqq \frac{L}{2}$

$$M_x = \frac{Px}{2} + M_2 - H_1 y_2$$

Apply Eq. (24-1) to obtain the moment at any section of the left column, and use Eqs. (12-19) to determine the axial and shearing forces in the left half of the arched girder. Moments and axial forces at corresponding sections in the right half of the frame are identical to those in the left half. Shearing forces at corresponding sections in the right half have the same numerical values as those in the left half, but are of the opposite sign.

24-10. Vertical Concentrated Load on Arched Girder



Obtain values of load constants S, T, and U from Table 16 or 19.

$$G = 1 + \frac{JS\psi'}{U} \qquad K = \frac{BS\psi'}{C}$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -PL\phi\left[\frac{T+GU}{F} \pm \frac{T-U}{2D}\right]$$

$$H_1 = H_4 = \frac{PL\phi}{Fh}[K + J(T+U)]$$

$$M_1 = M_2 + H_1 h \qquad M_4 = M_3 + H_1 h$$

$$V_1 = P\left[1 - \frac{m}{L} + \frac{\phi(T-U)}{D}\right] \qquad V_4 = P - V_1$$

For Notations and Constants, see Arts. 24-1 and 24-2

When $x \leqq m$

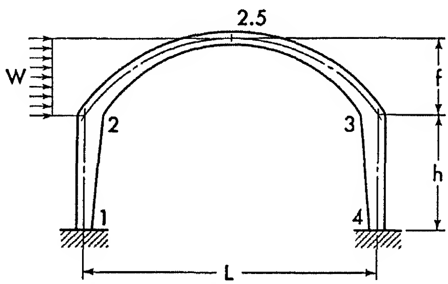
$$M_x = (Pn + M_3)\frac{x}{L} + M_2\left(1 - \frac{x}{L}\right) - H_1 y_2$$

When $x > m$

$$M_x = (Pm + M_2)\left(1 - \frac{x}{L}\right) + M_3\frac{x}{L} - H_1 y_2$$

Apply Eqs. (24-1) and (24-3) to obtain the moment at any section of the frame columns, and use Eqs. (12-20) through (12-23) to determine the axial and shearing forces in the arched girder.

24-11. Horizontal Uniform Load on Left Half of Arched Girder



W = Total load

Obtain values of load constants S, T, and U from Table 17 or 20.

$$G = 1 + \frac{JS\psi}{U} \qquad K = \frac{BS\psi}{C}$$

$$X = \beta_{12} - \alpha_{12}(J - 1)$$

$$Y = h(\alpha_{12} + \beta_{12}) - f\phi(T - U)$$

$$Z = \frac{B\alpha_{12} - A(\alpha_{12} + \beta_{12})}{C}$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -\frac{Wf\phi}{F}(T + GU) + \frac{WXh}{F} \pm \frac{WY}{2D}$$

$$H_4 = \frac{W\phi\psi}{F}[K + J(T + U)] + \frac{WZ}{F}$$

$$H_1 = -(W - H_4) \qquad V_4 = \frac{W}{L}\left(\frac{f}{2} + \frac{Y}{D}\right)$$

$$V_1 = -V_4 \qquad M_1 = M_2 + H_1 h$$

$$M_4 = M_3 + H_4 h$$

$$M_{2.5} = \frac{Wf}{4} + \frac{M_2 + M_3}{2} - H_4 f$$

When $x \leqq \frac{L}{2}$

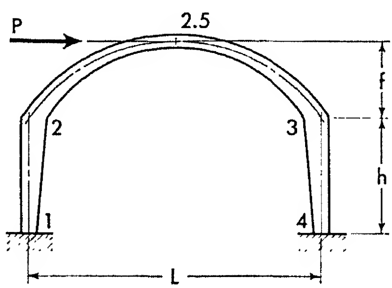
$$M_x = M_2 - V_4 x - \frac{W y_2^2}{2f} - H_1 y_2$$

When $x > \frac{L}{2}$

$$M_x = M_3 + V_4(L - x) - H_4 y_2$$

Apply Eqs. (24-1) and (24-3) to obtain the moment at any section of the frame columns, and use Eqs. (12-24) and (12-25) to determine the axial and shearing forces in the arched girder.

24-12. Horizontal Concentrated Load at Crown of Arched Girder



Obtain values of load constants T and U from Table 17 or 20.

$$Y = h(\alpha_{12} + \beta_{12}) - f\phi(T - U)$$

$$M_2 = \frac{PY}{2D} \qquad M_3 = -\frac{PY}{2D} \qquad H_4 = \frac{P}{2}$$

$$H_1 = -\frac{P}{2} \qquad V_4 = \frac{P}{L}\left(f + \frac{Y}{D}\right)$$

$$V_1 = -V_4 \qquad M_1 = M_2 + H_1 h$$

$$M_4 = M_3 + H_4 h \qquad M_{2.5} = 0$$

When $x \leqq \frac{L}{2}$

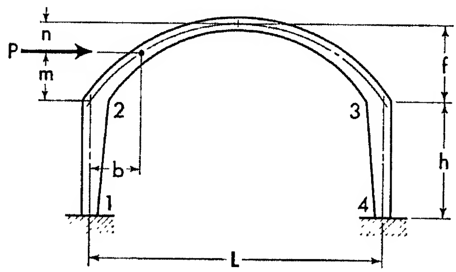
$$M_x = M_2 + \frac{Py_2}{2} - V_4 x$$

When $x > \frac{L}{2}$

$$M_x = -M_2 - \frac{Py_2}{2} + V_4(L - x)$$

Apply Eqs. (24-1) and (24-3) to obtain the moment at any section of the frame columns, and use Eqs. (12-26) through (12-28) to determine the axial and shearing forces in the arched girder.

24-13. Horizontal Concentrated Load at Any Point of Left Half of Arched Girder



Obtain values of load constants S, T, and U from Table 17 or 20.

$$G = 1 + \frac{JS\psi}{U} \qquad K = \frac{BS\psi}{C}$$

$$X = \beta_{12} - \alpha_{12}(J - 1)$$

$$Y = h(\alpha_{12} + \beta_{12}) - f\phi(T - U)$$

$$Z = \frac{B\alpha_{12} - A(\alpha_{12} + \beta_{12})}{C}$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = -\frac{Pf\phi}{F}(T + GU) + \frac{PXh}{F} \pm \frac{PY}{2D}$$

$$H_4 = \frac{P\phi\psi}{F}[K + J(T + U)] + \frac{PZ}{F}$$

$$H_1 = -(P - H_4) \qquad M_1 = M_2 + H_1h$$

$$M_4 = M_3 + H_4h$$

$$V_4 = \frac{P}{L}\left(m + \frac{Y}{D}\right) \qquad V_1 = -V_4$$

When $x \leqq b$

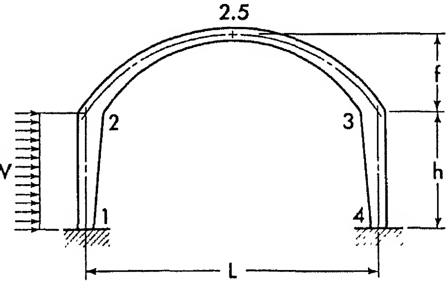
$$M_x = M_2 - V_4x - H_1y_2$$

When $x > b$

$$M_x = M_3 + V_4(L - x) - H_4y_2$$

Apply Eqs. (24-1) and (24-3) to obtain the moment at any section of the frame columns, and use Eqs. (12-29) through (12-31) to determine the axial and shearing forces in the arched girder.

24-14. Horizontal Uniform Load on Column



W = Total load

Obtain values of load constants R_{12} and R_{21} from applicable Charts 11 to 16.

$$G = \frac{B\alpha_{12}}{C} - J(\alpha_{12} + \beta_{12})$$

$$K = \beta_{12} - \alpha_{12}(J - 1) \qquad X = R_{21} - R_{12}(J - 1)$$

$$Y = \frac{BR_{12}}{C} - J(R_{12} + R_{21})$$

$$Z = (\alpha_{12} + \beta_{12}) - 2(R_{12} + R_{21})$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = \frac{Wh}{2F}(K - 2X) \pm \frac{WZh}{4D}$$

For Notations and Constants, see Arts. 24-1 and 24-2

$$H_4 = \frac{W}{2F}(G - 2Y) \qquad H_1 = -(W - H_4)$$

$$M_1 = M_2 - \frac{h}{2}(W - 2H_4) \qquad M_4 = M_3 + H_4h$$

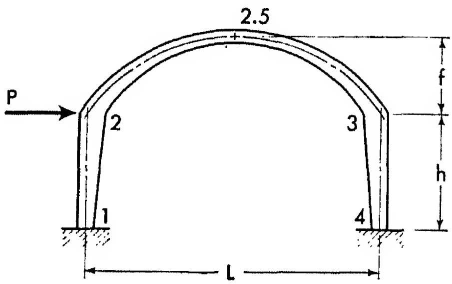
$$M_{2.5} = \frac{Wh}{2F}(K - 2X) - H_4f$$

$$V_4 = \frac{WZh}{2DL} \qquad V_1 = -V_4$$

$$M_{y_1} = \left(\frac{Wy_1}{2} + M_1\right)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (24-2) and (24-3) to obtain the moment at any section of frame members 2-3 and 3-4, and use Eqs. (12-4) and (12-5) to determine the axial and shearing forces in the arched girder.

24-15. Horizontal Concentrated Load at Joint 2



$$G = \frac{B\alpha_{12}}{C} - J(\alpha_{12} + \beta_{12})$$

$$K = \beta_{12} - \alpha_{12}(J - 1)$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = \frac{PKh}{F} \pm \frac{Ph(\alpha_{12} + \beta_{12})}{2D}$$

$$H_4 = \frac{PG}{F} \qquad H_1 = -(P - H_4)$$

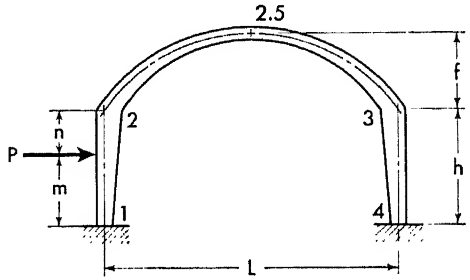
$$M_1 = M_2 - h(P - H_4) \qquad M_4 = M_3 + H_4h$$

$$M_{2.5} = \frac{P}{F}(Kh - Gf) \qquad V_4 = \frac{Ph}{DL}(\alpha_{12} + \beta_{12})$$

$$V_1 = -V_4 \qquad M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (24-2) and (24-3) to obtain the moment at any section of frame members 2-3 and 3-4, and use Eqs. (12-4) and (12-5) to determine the axial and shearing forces in the arched girder.

24-16. Horizontal Concentrated Load at Any Point of Column



Obtain values of load constants R_{12} and R_{21} from applicable Tables 1 to 8.

$$G = \frac{B\alpha_{12}}{C} - J(\alpha_{12} + \beta_{12})$$

$$K = \beta_{12} - \alpha_{12}(J - 1)$$

$$X = R_{21} - R_{12}(J - 1)$$

$$Y = \frac{BR_{12}}{C} - J(R_{12} + R_{21})$$

$$Z = m(\alpha_{12} + \beta_{12}) - h(R_{12} + R_{21})$$

$$\left.\begin{matrix} M_2 \\ M_3 \end{matrix}\right\rangle = \frac{P}{F}(Km - Xh) \pm \frac{PZ}{2D}$$

$$H_4 = \frac{P}{Fh}(Gm - Yh) \qquad H_1 = -(P - H_4)$$

$$M_1 = M_2 - Pm + H_4h \qquad M_4 = M_3 + H_4h$$

$$M_{2.5} = \frac{P}{F}(Km - Xh) - H_4f$$

$$V_4 = \frac{PZ}{DL} \qquad V_1 = -V_4$$

For Notations and Constants, see Arts. 24-1 and 24-2

When $y_1 \leqq m$

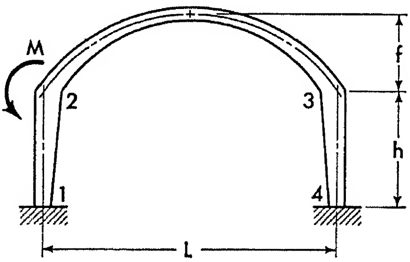
$$M_{y_1} = (Pn + M_2)\frac{y_1}{h} + M_1\left(1 - \frac{y_1}{h}\right)$$

When $y_1 > m$

$$M_{y_1} = (Pm + M_1)\left(1 - \frac{y_1}{h}\right) + M_2\frac{y_1}{h}$$

Apply Eqs. (24-2) and (24-3) to obtain the moment at any section of frame members 2-3 and 3-4, and use Eqs. (12-4) and (12-5) to determine the axial and shearing forces in the arched girder.

24-17. Moment Applied at Joint 2



$$G = 1 + \frac{J\psi'(\gamma_{23} + \delta_{23})}{2\beta_{23}}$$

$$K = \frac{B\psi'(\gamma_{23} + \delta_{23})}{2C}$$

$$\left.\begin{matrix} M_{21} \\ M_3 \end{matrix}\right\} = \frac{M\phi}{F}(\alpha_{23} + G\beta_{23}) \pm \frac{M\phi}{2D}(\alpha_{23} - \beta_{23})$$

$$H_1 = H_4 = -\frac{M\phi}{Fh}[K + J(\alpha_{23} + \beta_{23})]$$

$$M_{23} = -(M - M_{21}) \qquad M_1 = M_{21} + H_1h$$

$$M_4 = M_3 + H_4h$$

$$V_1 = \frac{M}{L}\left[1 - \frac{\phi(\alpha_{23} - \beta_{23})}{D}\right] \qquad V_4 = -V_1$$

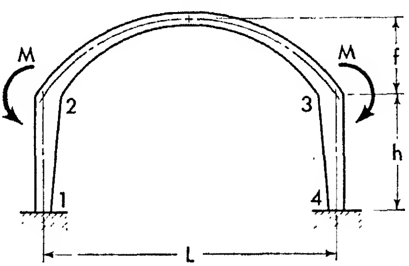
$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) + M_{21}\frac{y_1}{h}$$

$$M_x = M_{23}\left(1 - \frac{x}{L}\right) + M_3 \frac{x}{L} - H_4 y_2$$

$$M_{y_4} = M_4\left(1 - \frac{y_4}{h}\right) + M_3 \frac{y_4}{h}$$

Apply Eqs. (12-4) and (12-5) to determine the axial and shearing forces in the arched girder.

24-18. Two Equal Moments Applied at Joints 2 and 3



$$G = 1 + \frac{J\psi(\gamma_{23} + \delta_{23})}{2\beta_{23}}$$

$$K = \frac{B\psi(\gamma_{23} + \delta_{23})}{2C}$$

$$M_{21} = M_{34} = \frac{2M\phi}{F}(\alpha_{23} + G\beta_{23})$$

$$H_1 = H_4 = -\frac{2M\phi}{Fh}[K + J(\alpha_{23} + \beta_{23})]$$

$$M_{23} = M_{32} = -(M - M_{21})$$

$$M_1 = M_4 = M_{21} + H_1 h \qquad V_1 = V_4 = 0$$

$$M_{y_1} = M_1\left(1 - \frac{y_1}{h}\right) + M_{21}\frac{y_1}{h}$$

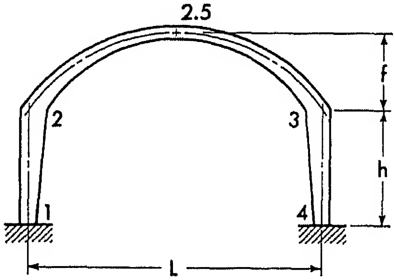
$$M_x = M_{23} - H_1 y_2$$

$$M_{y_4} = M_4\left(1 - \frac{y_4}{h}\right) + M_{34}\frac{y_4}{h}$$

Apply Eqs. (12-4) and (12-5) to determine the axial and shearing forces in the arched girder.

For Notations and Constants, see Arts. 24-1 and 24-2

24-19. Effect of Temperature Rise. Range t° for entire frame.



$$K = \frac{12L\varepsilon t^\circ}{CFh^2} E(\min I_{1\text{-}2})$$

$$M_2 = M_3 = -AK$$

$$M_1 = M_4 = K(B - A)$$

$$H_1 = H_4 = \frac{M_1 - M_2}{h} \qquad V_1 = V_4 = 0$$

$$M_{y_1} = M_1 - H_1y_1$$

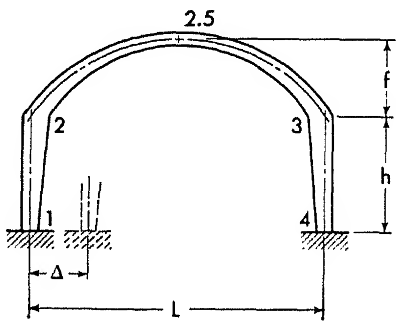
$$M_x = M_2 - H_1y_2$$

$$M_{y_4} = M_4 - H_1y_4$$

Apply Eqs. (12-4) and (12-5) to determine the axial and shearing forces in the arched girder.

Note: For temperature drop, introduce the value of t° with a negative sign.

24-20. Horizontal Displacement of One Support



$$K = \frac{12\Delta}{CFh^2} E(\min I_{1\text{-}2})$$

$$M_2 = M_3 = -AK$$

$$M_1 = M_4 = K(B - A)$$

$$H_1 = H_4 = \frac{M_1 - M_2}{h} \qquad V_1 = V_4 = 0$$

$$M_{y_1} = M_1 - H_1 y_1$$

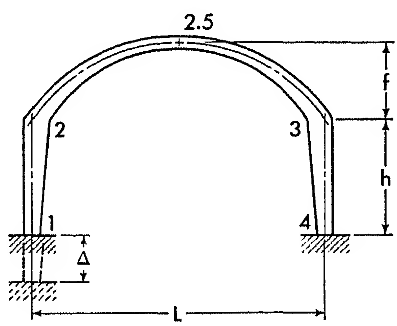
$$M_x = M_2 - H_1 y_2$$

$$M_{y_4} = M_4 - H_1 y_4$$

Apply Eqs. (12-4) and (12-5) to determine the axial and shearing forces in the arched girder.

Note: If the direction of the frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.

24-21. Vertical Settlement of One Support



$$K = \frac{12\Delta}{DLh} E(\min I_{1\text{-}2})$$

$$M_1 = M_2 = K$$

$$M_3 = M_4 = -K \qquad M_{2.5} = 0$$

$$H_1 = H_4 = 0 \qquad V_1 = -\frac{2K}{L} \qquad V_4 = -V_1$$

$$M_{y_1} = M_1 \qquad M_x = M_1 + V_1 x \qquad M_{y_4} = M_4$$

Apply Eqs. (12-4) and (12-5) to determine the axial and shearing forces in the arched girder.

Note: If the direction of the frame displacement is opposite to that shown on the sketch, introduce the value of Δ with a negative sign.

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APPENDIX

CONTENTS

EXPLANATORY NOTES TO CHARTS AND TABLES

1. In preparation of Charts 1 to 16, appearing on pp. 418 to 433, the graphs which appear in *Rahmentragwerke und Durchlauftrager* by R. Guldan, Vienna, 1943, copyright vested in the United States Attorney General, have been partially utilized, pursuant to License Agreement No. JA-1526 of the United States Attorney General.

2. Values of various constants as given in the tables and charts are for straight or arched members of constant width.

3. Curved haunches of the members with straight axis are outlined by the quadratic parabola. For the left-hand haunch the parabolic curve conforms to Eq. (8-1). Right-hand haunch is opposite hand.

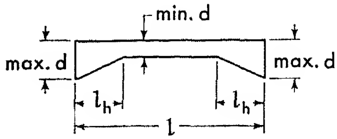
4. In Tables 1 to 8, for positioning of the concentrated load at point 0 or at point 12, the values of load constant R are zero.

5. Equations of various constants as given in headings of tables and charts always refer to the XY cartesian system of coordinates with the origin at the left end of the member. For details, see author's paper "Concept of Elastic Parameters," *Proc. Am. Concrete Inst.*, vol. 54, pp. 987–1008, 1958.

6. Values of various constants as given in tables for prime and quadratic arches are for arches having symmetric parabolic axes, conforming to Eq. (8-1).

7. Mathematical equations for load constants, as given in headings to Tables 16 and 20, are for the case of a horizontal concentrated load on the arch. The same equations are applicable to the horizontal uniformly distributed load on the arch upon the substitution of W for P.

CHART 1. VALUES OF ELASTIC PARAMETER α, for either end of symmetrical members with straight haunches



$$v = \frac{l_h}{l} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$$

For explanatory notes, see p. 417.
Values of parameter α are determined by the equation

$$\alpha = \frac{12}{l^3} \int \frac{I_0}{I} (l - x)^2 \, dx$$

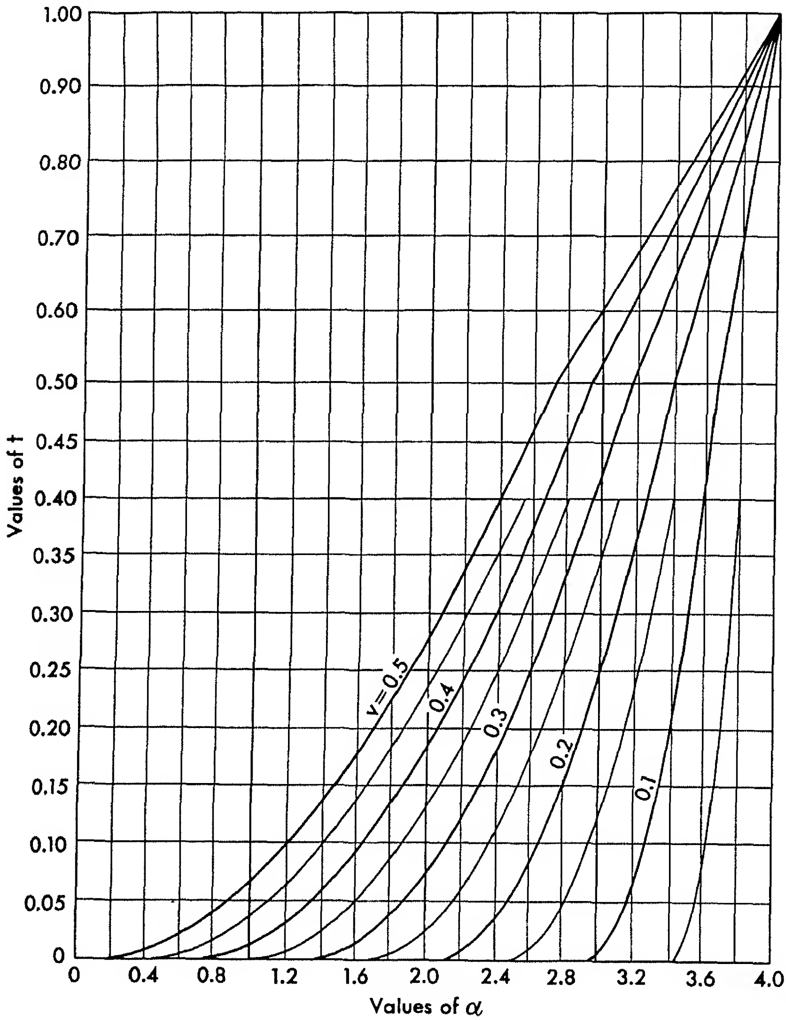
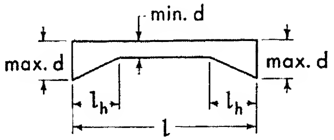


CHART 2. VALUES OF ELASTIC PARAMETER β, for either end of symmetrical members with straight haunches

$$v = \frac{l_h}{l} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$$



For explanatory notes, see p. 417.
Values of parameter β are determined by the equation

$$\beta = \frac{12}{l^3}\int \frac{I_0}{I}(l - x)x\,dx$$

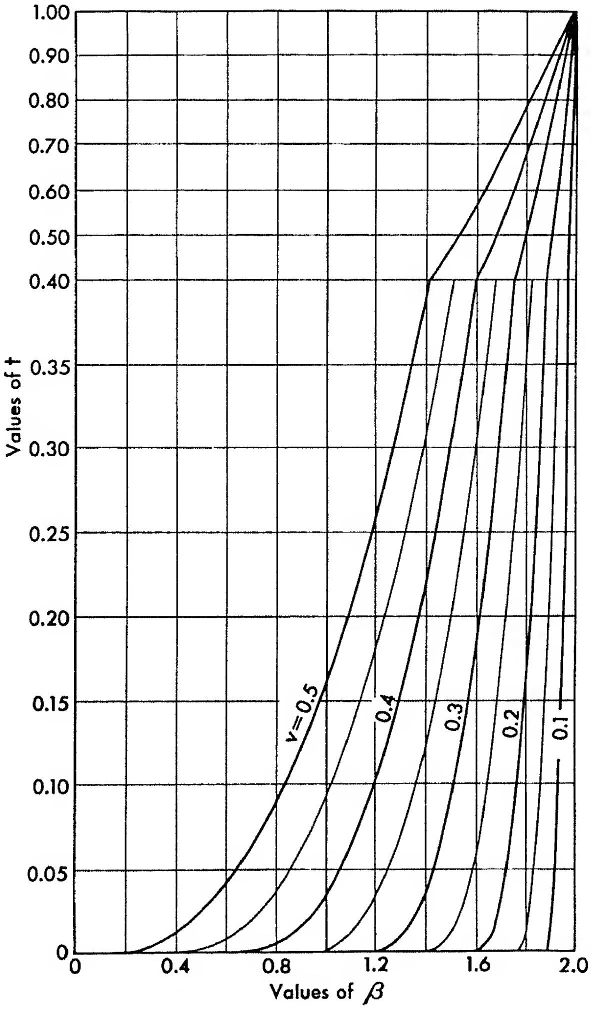
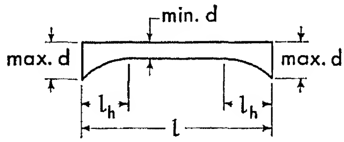


CHART 3. VALUES OF ELASTIC PARAMETER α, for either end of symmetrical members with parabolic haunches



$$v = \frac{l_h}{l} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$$

For explanatory notes, see p. 417.
Values of parameter α are determined by the equation

$$\alpha = \frac{12}{l^3}\int \frac{I_0}{I}(l - x)^2\,dx$$

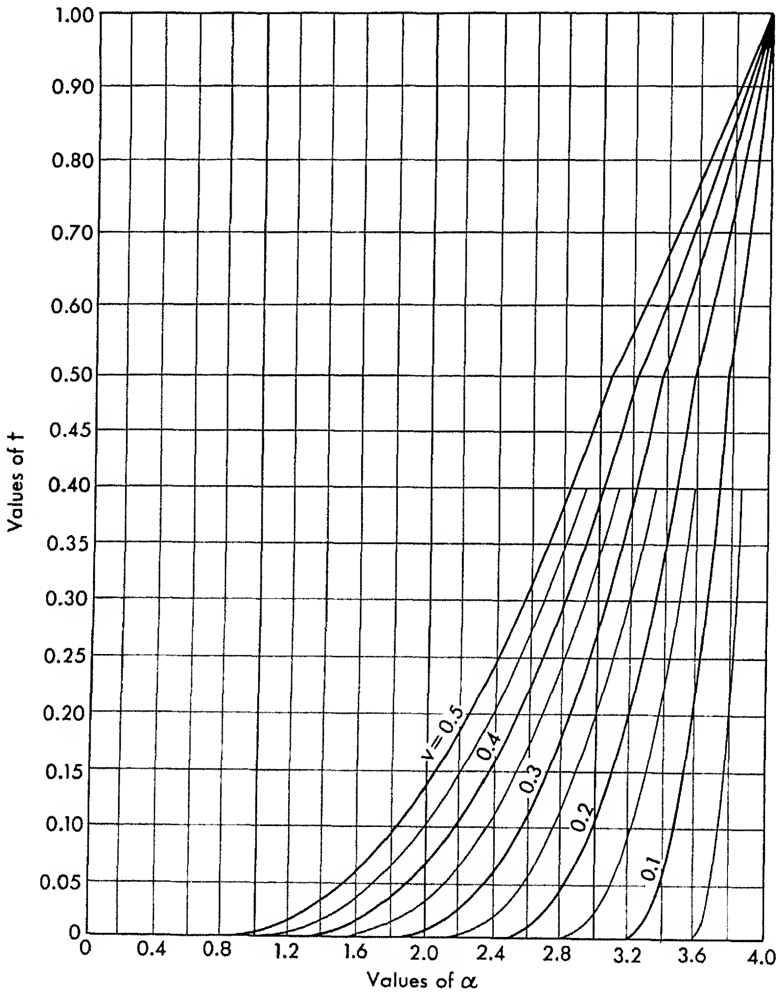
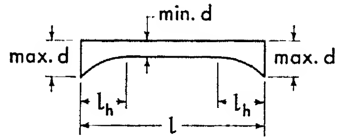


CHART 4. VALUES OF ELASTIC PARAMETER β, for either end of symmetrical members with parabolic haunches

$v = \frac{l_h}{l}$ $\qquad$ $t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$



For explanatory notes, see p. 417.
Values of parameter β are determined by the equation

$$\beta = \frac{12}{l^3} \int_{l}^{l_0} (l - x)x \, dx$$

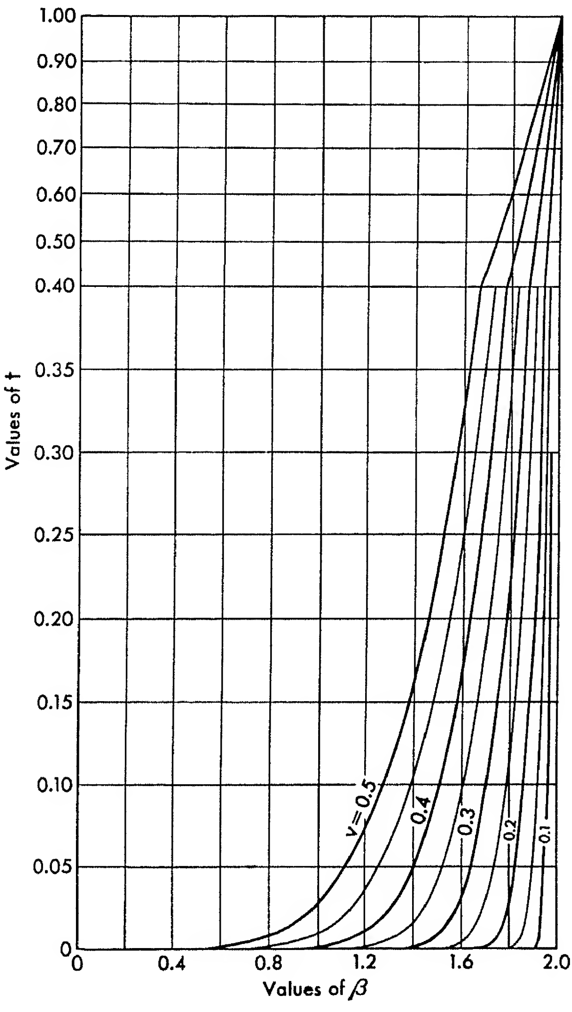
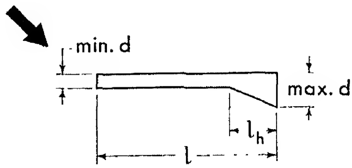


CHART 5. VALUES OF ELASTIC PARAMETER α, for the small end of members *with one straight haunch*

$$v = \frac{l_h}{l} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$$



For explanatory notes, see p. 417.
Values of parameter α are determined by the equation

$$\alpha = \frac{12}{l^3}\int \frac{I_0}{I}(l - x)^2\,dx$$

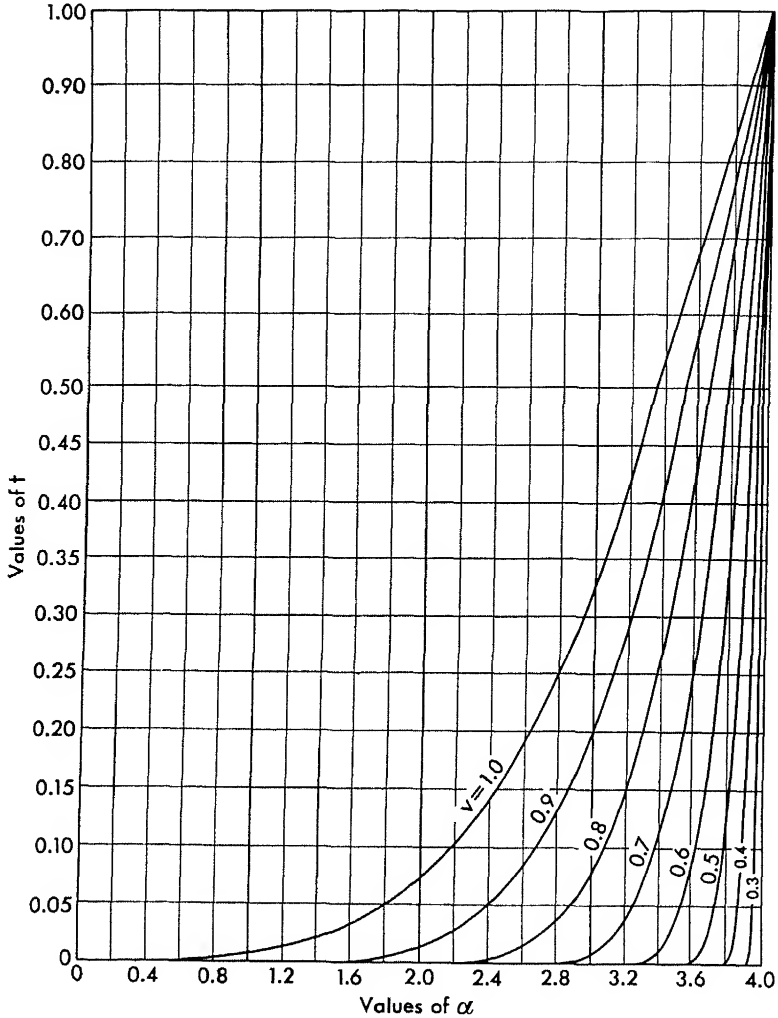
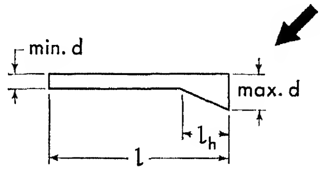


CHART 6. VALUES OF ELASTIC PARAMETER α, for the large end of members with one straight haunch



$v = \frac{l_h}{l}$ $\quad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$

For explanatory notes, see p. 417.

Values of parameter α are determined by the equation

$$\alpha = \frac{12}{l^3}\int \frac{I_0}{I} x^2\, dx$$

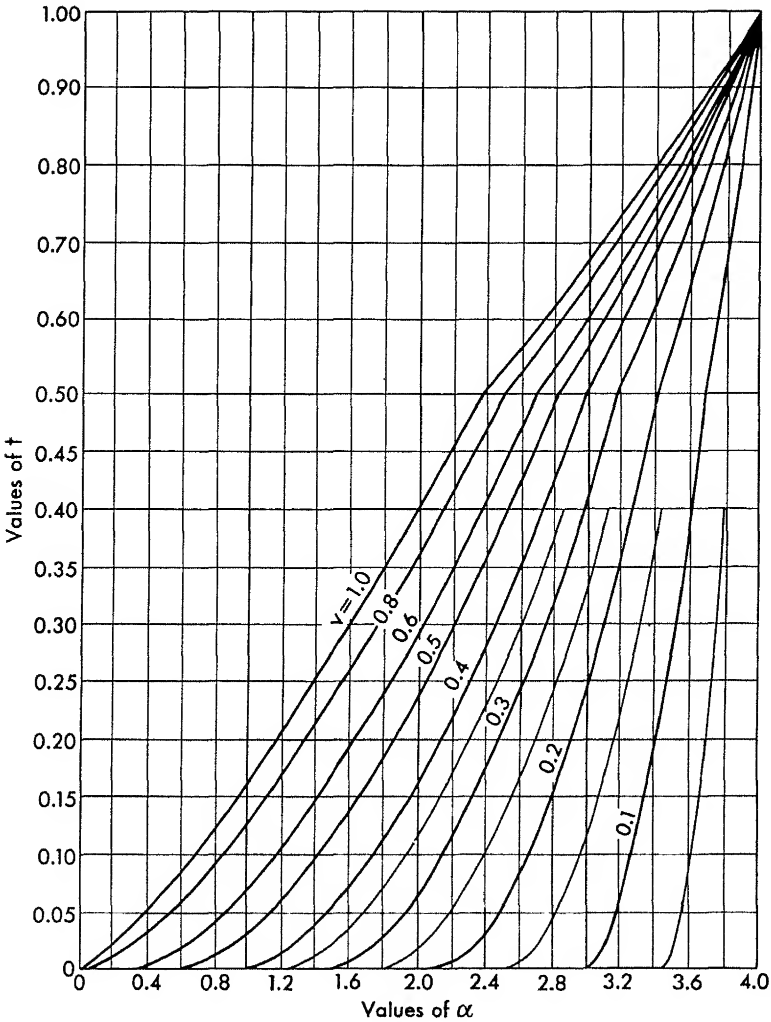
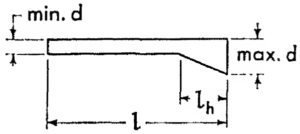


CHART 7. VALUES OF ELASTIC PARAMETER β, for either end of members with *one straight haunch*



$$v = \frac{l_h}{l} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$$

For explanatory notes, see p. 417.
Values of parameter β are determined by the equation

$$\beta = \frac{12}{l^3}\int \frac{I_0}{I}\,(l - x)x\,dx$$

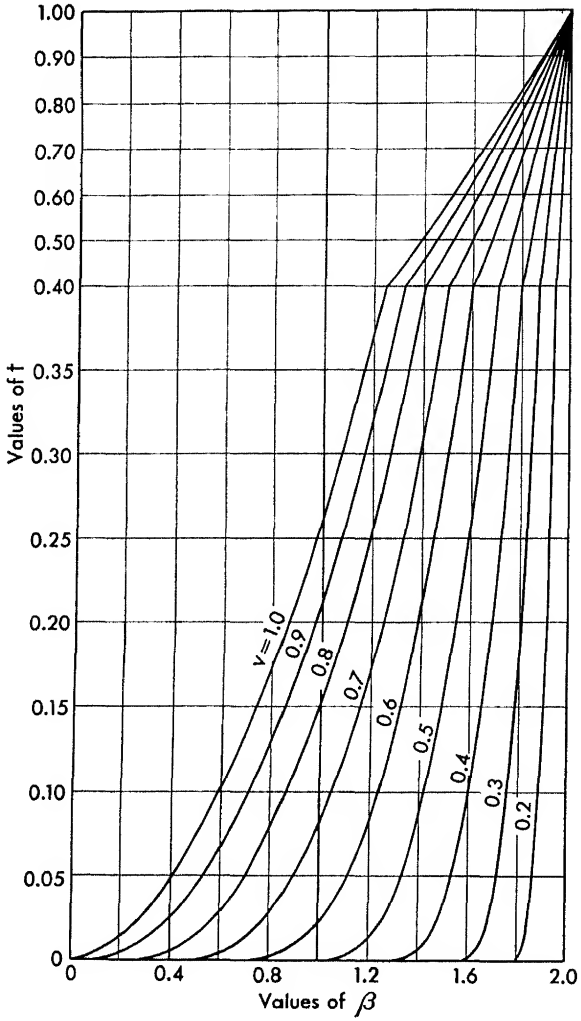
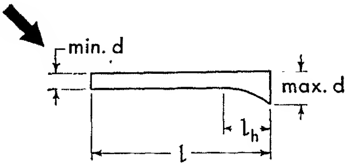


CHART 8. VALUES OF ELASTIC PARAMETER α, for the small end of members with one parabolic haunch

$$v = \frac{l_h}{l} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$$



For explanatory notes, see p. 417.
Values of parameter α are determined by the equation

$$\alpha = \frac{12}{l^3}\int \frac{I_0}{I}(l - x)^2\,dx$$

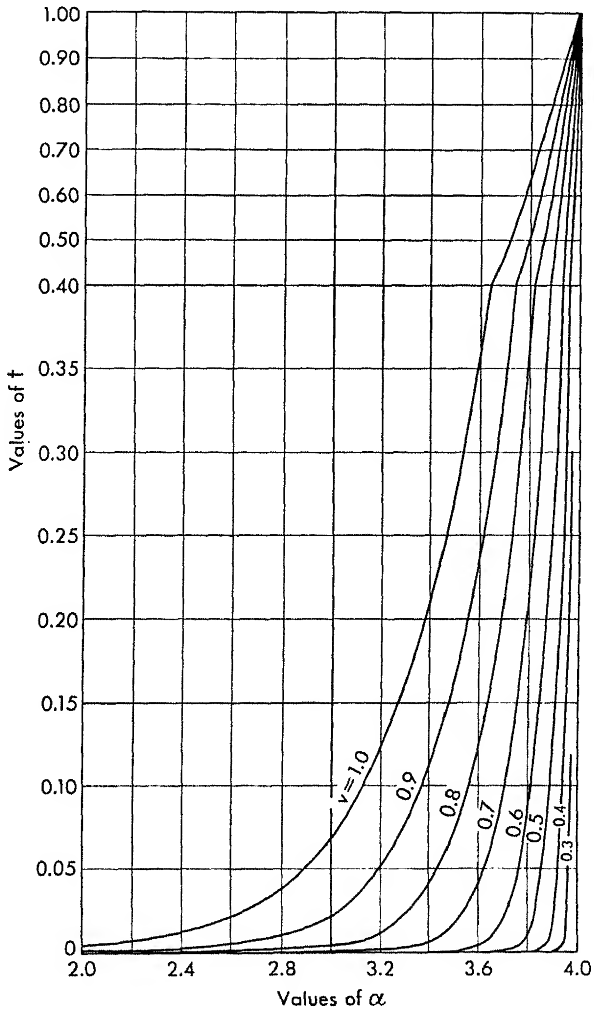
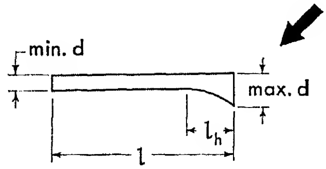


CHART 9. VALUES OF ELASTIC PARAMETER α, for the large end of members with one parabolic haunch



$$v = \frac{l_h}{l} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$$

For explanatory notes, see p. 417.
Values of parameter α are determined by the equation

$$\alpha = \frac{12}{l^3}\int \frac{I_0}{I}\, x^2\, dx$$

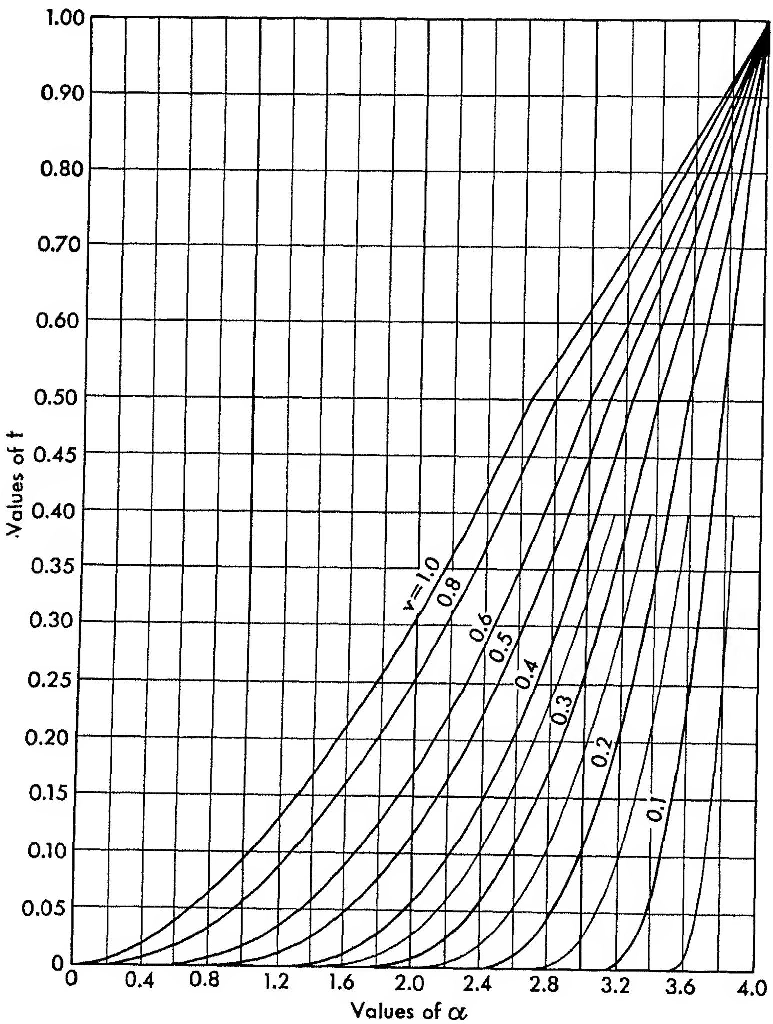
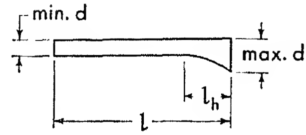


CHART 10. VALUES OF ELASTIC PARAMETER β, for either end of members with one parabolic haunch

$$v = \frac{l_h}{l} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$$



For explanatory notes, see p. 417.
Values of parameter β are determined by the equation

$$\beta = \frac{12}{l^3}\int^{l_0}\frac{I_0}{I}(l - x)x\,dx$$

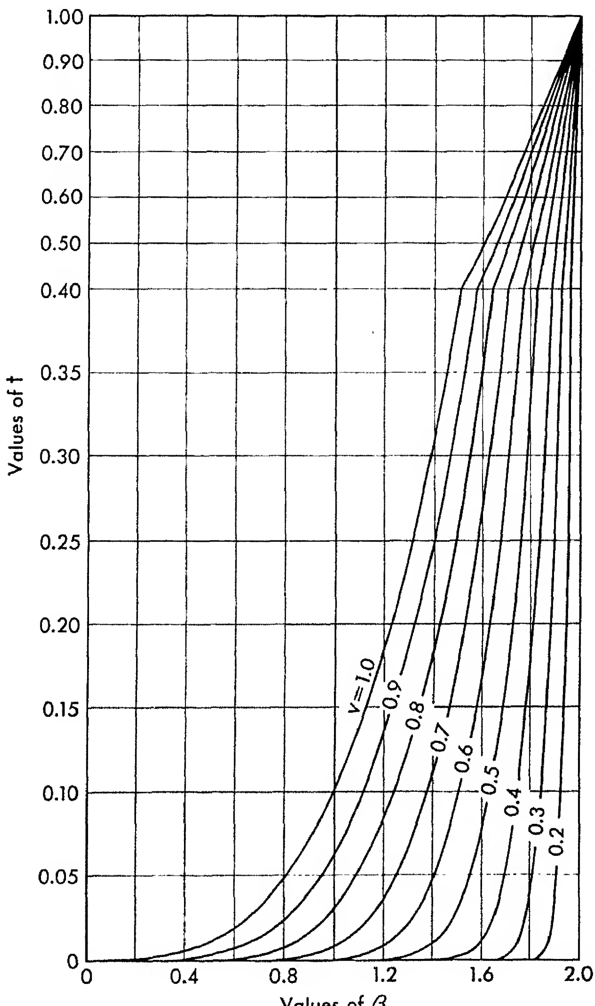
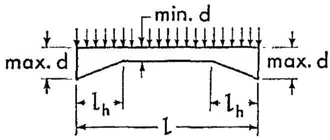


CHART 11. VALUES OF LOAD CONSTANT R, for either end of symmetrical members with straight haunches; uniformly distributed load



$$v = \frac{l_h}{l} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$$

For explanatory notes, see p. 417.
Values of constant R are determined by the equation

$$R = \frac{12}{Wl^3} \int \frac{I_0}{I} M(l - x)\, dx$$

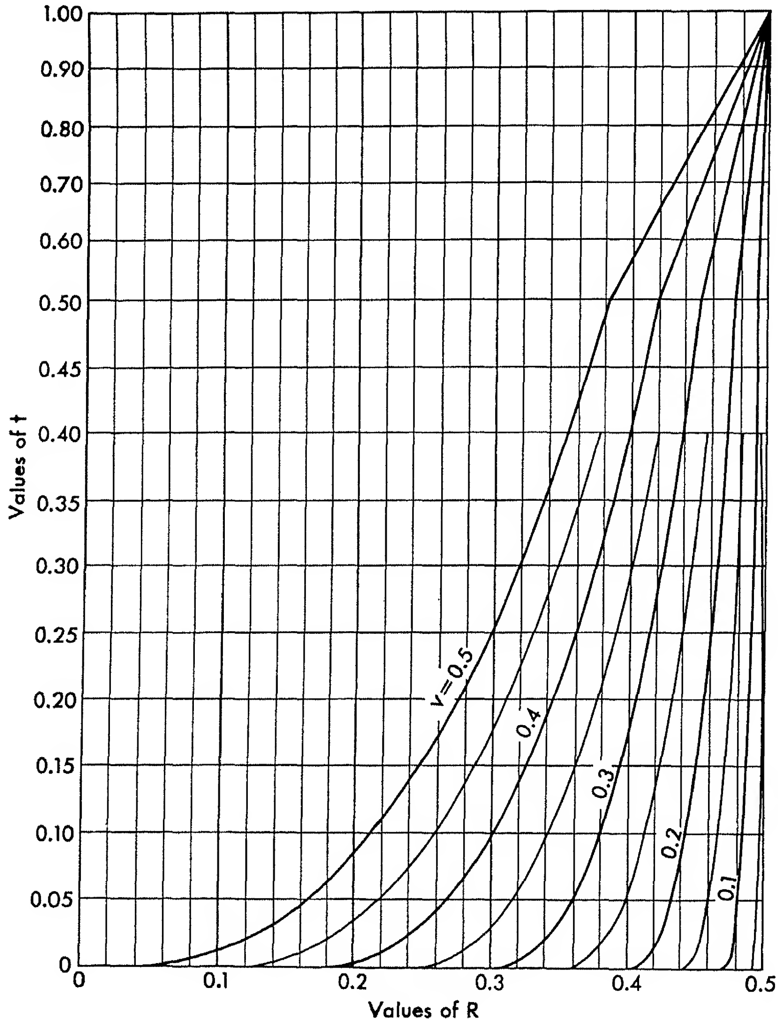
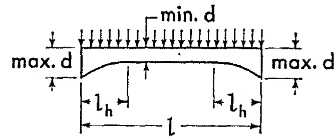


CHART 12. VALUES OF LOAD CONSTANT R, for either end of symmetrical members with parabolic haunches; uniformly distributed load

$$v = \frac{l_h}{l} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$$



For explanatory notes, see p. 417.
Values of constant R are determined by the equation

$$R = \frac{12}{Wl^3} \int \frac{I_c}{I} M(l - x)\, dx$$

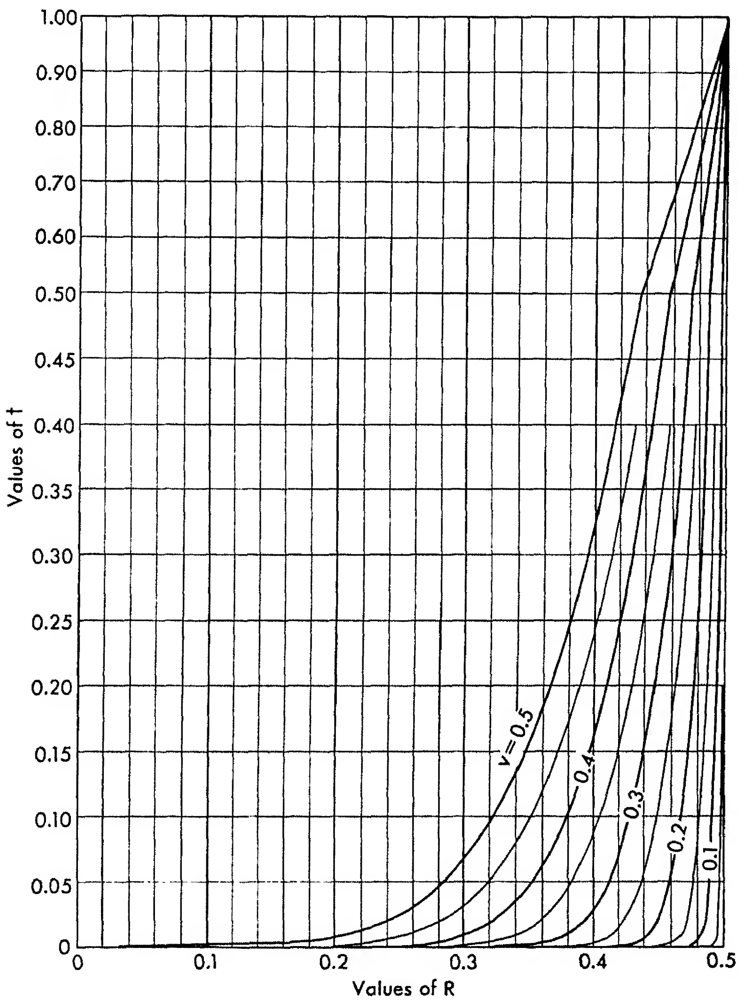
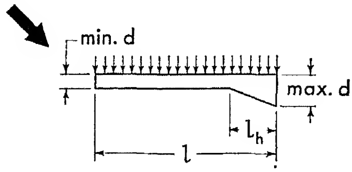


CHART 13. VALUES OF LOAD CONSTANT R, for the small end of members with one straight haunch; uniformly distributed load

$$v = \frac{l_h}{l} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$$



For explanatory notes, see p. 417.
Values of constant R are determined by the equation

$$R = \frac{12}{Wl^3}\int \frac{I_0}{I} M(l - x)\, dx$$

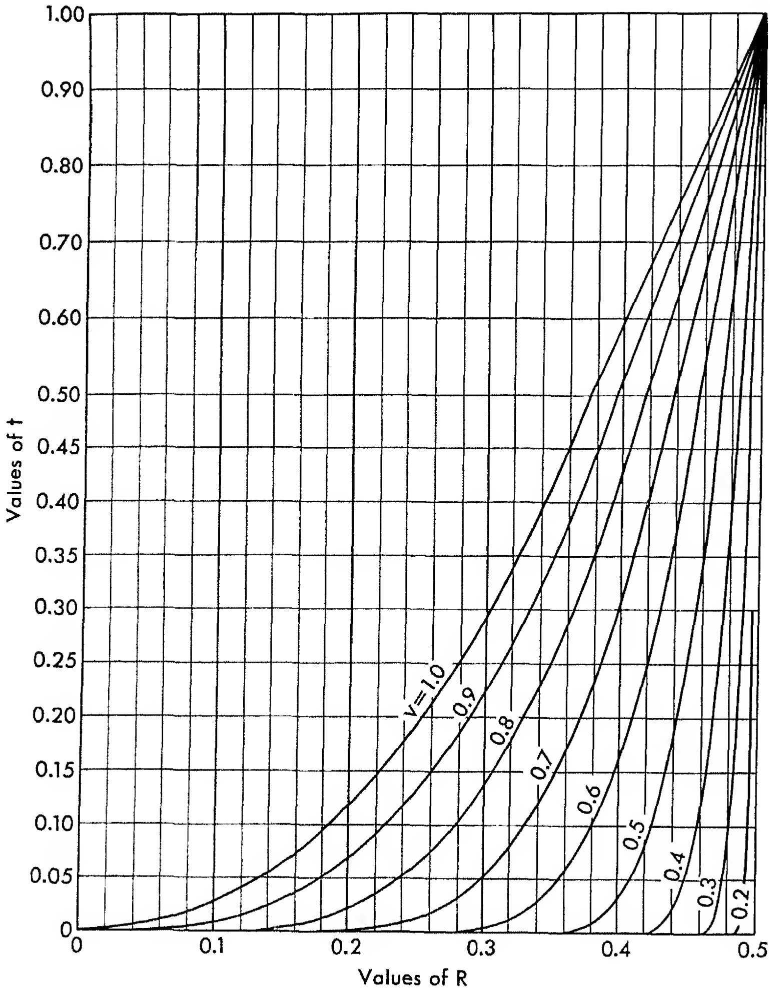
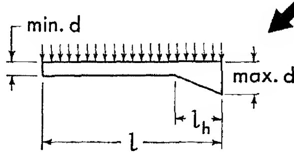


CHART 14. VALUES OF LOAD CONSTANT R, for the large end of members with one straight haunch; uniformly distributed load



$$v = \frac{l_h}{l} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$$

For explanatory notes, see p. 417.
Values of constant R are determined by the equation

$$R = \frac{12}{Wl^3}\int \frac{I_0}{I} Mx\,dx$$

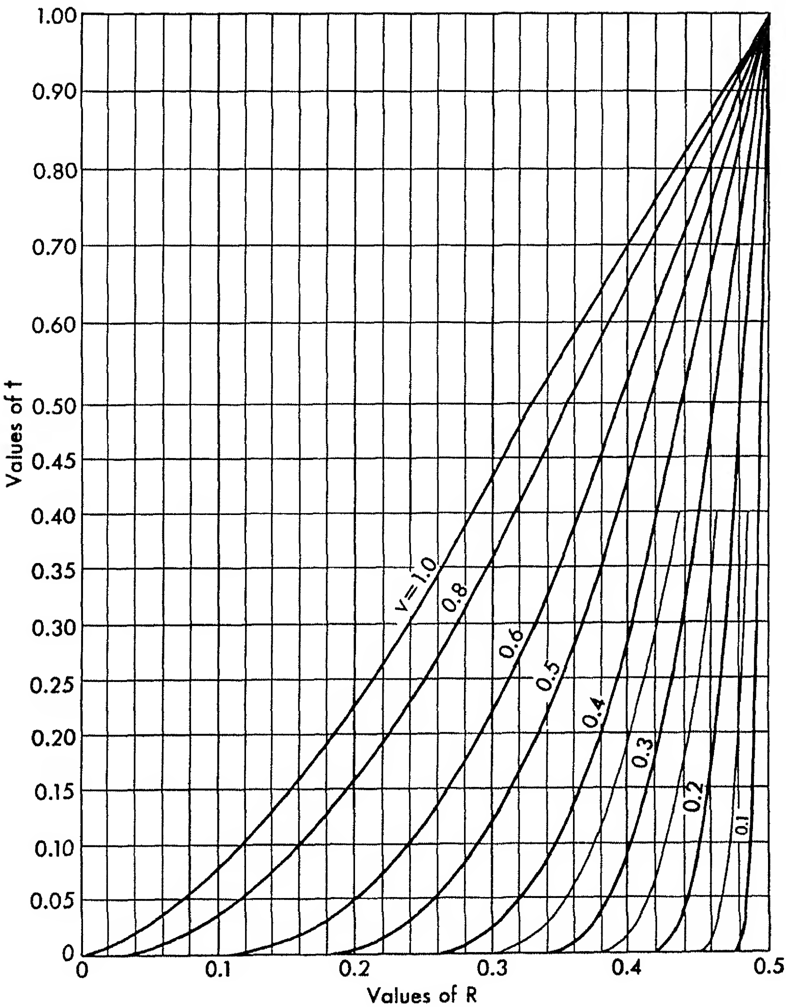
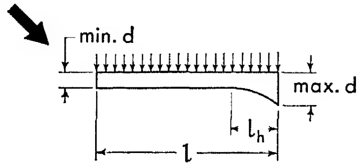


CHART 15. VALUES OF LOAD CONSTANT R, for the small end of members with one parabolic haunch; uniformly distributed load

$$v = \frac{l_h}{l} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$$



For explanatory notes, see p. 417.
Values of constant R are determined by the equation

$$R = \frac{12}{Wl^3}\int \frac{I_0}{I} M(l - x)\, dx$$

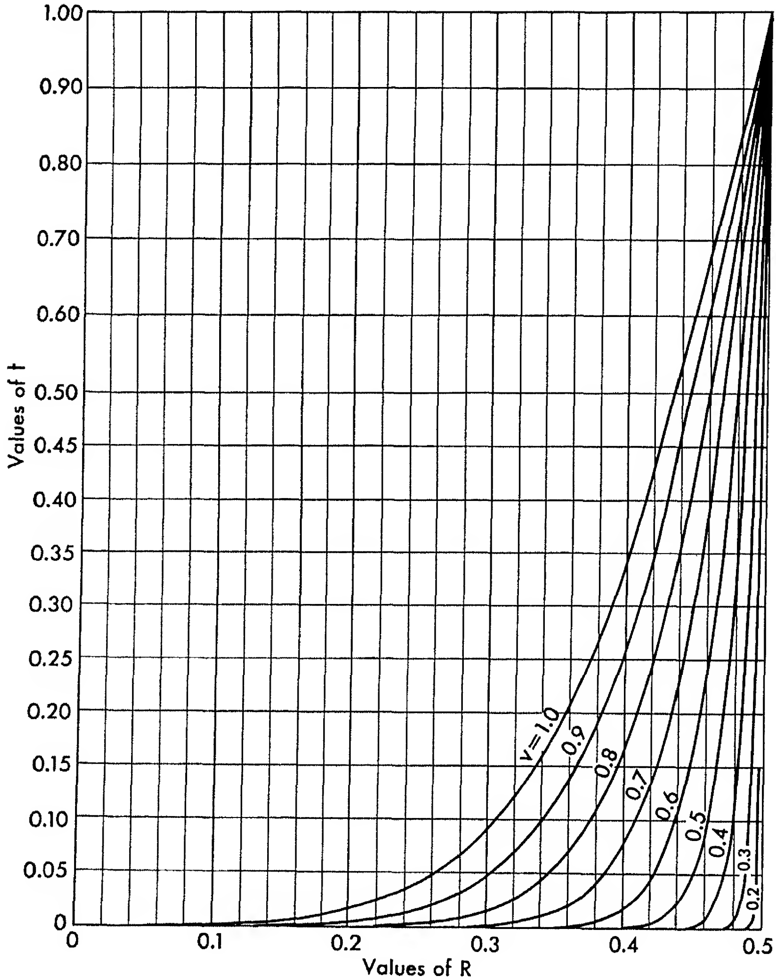
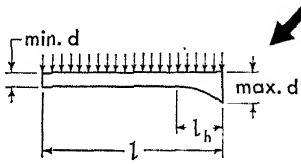


CHART 16. VALUES OF LOAD CONSTANT R, for the large end of members with one parabolic haunch; uniformly distributed load



$v = \frac{l_h}{l}$ $\qquad$ $t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$

For explanatory notes, see p. 417.
Values of constant R are determined by the equation

$$R = \frac{12}{Wl^3}\int \frac{I_0}{I} Mx\,dx$$

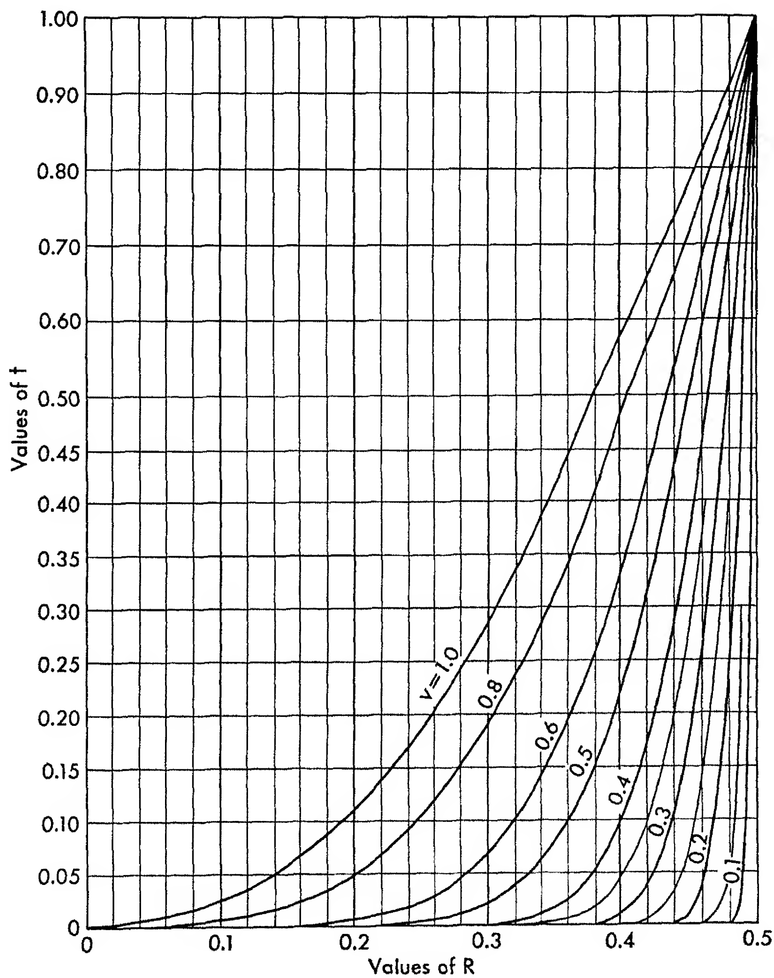
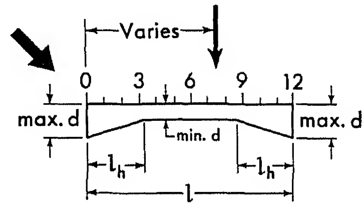


TABLE 1. VALUES OF LOAD CONSTANT R, for the left end of symmetrical members with straight haunches; concentrated load

$$v = \frac{l_h}{l} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$$

For explanatory notes, see p. 417.



Values of constant R are determined by the equation

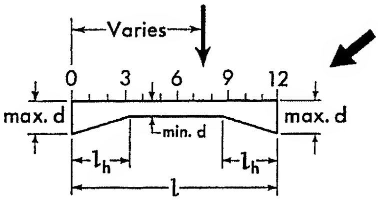
$$R = \frac{12}{Pl^3}\int \frac{I_0}{I} M(l - x)\, dx$$

v	t	Load at point										
		1	2	3	4	5	6	7	8	9	10	11
0.50	0.00											
	0.03	0.055	0.107	0.154	0.194	0.223	0.233	0.212	0.175	0.134	0.090	0.046
	0.05	0.068	0.133	0.190	0.235	0.268	0.276	0.252	0.211	0.162	0.109	0.054
	0.10	0.095	0.180	0.252	0.308	0.343	0.349	0.319	0.269	0.208	0.140	0.071
	0.20	0.132	0.245	0.335	0.401	0.437	0.438	0.402	0.341	0.265	0.181	0.091
	0.50	0.200	0.373	0.491	0.572	0.605	0.594	0.553	0.475	0.372	0.256	0.130
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166
0.40	0.00	0.050	0.102	0.152	0.203	0.252	0.270	0.246	0.197	0.148	0.098	0.049
	0.03	0.097	0.191	0.280	0.356	0.410	0.419	0.388	0.323	0.246	0.164	0.083
	0.05	0.109	0.214	0.307	0.388	0.440	0.448	0.413	0.346	0.264	0.178	0.089
	0.10	0.131	0.252	0.355	0.438	0.488	0.492	0.455	0.384	0.295	0.200	0.100
	0.20	0.161	0.304	0.420	0.505	0.552	0.551	0.509	0.431	0.335	0.227	0.114
	0.50	0.227	0.401	0.536	0.625	0.661	0.652	0.600	0.515	0.406	0.280	0.143
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166
0.35	0.00	0.077	0.155	0.232	0.308	0.370	0.382	0.354	0.290	0.218	0.146	0.072
	0.03	0.120	0.235	0.344	0.437	0.492	0.497	0.460	0.389	0.296	0.199	0.100
	0.05	0.131	0.254	0.370	0.462	0.514	0.518	0.479	0.406	0.311	0.209	0.104
	0.10	0.150	0.289	0.410	0.504	0.553	0.553	0.511	0.436	0.335	0.227	0.114
	0.20	0.178	0.335	0.462	0.556	0.601	0.598	0.551	0.472	0.366	0.248	0.125
	0.50	0.235	0.421	0.562	0.650	0.686	0.676	0.620	0.533	0.421	0.288	0.148
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166
0.30	0.00	0.104	0.211	0.316	0.418	0.475	0.480	0.449	0.377	0.284	0.190	0.095
	0.03	0.144	0.283	0.413	0.515	0.564	0.565	0.523	0.448	0.344	0.230	0.115
	0.05	0.154	0.300	0.433	0.534	0.581	0.580	0.536	0.458	0.355	0.239	0.119
	0.10	0.169	0.329	0.466	0.564	0.610	0.605	0.562	0.480	0.372	0.252	0.126
	0.20	0.193	0.366	0.508	0.604	0.646	0.640	0.590	0.505	0.395	0.269	0.136
	0.50	0.244	0.440	0.580	0.672	0.707	0.695	0.638	0.548	0.433	0.300	0.152
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166

TABLE 1 (Continued)

v	t	Load at point										
		1	2	3	4	5	6	7	8	9	10	11
0.25	0.00	0.134	0.271	0.407	0.512	0.562	0.563	0.522	0.448	0.344	0.229	0.114
	0.03	0.170	0.334	0.482	0.582	0.625	0.620	0.576	0.494	0.385	0.259	0.130
	0.05	0.178	0.347	0.497	0.595	0.638	0.632	0.584	0.503	0.392	0.265	0.132
	0.10	0.192	0.371	0.520	0.616	0.658	0.650	0.601	0.517	0.404	0.275	0.138
	0.20	0.214	0.400	0.551	0.644	0.683	0.672	0.620	0.533	0.419	0.286	0.144
	0.50	0.253	0.456	0.599	0.690	0.727	0.714	0.655	0.562	0.444	0.304	0.158
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166
0.20	0.00	0.167	0.337	0.492	0.593	0.636	0.630	0.584	0.502	0.394	0.264	0.132
	0.03	0.194	0.377	0.539	0.637	0.677	0.667	0.617	0.532	0.418	0.277	0.140
	0.05	0.199	0.388	0.548	0.646	0.684	0.674	0.623	0.536	0.422	0.281	0.142
	0.10	0.210	0.406	0.568	0.660	0.697	0.686	0.632	0.545	0.430	0.289	0.144
	0.20	0.226	0.424	0.584	0.678	0.719	0.701	0.646	0.556	0.438	0.298	0.148
	0.50	0.259	0.464	0.619	0.707	0.743	0.727	0.668	0.572	0.454	0.311	0.160
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166

TABLE 2. VALUES OF LOAD CONSTANT R, for the right end of symmetrical members with straight haunches; concentrated load



$$v = \frac{l_h}{l} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$$

For explanatory notes, see p. 417.

Values of constant R are determined by the equation

$$R = \frac{12}{Pl^3}\int \frac{I_0}{I} Mx\,dx$$

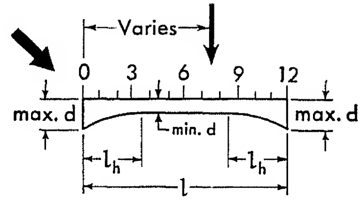
v	t	Load at point										
		1	2	3	4	5	6	7	8	9	10	11
0.50	0.00											
	0.03	0.046	0.090	0.134	0.175	0.212	0.233	0.223	0.194	0.154	0.107	0.055
	0.05	0.054	0.109	0.162	0.211	0.252	0.276	0.268	0.235	0.190	0.133	0.068
	0.10	0.071	0.140	0.208	0.269	0.319	0.349	0.343	0.308	0.252	0.180	0.095
	0.20	0.091	0.181	0.265	0.341	0.402	0.438	0.437	0.401	0.335	0.245	0.132
	0.50	0.130	0.256	0.372	0.475	0.553	0.594	0.605	0.572	0.491	0.373	0.200
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293
0.40	0.00	0.049	0.098	0.148	0.197	0.246	0.270	0.252	0.203	0.152	0.102	0.050
	0.03	0.083	0.164	0.246	0.323	0.388	0.419	0.410	0.356	0.280	0.191	0.097
	0.05	0.089	0.178	0.264	0.346	0.413	0.448	0.440	0.388	0.307	0.214	0.109
	0.10	0.100	0.200	0.295	0.384	0.455	0.492	0.488	0.438	0.355	0.252	0.131
	0.20	0.114	0.227	0.335	0.431	0.509	0.551	0.552	0.505	0.420	0.304	0.161
	0.50	0.143	0.280	0.406	0.515	0.600	0.652	0.661	0.625	0.536	0.401	0.227
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293
0.35	0.00	0.072	0.146	0.218	0.290	0.354	0.382	0.370	0.308	0.232	0.155	0.077
	0.03	0.100	0.199	0.296	0.389	0.460	0.497	0.492	0.437	0.344	0.235	0.120
	0.05	0.104	0.209	0.311	0.406	0.479	0.518	0.514	0.462	0.370	0.254	0.131
	0.10	0.114	0.227	0.335	0.436	0.511	0.553	0.553	0.504	0.410	0.289	0.150
	0.20	0.125	0.248	0.366	0.472	0.551	0.598	0.601	0.556	0.462	0.335	0.178
	0.50	0.148	0.288	0.421	0.533	0.620	0.676	0.686	0.650	0.562	0.421	0.235
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293
0.30	0.00	0.095	0.190	0.284	0.377	0.449	0.480	0.475	0.418	0.316	0.211	0.104
	0.03	0.115	0.230	0.344	0.448	0.523	0.565	0.564	0.515	0.413	0.283	0.144
	0.05	0.119	0.239	0.355	0.458	0.536	0.580	0.581	0.534	0.433	0.300	0.154
	0.10	0.126	0.252	0.372	0.480	0.562	0.605	0.610	0.564	0.466	0.329	0.169
	0.20	0.136	0.269	0.395	0.505	0.590	0.640	0.646	0.604	0.508	0.366	0.193
	0.50	0.152	0.300	0.433	0.548	0.638	0.695	0.707	0.672	0.580	0.440	0.244
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293

TABLE 2 (Continued)

v	t	Load at point										
		1	2	3	4	5	6	7	8	9	10	11
0.25	0.00	0.114	0.229	0.344	0.448	0.522	0.563	0.562	0.512	0.407	0.271	0.134
	0.03	0.130	0.259	0.385	0.494	0.576	0.620	0.625	0.582	0.482	0.334	0.170
	0.05	0.132	0.265	0.392	0.503	0.584	0.632	0.638	0.595	0.497	0.347	0.178
	0.10	0.138	0.275	0.404	0.517	0.601	0.650	0.658	0.616	0.520	0.371	0.192
	0.20	0.144	0.286	0.419	0.533	0.620	0.672	0.683	0.644	0.551	0.400	0.214
	0.50	0.158	0.304	0.444	0.562	0.655	0.714	0.727	0.690	0.599	0.456	0.253
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293
0.20	0.00	0.132	0.264	0.394	0.502	0.584	0.630	0.636	0.593	0.492	0.337	0.167
	0.03	0.140	0.277	0.418	0.532	0.617	0.667	0.677	0.637	0.539	0.377	0.194
	0.05	0.142	0.281	0.422	0.536	0.623	0.674	0.684	0.646	0.548	0.388	0.199
	0.10	0.144	0.289	0.430	0.545	0.632	0.686	0.697	0.660	0.568	0.406	0.210
	0.20	0.148	0.298	0.438	0.556	0.646	0.701	0.719	0.678	0.584	0.424	0.226
	0.50	0.160	0.311	0.454	0.572	0.668	0.727	0.743	0.707	0.619	0.464	0.259
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293

TABLE 3. VALUES OF LOAD CONSTANT R, for the left end of symmetrical members with parabolic haunches; concentrated load

$$v = \frac{l_h}{l} \qquad t = \left(\frac{\text{min } d}{\text{max } d}\right)^3$$



For explanatory notes, see p. 417.

Values of constant R are determined by the equation

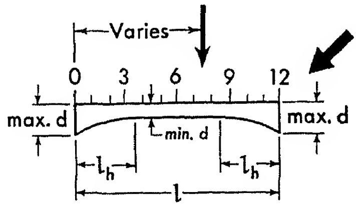
$$R = \frac{12}{Pl^3}\int \frac{I_0}{I} M(l - x)\, dx$$

v	t	Load at point										
		1	2	3	4	5	6	7	8	9	10	11
0.50	0.03	0.106	0.208	0.300	0.376	0.421	0.427	0.394	0.332	0.254	0.172	0.085
	0.05	0.121	0.234	0.335	0.414	0.460	0.463	0.427	0.361	0.278	0.187	0.094
	0.10	0.146	0.277	0.389	0.472	0.516	0.517	0.476	0.404	0.313	0.214	0.107
	0.20	0.176	0.330	0.455	0.539	0.581	0.577	0.532	0.454	0.354	0.241	0.121
	0.50	0.235	0.419	0.558	0.647	0.679	0.668	0.614	0.528	0.414	0.283	0.144
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166
0.40	0.03	0.142	0.277	0.400	0.494	0.542	0.545	0.504	0.428	0.331	0.222	0.112
	0.05	0.154	0.299	0.426	0.520	0.568	0.566	0.524	0.446	0.347	0.234	0.116
	0.10	0.174	0.334	0.468	0.560	0.606	0.600	0.556	0.475	0.368	0.252	0.126
	0.20	0.199	0.377	0.514	0.607	0.646	0.640	0.590	0.505	0.396	0.270	0.136
	0.50	0.245	0.439	0.583	0.677	0.712	0.697	0.642	0.550	0.432	0.298	0.150
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166
0.35	0.03	0.161	0.314	0.450	0.548	0.594	0.593	0.548	0.469	0.364	0.246	0.122
	0.05	0.172	0.332	0.473	0.569	0.613	0.611	0.564	0.484	0.376	0.254	0.127
	0.10	0.191	0.362	0.505	0.600	0.642	0.636	0.587	0.503	0.394	0.266	0.134
	0.20	0.212	0.397	0.544	0.636	0.676	0.665	0.613	0.526	0.413	0.283	0.143
	0.50	0.251	0.449	0.601	0.690	0.725	0.708	0.655	0.560	0.443	0.301	0.152
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166
0.30	0.03	0.185	0.353	0.502	0.598	0.641	0.635	0.586	0.504	0.394	0.266	0.133
	0.05	0.191	0.367	0.517	0.613	0.654	0.648	0.598	0.514	0.402	0.274	0.137
	0.10	0.205	0.392	0.542	0.637	0.677	0.666	0.614	0.529	0.416	0.283	0.142
	0.20	0.224	0.421	0.571	0.664	0.700	0.689	0.634	0.546	0.430	0.294	0.148
	0.50	0.257	0.460	0.616	0.703	0.737	0.719	0.664	0.571	0.452	0.307	0.156
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166

TABLE 3 (Continued)

v	t	Load at point										
		1	2	3	4	5	6	7	8	9	10	11
0.25	0.03	0.202	0.392	0.546	0.641	0.679	0.671	0.618	0.533	0.419	0.286	0.143
	0.05	0.210	0.404	0.558	0.653	0.689	0.678	0.626	0.540	0.425	0.290	0.145
	0.10	0.222	0.424	0.576	0.667	0.703	0.692	0.638	0.550	0.432	0.296	0.149
	0.20	0.240	0.445	0.596	0.686	0.720	0.707	0.652	0.562	0.443	0.302	0.154
	0.50	0.264	0.467	0.624	0.715	0.749	0.730	0.673	0.577	0.455	0.307	0.158
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166
0.20	0.03	0.224	0.427	0.582	0.676	0.712	0.698	0.644	0.554	0.438	0.298	0.150
	0.05	0.229	0.437	0.589	0.683	0.718	0.704	0.649	0.559	0.442	0.301	0.151
	0.10	0.241	0.449	0.601	0.694	0.727	0.713	0.656	0.565	0.446	0.306	0.154
	0.20	0.251	0.462	0.614	0.706	0.738	0.722	0.665	0.572	0.452	0.312	0.156
	0.50	0.270	0.479	0.634	0.725	0.757	0.734	0.679	0.584	0.460	0.317	0.161
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166

TABLE 4. VALUES OF LOAD CONSTANT R, for the right end of symmetrical members with parabolic haunches; concentrated load



$$v = \frac{l_h}{l} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$$

For explanatory notes, see p. 417.

Values of constant R are determined by the equation

$$R = \frac{12}{Pl^3}\int \frac{I_0}{I} Mx\, dx$$

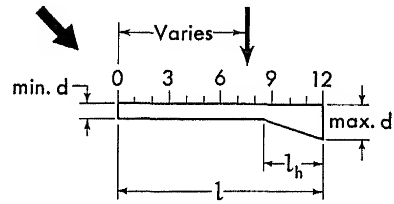
v	t	Load at point 1	2	3	4	5	6	7	8	9	10	11
0.50	0.03	0.085	0.172	0.254	0.332	0.394	0.427	0.421	0.376	0.300	0.208	0.106
	0.05	0.094	0.187	0.278	0.361	0.427	0.463	0.460	0.414	0.335	0.234	0.121
	0.10	0.107	0.214	0.313	0.404	0.476	0.517	0.516	0.472	0.389	0.277	0.146
	0.20	0.121	0.241	0.354	0.454	0.532	0.577	0.581	0.539	0.455	0.330	0.176
	0.50	0.144	0.283	0.414	0.528	0.614	0.668	0.679	0.647	0.558	0.419	0.235
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293
0.40	0.03	0.112	0.222	0.331	0.428	0.504	0.545	0.542	0.494	0.400	0.277	0.142
	0.05	0.116	0.234	0.347	0.446	0.524	0.566	0.568	0.520	0.426	0.299	0.154
	0.10	0.126	0.252	0.368	0.475	0.556	0.600	0.606	0.560	0.468	0.334	0.174
	0.20	0.136	0.270	0.396	0.505	0.590	0.640	0.646	0.607	0.514	0.377	0.199
	0.50	0.150	0.298	0.432	0.550	0.642	0.697	0.712	0.677	0.583	0.439	0.245
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293
0.35	0.03	0.122	0.246	0.364	0.469	0.548	0.593	0.594	0.548	0.450	0.314	0.161
	0.05	0.127	0.254	0.376	0.484	0.564	0.611	0.613	0.569	0.473	0.332	0.172
	0.10	0.134	0.266	0.394	0.503	0.587	0.636	0.642	0.600	0.505	0.362	0.191
	0.20	0.143	0.283	0.413	0.526	0.613	0.665	0.676	0.636	0.544	0.397	0.212
	0.50	0.152	0.301	0.443	0.560	0.655	0.708	0.725	0.690	0.601	0.449	0.251
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293
0.30	0.03	0.133	0.266	0.394	0.504	0.586	0.635	0.641	0.598	0.502	0.353	0.185
	0.05	0.137	0.274	0.402	0.514	0.598	0.648	0.654	0.613	0.517	0.367	0.191
	0.10	0.142	0.283	0.416	0.529	0.614	0.666	0.677	0.637	0.542	0.392	0.205
	0.20	0.148	0.294	0.430	0.546	0.634	0.689	0.700	0.664	0.571	0.421	0.224
	0.50	0.156	0.307	0.452	0.571	0.664	0.719	0.737	0.703	0.616	0.460	0.257
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293

TABLE 4 (Continued)

v	t	Load at point										
		1	2	3	4	5	6	7	8	9	10	11
0.25	0.03	0.143	0.286	0.419	0.533	0.618	0.671	0.679	0.641	0.546	0.392	0.202
	0.05	0.145	0.290	0.425	0.540	0.626	0.678	0.689	0.653	0.558	0.404	0.210
	0.10	0.149	0.296	0.432	0.550	0.638	0.692	0.703	0.667	0.576	0.424	0.222
	0.20	0.154	0.302	0.443	0.562	0.652	0.707	0.720	0.686	0.596	0.445	0.240
	0.50	0.158	0.307	0.455	0.577	0.673	0.730	0.749	0.715	0.624	0.467	0.264
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293
0.20	0.03	0.150	0.298	0.438	0.554	0.644	0.698	0.712	0.676	0.582	0.427	0.224
	0.05	0.151	0.301	0.442	0.559	0.649	0.704	0.718	0.683	0.589	0.437	0.229
	0.10	0.154	0.306	0.446	0.565	0.656	0.713	0.727	0.694	0.601	0.449	0.241
	0.20	0.156	0.312	0.452	0.572	0.665	0.722	0.738	0.706	0.614	0.462	0.251
	0.50	0.161	0.317	0.460	0.584	0.679	0.734	0.757	0.725	0.634	0.479	0.270
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293

TABLE 5. VALUES OF LOAD CONSTANT R, for the small end of members with one straight haunch; concentrated load

$$v = \frac{l_h}{l} \qquad t = \left(\frac{\min d}{\max d}\right)^3$$



For explanatory notes, see p. 417.

Values of constant R are determined by the equation

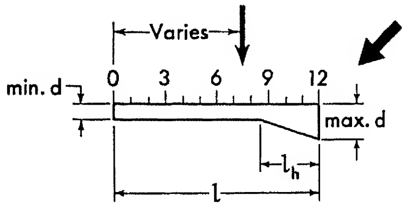
$$R = \frac{12}{Pl^3}\int_{i}^{l_b} M(l - x)\, dx$$

v	t	Load at point 1	2	3	4	5	6	7	8	9	10	11
1.00	0.00											
	0.03	0.095	0.143	0.163	0.166	0.158	0.143	0.125	0.101	0.078	0.053	0.028
	0.05	0.114	0.176	0.205	0.212	0.204	0.187	0.162	0.133	0.102	0.070	0.035
	0.10	0.145	0.230	0.276	0.290	0.284	0.264	0.232	0.192	0.148	0.100	0.053
	0.20	0.181	0.298	0.365	0.392	0.391	0.367	0.326	0.275	0.212	0.145	0.072
	0.50	0.239	0.406	0.512	0.566	0.578	0.554	0.502	0.426	0.332	0.228	0.116
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166
0.50	0.00	0.251	0.426	0.534	0.574	0.563	0.500	0.416	0.332	0.250	0.167	0.083
	0.03	0.268	0.458	0.578	0.638	0.642	0.596	0.515	0.419	0.317	0.211	0.106
	0.05	0.270	0.463	0.587	0.649	0.655	0.612	0.533	0.434	0.330	0.221	0.110
	0.10	0.274	0.472	0.600	0.665	0.676	0.637	0.560	0.460	0.352	0.236	0.119
	0.20	0.278	0.481	0.613	0.684	0.698	0.664	0.592	0.492	0.378	0.256	0.128
	0.50	0.286	0.496	0.634	0.714	0.736	0.709	0.650	0.551	0.428	0.293	0.149
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166
0.40	0.00	0.272	0.479	0.593	0.656	0.660	0.623	0.540	0.432	0.324	0.216	0.108
	0.03	0.280	0.484	0.618	0.689	0.704	0.672	0.598	0.491	0.373	0.248	0.125
	0.05	0.280	0.485	0.619	0.694	0.710	0.679	0.606	0.500	0.380	0.254	0.127
	0.10	0.283	0.491	0.626	0.702	0.721	0.692	0.622	0.516	0.394	0.266	0.133
	0.20	0.286	0.494	0.635	0.710	0.733	0.706	0.638	0.535	0.412	0.278	0.140
	0.50	0.289	0.503	0.643	0.727	0.752	0.730	0.665	0.566	0.443	0.304	0.156
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166
0.35	0.00	0.277	0.480	0.613	0.685	0.700	0.668	0.589	0.479	0.359	0.240	0.119
	0.03	0.284	0.492	0.630	0.707	0.726	0.697	0.629	0.523	0.396	0.265	0.133
	0.05	0.286	0.494	0.632	0.709	0.730	0.702	0.634	0.529	0.404	0.270	0.136
	0.10	0.287	0.497	0.636	0.715	0.738	0.710	0.643	0.540	0.414	0.280	0.140
	0.20	0.288	0.500	0.642	0.721	0.745	0.721	0.654	0.554	0.428	0.289	0.145
	0.50	0.290	0.505	0.647	0.732	0.758	0.737	0.672	0.575	0.451	0.310	0.158
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166

TABLE 5 (Continued)

v	t	Load at point										
		1	2	3	4	5	6	7	8	9	10	11
0.30	0.00	0.278	0.488	0.630	0.700	0.721	0.694	0.624	0.521	0.392	0.262	0.131
	0.03	0.287	0.498	0.641	0.720	0.743	0.716	0.650	0.550	0.420	0.281	0.140
	0.05	0.288	0.500	0.641	0.721	0.744	0.720	0.655	0.553	0.425	0.286	0.143
	0.10	0.289	0.502	0.644	0.725	0.750	0.725	0.660	0.560	0.433	0.293	0.146
	0.20	0.290	0.504	0.647	0.728	0.754	0.732	0.667	0.569	0.442	0.299	0.150
	0.50	0.292	0.506	0.649	0.734	0.762	0.742	0.678	0.581	0.458	0.316	0.160
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166
0.25	0.00	0.287	0.493	0.640	0.715	0.742	0.714	0.655	0.553	0.422	0.282	0.140
	0.03	0.290	0.503	0.646	0.728	0.755	0.732	0.667	0.568	0.439	0.295	0.148
	0.05	0.290	0.504	0.647	0.730	0.755	0.733	0.668	0.570	0.443	0.299	0.149
	0.10	0.291	0.504	0.649	0.731	0.758	0.737	0.673	0.574	0.448	0.304	0.151
	0.20	0.292	0.506	0.650	0.734	0.762	0.739	0.678	0.578	0.454	0.308	0.155
	0.50	0.292	0.508	0.652	0.738	0.766	0.745	0.683	0.586	0.461	0.322	0.163
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166
0.20	0.00	0.292	0.504	0.648	0.728	0.757	0.734	0.672	0.575	0.444	0.299	0.149
	0.03	0.292	0.506	0.649	0.734	0.762	0.739	0.678	0.580	0.454	0.308	0.158
	0.05	0.292	0.506	0.649	0.735	0.762	0.742	0.678	0.581	0.456	0.313	0.160
	0.10	0.292	0.507	0.650	0.736	0.763	0.743	0.680	0.583	0.458	0.317	0.161
	0.20	0.292	0.508	0.651	0.737	0.766	0.745	0.683	0.586	0.461	0.318	0.163
	0.50	0.293	0.509	0.653	0.739	0.768	0.748	0.685	0.589	0.464	0.322	0.164
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166
0.10	0.00	0.293	0.508	0.653	0.740	0.769	0.749	0.686	0.590	0.467	0.322	0.162
	0.03	0.293	0.509	0.653	0.740	0.769	0.749	0.688	0.592	0.467	0.322	0.162
	0.05	0.293	0.509	0.653	0.740	0.769	0.749	0.688	0.592	0.467	0.323	0.163
	0.10	0.293	0.509	0.653	0.740	0.769	0.749	0.688	0.592	0.468	0.323	0.163
	0.20	0.293	0.509	0.653	0.740	0.769	0.749	0.688	0.592	0.468	0.323	0.164
	0.50	0.293	0.509	0.654	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.165
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166

TABLE 6. VALUES OF LOAD CONSTANT R, for the large end of members with one straight haunch; concentrated load



$$v = \frac{l_h}{l} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$$

For explanatory notes, see p. 417.

Values of constant R are determined by the equation

$$R = \frac{12}{Pl^3}\int^{l_0} \frac{l_0}{I} Mx\,dx$$

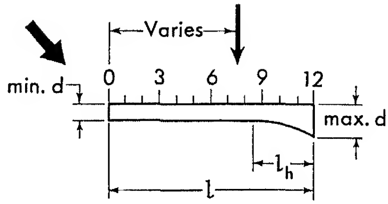
v	t	Load at point										
		1	2	3	4	5	6	7	8	9	10	11
1.00	0.00											
	0.03	0.025	0.047	0.062	0.073	0.078	0.079	0.074	0.066	0.054	0.040	0.022
	0.05	0.034	0.064	0.086	0.102	0.110	0.112	0.107	0.096	0.080	0.058	0.031
	0.10	0.049	0.094	0.131	0.157	0.173	0.179	0.173	0.158	0.133	0.097	0.053
	0.20	0.072	0.138	0.196	0.239	0.268	0.281	0.277	0.256	0.218	0.162	0.090
	0.50	0.118	0.230	0.330	0.410	0.469	0.504	0.508	0.480	0.418	0.319	0.182
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293
0.50	0.00	0.082	0.157	0.220	0.259	0.272	0.251	0.209	0.167	0.125	0.084	0.042
	0.03	0.104	0.203	0.288	0.350	0.385	0.386	0.352	0.296	0.230	0.157	0.080
	0.05	0.109	0.212	0.301	0.370	0.409	0.414	0.383	0.328	0.259	0.180	0.091
	0.10	0.119	0.228	0.325	0.401	0.449	0.462	0.437	0.383	0.310	0.218	0.114
	0.20	0.128	0.250	0.356	0.443	0.500	0.524	0.509	0.458	0.378	0.272	0.145
	0.50	0.148	0.286	0.412	0.516	0.594	0.636	0.638	0.598	0.514	0.385	0.216
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293
0.40	0.00	0.107	0.208	0.293	0.359	0.394	0.398	0.359	0.288	0.216	0.144	0.072
	0.03	0.125	0.241	0.343	0.425	0.479	0.498	0.476	0.408	0.319	0.218	0.110
	0.05	0.127	0.247	0.353	0.437	0.496	0.517	0.498	0.434	0.342	0.236	0.121
	0.10	0.132	0.258	0.370	0.460	0.522	0.551	0.539	0.479	0.384	0.271	0.142
	0.20	0.140	0.272	0.391	0.490	0.559	0.594	0.588	0.535	0.440	0.318	0.169
	0.50	0.152	0.298	0.430	0.540	0.624	0.672	0.678	0.636	0.546	0.408	0.229
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293
0.35	0.00	0.118	0.230	0.332	0.404	0.454	0.469	0.442	0.362	0.275	0.184	0.091
	0.03	0.133	0.258	0.370	0.458	0.521	0.550	0.535	0.473	0.371	0.254	0.130
	0.05	0.136	0.263	0.377	0.469	0.534	0.565	0.554	0.494	0.392	0.270	0.138
	0.10	0.140	0.271	0.391	0.487	0.557	0.593	0.586	0.530	0.430	0.301	0.157
	0.20	0.145	0.282	0.407	0.510	0.586	0.626	0.626	0.576	0.479	0.344	0.182
	0.50	0.155	0.304	0.438	0.551	0.637	0.688	0.697	0.656	0.566	0.422	0.236
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293

TABLE 6 (Continued)

v	t	Load at point										
		1	2	3	4	5	6	7	8	9	10	11
0.30	0.00	0.127	0.251	0.362	0.448	0.506	0.535	0.516	0.451	0.343	0.229	0.114
	0.03	0.140	0.272	0.394	0.492	0.560	0.596	0.592	0.538	0.431	0.295	0.149
	0.05	0.143	0.277	0.398	0.498	0.572	0.610	0.605	0.553	0.446	0.310	0.158
	0.10	0.146	0.286	0.408	0.512	0.590	0.631	0.630	0.581	0.479	0.336	0.174
	0.20	0.150	0.294	0.422	0.528	0.611	0.656	0.660	0.616	0.516	0.372	0.198
	0.50	0.157	0.308	0.445	0.562	0.649	0.703	0.715	0.678	0.587	0.440	0.241
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293
0.25	0.00	0.140	0.269	0.390	0.486	0.557	0.590	0.590	0.533	0.422	0.282	0.140
	0.03	0.148	0.288	0.415	0.520	0.598	0.641	0.641	0.594	0.491	0.344	0.173
	0.05	0.149	0.292	0.419	0.524	0.604	0.649	0.650	0.604	0.505	0.353	0.180
	0.10	0.151	0.296	0.426	0.535	0.618	0.664	0.668	0.626	0.527	0.374	0.196
	0.20	0.155	0.302	0.436	0.550	0.634	0.684	0.692	0.652	0.557	0.403	0.215
	0.50	0.160	0.313	0.451	0.570	0.661	0.716	0.731	0.696	0.606	0.458	0.252
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293
0.20	0.00	0.149	0.288	0.416	0.522	0.602	0.648	0.650	0.606	0.499	0.342	0.170
	0.03	0.154	0.300	0.432	0.544	0.628	0.677	0.685	0.643	0.547	0.390	0.202
	0.05	0.155	0.301	0.436	0.548	0.632	0.683	0.691	0.652	0.556	0.401	0.206
	0.10	0.156	0.305	0.440	0.554	0.641	0.694	0.703	0.665	0.571	0.416	0.217
	0.20	0.158	0.310	0.446	0.563	0.652	0.706	0.718	0.682	0.589	0.436	0.233
	0.50	0.162	0.317	0.457	0.578	0.671	0.728	0.744	0.712	0.623	0.474	0.262
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293
0.10	0.00	0.164	0.316	0.454	0.574	0.668	0.724	0.738	0.703	0.616	0.462	0.242
	0.03	0.164	0.318	0.458	0.580	0.672	0.731	0.748	0.715	0.628	0.476	0.263
	0.05	0.164	0.318	0.460	0.581	0.674	0.732	0.749	0.718	0.630	0.480	0.264
	0.10	0.165	0.319	0.461	0.582	0.676	0.734	0.752	0.720	0.634	0.484	0.268
	0.20	0.165	0.320	0.463	0.584	0.679	0.738	0.756	0.725	0.638	0.490	0.274
	0.50	0.166	0.322	0.466	0.589	0.684	0.744	0.763	0.733	0.648	0.499	0.288
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293

TABLE 7. VALUES OF LOAD CONSTANT R, for the small end of members with one parabolic haunch; concentrated load

$$v = \frac{l_h}{l} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$$



For explanatory notes, see p. 417.

Values of constant R are determined by the equation

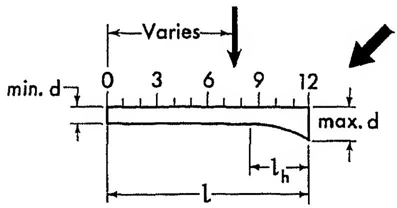
$$R = \frac{12}{Pl^3}\int \frac{I_0}{I} M(l - x)\, dx$$

v	t	Load at point 1	2	3	4	5	6	7	8	9	10	11
1.00	0.03	0.187	0.301	0.359	0.373	0.358	0.324	0.280	0.228	0.173	0.116	0.058
	0.05	0.202	0.329	0.396	0.416	0.406	0.371	0.323	0.264	0.202	0.136	0.067
	0.10	0.221	0.367	0.450	0.484	0.478	0.443	0.388	0.322	0.246	0.167	0.083
	0.20	0.242	0.407	0.509	0.554	0.556	0.523	0.466	0.389	0.300	0.204	0.102
	0.50	0.270	0.463	0.590	0.658	0.673	0.647	0.584	0.497	0.389	0.266	0.136
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166
0.50	0.03	0.277	0.479	0.611	0.682	0.696	0.665	0.592	0.490	0.372	0.250	0.125
	0.05	0.282	0.487	0.622	0.695	0.713	0.682	0.610	0.506	0.386	0.260	0.130
	0.10	0.284	0.491	0.629	0.704	0.724	0.696	0.625	0.523	0.404	0.272	0.137
	0.20	0.284	0.497	0.637	0.715	0.738	0.712	0.644	0.544	0.421	0.284	0.144
	0.50	0.290	0.504	0.646	0.730	0.756	0.733	0.668	0.571	0.446	0.306	0.156
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166
0.40	0.03	0.286	0.496	0.636	0.715	0.737	0.710	0.642	0.540	0.413	0.277	0.139
	0.05	0.287	0.498	0.640	0.718	0.739	0.714	0.648	0.545	0.419	0.283	0.142
	0.10	0.288	0.502	0.643	0.722	0.748	0.722	0.658	0.556	0.430	0.292	0.146
	0.20	0.289	0.504	0.646	0.727	0.752	0.731	0.666	0.565	0.440	0.300	0.151
	0.50	0.292	0.506	0.649	0.734	0.762	0.742	0.678	0.580	0.454	0.313	0.157
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166
0.35	0.03	0.289	0.502	0.644	0.722	0.746	0.722	0.658	0.557	0.430	0.289	0.144
	0.05	0.289	0.502	0.644	0.725	0.749	0.726	0.661	0.560	0.434	0.293	0.146
	0.10	0.289	0.503	0.648	0.728	0.754	0.731	0.667	0.568	0.442	0.300	0.150
	0.20	0.290	0.505	0.650	0.732	0.760	0.738	0.674	0.574	0.450	0.306	0.155
	0.50	0.292	0.508	0.651	0.737	0.764	0.744	0.682	0.583	0.461	0.316	0.163
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166

TABLE 7 (Continued)

v	t	Load at point										
		1	2	3	4	5	6	7	8	9	10	11
0.30	0.03	0.290	0.503	0.647	0.730	0.755	0.733	0.668	0.570	0.444	0.299	0.150
	0.05	0.291	0.505	0.649	0.732	0.757	0.736	0.671	0.572	0.446	0.302	0.151
	0.10	0.292	0.506	0.652	0.733	0.761	0.739	0.676	0.577	0.451	0.306	0.154
	0.20	0.292	0.508	0.653	0.737	0.763	0.742	0.679	0.581	0.456	0.312	0.157
	0.50	0.293	0.508	0.655	0.738	0.767	0.746	0.684	0.588	0.463	0.318	0.164
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166
0.25	0.03	0.292	0.506	0.652	0.734	0.763	0.739	0.677	0.581	0.454	0.308	0.155
	0.05	0.293	0.506	0.653	0.735	0.764	0.743	0.678	0.582	0.456	0.311	0.156
	0.10	0.293	0.507	0.653	0.737	0.766	0.744	0.682	0.584	0.460	0.313	0.157
	0.20	0.293	0.508	0.654	0.739	0.767	0.746	0.683	0.587	0.462	0.318	0.160
	0.50	0.293	0.509	0.655	0.739	0.768	0.748	0.686	0.590	0.466	0.319	0.166
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166
0.20	0.03	0.292	0.508	0.652	0.738	0.766	0.745	0.683	0.586	0.461	0.318	0.158
	0.05	0.292	0.508	0.652	0.738	0.766	0.745	0.684	0.587	0.462	0.318	0.161
	0.10	0.293	0.508	0.652	0.738	0.767	0.746	0.685	0.588	0.463	0.319	0.162
	0.20	0.293	0.509	0.653	0.739	0.768	0.748	0.686	0.589	0.466	0.323	0.162
	0.50	0.293	0.509	0.654	0.740	0.769	0.749	0.688	0.592	0.467	0.323	0.164
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166
0.10	0.03	0.293	0.509	0.653	0.740	0.769	0.749	0.688	0.592	0.468	0.323	0.162
	0.05	0.293	0.509	0.653	0.740	0.769	0.750	0.688	0.592	0.468	0.323	0.162
	0.10	0.293	0.509	0.653	0.740	0.769	0.750	0.689	0.592	0.468	0.324	0.163
	0.20	0.293	0.509	0.654	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.164
	0.50	0.293	0.509	0.654	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.165
	1.00	0.293	0.509	0.656	0.740	0.769	0.750	0.689	0.593	0.468	0.324	0.166

TABLE 8. VALUES OF LOAD CONSTANT R, *for the large end of members with one parabolic haunch; concentrated load*



$$v = \frac{l_h}{l} \qquad t = \left(\frac{\text{min d}}{\text{max d}}\right)^3$$

For explanatory notes, see p. 417.

Values of constant R are determined by the equation

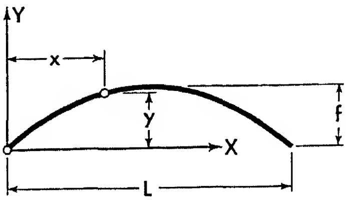
$$R = \frac{12}{Pl^3}\int \frac{I_0}{I} Mx\,dx$$

v	t	Load at point										
		1	2	3	4	5	6	7	8	9	10	11
1.00	0.03	0.058	0.108	0.146	0.172	0.184	0.181	0.168	0.146	0.116	0.082	0.043
	0.05	0.067	0.127	0.175	0.209	0.224	0.227	0.212	0.187	0.150	0.107	0.056
	0.10	0.083	0.158	0.222	0.268	0.295	0.301	0.289	0.259	0.214	0.155	0.083
	0.20	0.101	0.197	0.278	0.343	0.385	0.401	0.394	0.360	0.302	0.222	0.121
	0.50	0.133	0.262	0.373	0.468	0.536	0.574	0.576	0.542	0.467	0.352	0.194
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293
0.50	0.03	0.126	0.241	0.344	0.426	0.481	0.503	0.484	0.424	0.336	0.233	0.119
	0.05	0.130	0.252	0.360	0.448	0.508	0.533	0.517	0.461	0.370	0.257	0.133
	0.10	0.137	0.264	0.379	0.473	0.540	0.571	0.562	0.508	0.418	0.295	0.155
	0.20	0.144	0.278	0.402	0.504	0.577	0.617	0.613	0.566	0.474	0.344	0.184
	0.50	0.155	0.302	0.436	0.550	0.636	0.686	0.695	0.658	0.569	0.426	0.238
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293
0.40	0.03	0.139	0.269	0.386	0.482	0.551	0.584	0.578	0.521	0.419	0.293	0.149
	0.05	0.142	0.275	0.395	0.494	0.565	0.601	0.596	0.544	0.443	0.312	0.161
	0.10	0.146	0.284	0.408	0.511	0.587	0.630	0.628	0.581	0.481	0.343	0.179
	0.20	0.151	0.294	0.424	0.533	0.613	0.660	0.664	0.620	0.523	0.382	0.204
	0.50	0.158	0.310	0.448	0.564	0.653	0.707	0.720	0.679	0.589	0.443	0.247
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293
0.35	0.03	0.144	0.281	0.403	0.506	0.581	0.619	0.618	0.568	0.464	0.324	0.166
	0.05	0.146	0.287	0.410	0.514	0.592	0.635	0.635	0.584	0.484	0.341	0.175
	0.10	0.150	0.293	0.421	0.528	0.610	0.655	0.659	0.613	0.514	0.368	0.193
	0.20	0.155	0.301	0.433	0.546	0.630	0.678	0.688	0.644	0.550	0.403	0.216
	0.50	0.160	0.313	0.451	0.570	0.660	0.716	0.731	0.695	0.604	0.452	0.256
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293

TABLE 8 (Continued)

v	t	Load at point										
		1	2	3	4	5	6	7	8	9	10	11
0.30	0.03	0.150	0.290	0.420	0.527	0.607	0.650	0.654	0.610	0.509	0.359	0.182
	0.05	0.151	0.295	0.426	0.534	0.614	0.662	0.667	0.624	0.523	0.374	0.193
	0.10	0.154	0.301	0.433	0.544	0.629	0.678	0.685	0.644	0.547	0.397	0.208
	0.20	0.157	0.307	0.443	0.557	0.643	0.696	0.708	0.670	0.575	0.424	0.227
	0.50	0.161	0.316	0.456	0.576	0.667	0.725	0.740	0.707	0.618	0.463	0.262
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293
0.25	0.03	0.155	0.301	0.430	0.547	0.631	0.680	0.689	0.647	0.551	0.396	0.203
	0.05	0.156	0.304	0.437	0.551	0.637	0.688	0.697	0.658	0.563	0.407	0.212
	0.10	0.157	0.312	0.444	0.558	0.648	0.698	0.710	0.672	0.580	0.425	0.224
	0.20	0.160	0.316	0.450	0.568	0.658	0.712	0.725	0.691	0.599	0.446	0.240
	0.50	0.162	0.318	0.460	0.581	0.673	0.732	0.749	0.716	0.629	0.480	0.268
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293
0.20	0.03	0.158	0.308	0.445	0.562	0.650	0.703	0.715	0.679	0.587	0.434	0.221
	0.05	0.159	0.311	0.448	0.565	0.654	0.709	0.721	0.685	0.594	0.439	0.230
	0.10	0.160	0.313	0.451	0.570	0.660	0.716	0.731	0.696	0.606	0.452	0.238
	0.20	0.161	0.316	0.456	0.576	0.667	0.725	0.740	0.707	0.618	0.472	0.251
	0.50	0.164	0.320	0.463	0.584	0.679	0.738	0.756	0.725	0.638	0.485	0.271
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293
0.10	0.03	0.163	0.320	0.462	0.584	0.678	0.738	0.756	0.725	0.638	0.488	0.270
	0.05	0.163	0.321	0.463	0.586	0.679	0.739	0.757	0.726	0.640	0.491	0.275
	0.10	0.164	0.322	0.464	0.587	0.682	0.742	0.760	0.728	0.643	0.494	0.280
	0.20	0.165	0.322	0.466	0.588	0.683	0.744	0.762	0.732	0.647	0.498	0.283
	0.50	0.166	0.323	0.467	0.590	0.686	0.746	0.766	0.737	0.652	0.504	0.288
	1.00	0.166	0.324	0.468	0.593	0.689	0.750	0.769	0.740	0.656	0.509	0.293

TABLE 9a. COORDINATES OF PARABOLIC ARCH AXIS



Coordinates are determined by the equation

$$y = 4f\left(1 - \frac{x}{L}\right)\frac{x}{L}$$

x	y	x	y	x	y	x	y	x	y
0.01L	0.040f	0.21L	0.664f	0.41L	0.968f	0.61L	0.952f	0.81L	0.616f
0.02L	0.078f	0.22L	0.686f	0.42L	0.974f	0.62L	0.942f	0.82L	0.590f
0.03L	0.116f	0.23L	0.708f	0.43L	0.980f	0.63L	0.932f	0.83L	0.564f
0.04L	0.154f	0.24L	0.730f	0.44L	0.986f	0.64L	0.922f	0.84L	0.538f
0.05L	0.190f	0.25L	0.750f	0.45L	0.990f	0.65L	0.910f	0.85L	0.510f
0.06L	0.226f	0.26L	0.770f	0.46L	0.994f	0.66L	0.898f	0.86L	0.482f
0.07L	0.260f	0.27L	0.788f	0.47L	0.996f	0.67L	0.884f	0.87L	0.452f
0.08L	0.294f	0.28L	0.806f	0.48L	0.998f	0.68L	0.870f	0.88L	0.422f
0.09L	0.328f	0.29L	0.824f	0.49L	0.999f	0.69L	0.856f	0.89L	0.392f
0.10L	0.360f	0.30L	0.840f	0.50L	1.000f	0.70L	0.840f	0.90L	0.360f
0.11L	0.392f	0.31L	0.856f	0.51L	0.999f	0.71L	0.824f	0.91L	0.328f
0.12L	0.422f	0.32L	0.870f	0.52L	0.998f	0.72L	0.806f	0.92L	0.294f
0.13L	0.452f	0.33L	0.884f	0.53L	0.996f	0.73L	0.788f	0.93L	0.260f
0.14L	0.482f	0.34L	0.898f	0.54L	0.994f	0.74L	0.770f	0.94L	0.226f
0.15L	0.510f	0.35L	0.910f	0.55L	0.990f	0.75L	0.750f	0.95L	0.190f
0.16L	0.538f	0.36L	0.922f	0.56L	0.986f	0.76L	0.730f	0.96L	0.154f
0.17L	0.564f	0.37L	0.932f	0.57L	0.980f	0.77L	0.708f	0.97L	0.116f
0.18L	0.590f	0.38L	0.942f	0.58L	0.974f	0.78L	0.686f	0.98L	0.078f
0.19L	0.616f	0.39L	0.952f	0.59L	0.968f	0.79L	0.664f	0.99L	0.040f
0.20L	0.640f	0.40L	0.960f	0.60L	0.960f	0.80L	0.640f	1.00L	0.000f

TABLE 9b. LENGTH CONSTANTS OF PARABOLIC ARCHES AND RELATED DATA

For explanatory notes, see p. 417.

Length of parabolic arch axes are determined by the equation

$$s = \left\{\sqrt{1+\left(\frac{4f}{L}\right)^2} + \frac{L}{4f}\log_e\left[\frac{4f}{L}+\sqrt{1+\left(\frac{4f}{L}\right)^2}\right]\right\}\frac{L}{2}$$

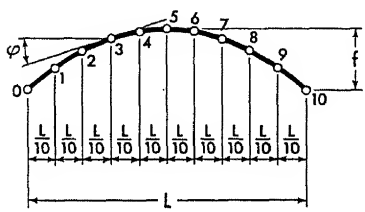
Arch ratio of f/L	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50	0.60
Length of parabolic arch axis (s)	1.000L	1.007L	1.026L	1.057L	1.098L	1.148L	1.204L	1.334L	1.479L	1.635L
Abscissa of a point, on parabolic arch axis, corresponding to the quarter axis length	0.250L	0.249L	0.246L	0.241L	0.234L	0.226L	0.219L	0.205L	0.195L	0.186L

TABLE 10. VALUES OF ANGLE OF INCLINATION φ, of parabolic arch axis for various ratios of f/L

For explanatory notes, see p. 417.

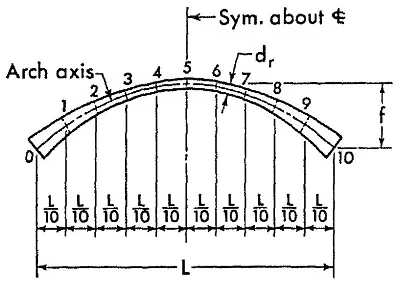
Angle of inclination of arch axis for points other than those tabulated may be found by the equation

$$\tan\varphi = \frac{4f}{L}\left(1-\frac{2x}{L}\right)$$



Arch ratio f/L	Sections					
	0 and 10	1 and 9	2 and 8	3 and 7	4 and 6	5
0.05	11°19′	9°05′	6°51′	4°34′	2°17′	0
0.10	21°48′	17°45′	13°30′	9°05′	4°34′	0
0.15	30°58′	25°38′	19°48′	13°30′	6°51′	0
0.20	38°40′	32°37′	25°38′	17°45′	9°05′	0
0.25	45°00′	38°40′	30°58′	21°48′	11°19′	0
0.30	50°12′	43°50′	35°45′	25°38′	13°30′	0
0.35	54°28′	48°14′	40°02′	29°15′	15°39′	0
0.40	58°00′	52°00′	43°50′	32°37′	17°45′	0
0.45	60°57′	55°13′	47°12′	35°45′	19°48′	0
0.50	63°26′	58°00′	50°12′	38°40′	21°48′	0
0.55	65°33′	60°24′	52°51′	42°21′	23°45′	0
0.60	67°23′	62°29′	55°13′	43°50′	25°38′	0

TABLE 11. PRIME ARCHES. RELATIVE VALUES OF ARCH THICKNESS d_r



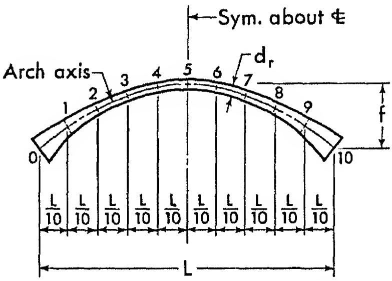
For explanatory notes, see p. 417. Tabular values are determined by Eq. (20-1).

Arch rise to span ratio f/L	Arch k value	Sections					
		0	1	2	3	4	5
0.00	5.00	0.585	0.620	0.665	0.728	0.822	1.000
	3.00	0.693	0.727	0.769	0.822	0.894	1.000
	2.00	0.794	0.822	0.855	0.894	0.941	1.000
	1.00	1.000	1.000	1.000	1.000	1.000	1.000
	0.80	1.077	1.060	1.043	1.028	1.014	1.000
	0.60	1.186	1.137	1.096	1.060	1.028	1.000
	0.40	1.357	1.244	1.161	1.096	1.043	1.000
	0.30	1.494	1.315	1.199	1.116	1.052	1.000
	0.20	1.710	1.406	1.244	1.137	1.060	1.000
	0.10	2.155	1.529	1.295	1.161	1.068	1.000
	0.06	2.556	1.592	1.319	1.170	1.072	1.000
	0.02	3.684	1.667	1.344	1.180	1.076	1.000
0.10	5.00	0.600	0.630	0.671	0.731	0.822	1.000
	3.00	0.711	0.739	0.775	0.826	0.894	1.000
	2.00	0.814	0.836	0.863	0.898	0.941	1.000
	1.00	1.025	1.016	1.009	1.004	1.001	1.000
	0.80	1.104	1.077	1.053	1.033	1.015	1.000
	0.60	1.215	1.156	1.106	1.064	1.029	1.000
	0.40	1.391	1.264	1.172	1.100	1.045	1.000
	0.30	1.531	1.336	1.210	1.120	1.053	1.000
	0.20	1.753	1.429	1.255	1.142	1.062	1.000
	0.10	2.206	1.554	1.307	1.166	1.070	1.000
	0.06	2.619	1.618	1.331	1.175	1.073	1.000
	0.02	3.776	1.694	1.356	1.185	1.077	1.000
0.20	5.00	0.635	0.657	0.689	0.739	0.826	1.000
	3.00	0.753	0.770	0.797	0.836	0.898	1.000
	2.00	0.862	0.871	0.885	0.909	0.946	1.000
	1.00	1.086	1.059	1.035	1.016	1.004	1.000
	0.80	1.170	1.122	1.080	1.045	1.018	1.000
	0.60	1.287	1.204	1.134	1.077	1.033	1.000
	0.40	1.474	1.317	1.201	1.114	1.048	1.000
	0.30	1.622	1.392	1.241	1.134	1.056	1.000
	0.20	1.875	1.488	1.287	1.156	1.064	1.000
	0.10	2.340	1.618	1.341	1.180	1.073	1.000
	0.06	2.773	1.685	1.365	1.190	1.076	1.000
	0.02	4.000	1.765	1.391	1.200	1.080	1.000

TABLE 11 (Continued)

Arch rise to span ratio f/L	Arch k value	Sections					
		0	1	2	3	4	5
0.30	5.00	0.678	0.691	0.713	0.753	0.829	1.000
	3.00	0.804	0.810	0.825	0.851	0.902	1.000
	2.00	0.920	0.920	0.920	0.926	0.950	1.000
	1.00	1.160	1.115	1.072	1.035	1.009	1.000
	0.80	1.250	1.182	1.119	1.064	1.023	1.000
	0.60	1.376	1.268	1.175	1.097	1.038	1.000
	0.40	1.575	1.387	1.244	1.134	1.053	1.000
	0.30	1.733	1.466	1.285	1.155	1.061	1.000
	0.20	1.984	1.567	1.334	1.177	1.070	1.000
	0.10	2.500	1.704	1.389	1.201	1.078	1.000
	0.06	2.964	1.775	1.414	1.212	1.082	1.000
	0.02	4.274	1.858	1.441	1.222	1.086	1.000
0.40	1.00	1.236	1.175	1.115	1.059	1.016	1.000
	0.80	1.331	1.246	1.163	1.089	1.030	1.000
	0.60	1.465	1.337	1.222	1.122	1.045	1.000
	0.40	1.677	1.462	1.294	1.160	1.061	1.000
	0.30	1.846	1.545	1.337	1.182	1.069	1.000
	0.20	2.112	1.652	1.387	1.204	1.077	1.000
	0.10	2.662	1.796	1.444	1.229	1.086	1.000
	0.06	3.156	1.871	1.470	1.239	1.090	1.000
	0.02	4.553	1.959	1.498	1.250	1.094	1.000
0.50	1.00	1.308	1.236	1.160	1.086	1.025	1.000
	0.80	1.409	1.310	1.211	1.116	1.039	1.000
	0.60	1.550	1.405	1.271	1.151	1.054	1.000
	0.40	1.775	1.537	1.346	1.190	1.070	1.000
	0.30	1.953	1.625	1.391	1.211	1.078	1.000
	0.20	2.236	1.737	1.443	1.235	1.086	1.000
	0.10	2.817	1.889	1.503	1.260	1.095	1.000
	0.06	3.340	1.967	1.530	1.271	1.098	1.000
	0.02	4.817	2.059	1.559	1.282	1.103	1.000
0.60	1.00	1.375	1.294	1.206	1.115	1.035	1.000
	0.80	1.481	1.371	1.258	1.147	1.050	1.000
	0.60	1.630	1.471	1.321	1.182	1.064	1.000
	0.40	1.866	1.609	1.399	1.222	1.080	1.000
	0.30	2.054	1.701	1.446	1.244	1.089	1.000
	0.20	2.352	1.818	1.500	1.268	1.097	1.000
	0.10	2.963	1.977	1.562	1.294	1.105	1.000
	0.06	3.513	2.059	1.590	1.305	1.110	1.000
	0.02	5.066	2.156	1.621	1.316	1.113	1.000

TABLE 12. QUADRATIC ARCHES. RELATIVE VALUES OF ARCH THICKNESS d_r



For explanatory notes, see p. 417. Values marked by asterisk should be used for analysis of arches of constant section.
Tabular values are determined by Eq. (20-2).

Arch rise to span ratio f/L	Arch k value	Sections						
		0	1	2	3	4	5	
0.00	5.00	0.585	0.655	0.743	0.848	0.952	1.000	
	3.00	0.693	0.760	0.834	0.912	0.975	1.000	
	1.00	1.000	1.000	1.000	1.000	1.000	1.000	*
	0.80	1.077	1.047	1.025	1.011	1.003	1.000	
	0.60	1.186	1.103	1.053	1.022	1.005	1.000	
	0.40	1.357	1.175	1.085	1.034	1.008	1.000	
	0.30	1.494	1.219	1.102	1.040	1.010	1.000	
	0.20	1.710	1.270	1.120	1.047	1.011	1.000	
	0.10	2.155	1.331	1.139	1.053	1.012	1.000	
	0.02	3.684	1.390	1.156	1.058	1.013	1.000	
0.10	5.00	0.600	0.666	0.750	0.852	0.952	1.000	
	3.00	0.712	0.772	0.842	0.917	0.975	1.000	
	2.00	0.814	0.862	0.911	0.958	0.987	1.000	
	1.08	0.999	1.000	1.000	1.000	1.000	1.000	*
	1.00	1.025	1.016	1.009	1.004	1.001	1.000	
	0.80	1.104	1.064	1.035	1.015	1.003	1.000	
	0.60	1.215	1.122	1.063	1.027	1.006	1.000	
	0.40	1.391	1.195	1.095	1.039	1.009	1.000	
	0.30	1.531	1.239	1.112	1.045	1.011	1.000	
	0.20	1.753	1.291	1.130	1.051	1.012	1.000	
	0.10	2.206	1.353	1.150	1.058	1.013	1.000	
	0.06	2.619	1.381	1.158	1.060	1.014	1.000	
	0.02	3.776	1.412	1.167	1.063	1.014	1.000	
0.20	5.00	0.635	0.694	0.770	0.863	0.956	1.000	
	3.00	0.753	0.804	0.864	0.926	0.980	1.000	
	2.00	0.862	0.898	0.936	0.968	0.991	1.000	
	1.28	1.000	1.000	1.000	1.000	1.000	1.000	*
	1.00	1.086	1.059	1.035	1.016	1.004	1.000	
	0.80	1.170	1.108	1.061	1.028	1.007	1.000	
	0.60	1.287	1.169	1.090	1.039	1.010	1.000	
	0.40	1.474	1.245	1.123	1.051	1.012	1.000	
	0.30	1.622	1.291	1.140	1.057	1.014	1.000	
	0.20	1.857	1.345	1.159	1.064	1.015	1.000	
	0.10	2.340	1.410	1.180	1.071	1.017	1.000	
	0.06	2.773	1.439	1.188	1.073	1.017	1.000	
	0.02	4.000	1.471	1.197	1.076	1.018	1.000	

TABLE 12 (Continued)

Arch rise to span ratio f/L	Arch k value	Sections						
		0	1	2	3	4	5	
0.30	5.00	0.678	0.731	0.797	0.878	0.956	1.000	
	3.00	0.804	0.846	0.895	0.944	0.980	1.000	
	1.56	1.000	1.006	1.008	1.005	1.002	1.000	*
	1.00	1.160	1.115	1.072	1.029	1.009	1.000	
	0.80	1.250	1.167	1.099	1.046	1.012	1.000	
	0.60	1.376	1.230	1.129	1.058	1.015	1.000	
	0.40	1.575	1.310	1.163	1.071	1.018	1.000	
	0.30	1.733	1.359	1.181	1.077	1.019	1.000	
	0.20	1.984	1.416	1.201	1.084	1.021	1.000	
	0.10	2.500	1.484	1.222	1.090	1.022	1.000	
	0.06	2.964	1.515	1.230	1.093	1.023	1.000	
	0.02	4.274	1.549	1.239	1.096	1.023	1.000	
0.40	5.00	0.722	0.770	0.828	0.898	0.967	1.000	
	3.00	0.856	0.893	0.930	0.965	0.991	1.000	
	1.90	1.000	1.012	1.017	1.013	1.005	1.000	*
	1.00	1.236	1.175	1.115	1.059	1.016	1.000	
	0.80	1.331	1.230	1.144	1.070	1.019	1.000	
	0.60	1.465	1.297	1.174	1.082	1.022	1.000	
	0.40	1.677	1.382	1.209	1.095	1.025	1.000	
	0.30	1.846	1.433	1.228	1.102	1.026	1.000	
	0.20	2.112	1.493	1.249	1.108	1.028	1.000	
	0.10	2.662	1.565	1.271	1.115	1.029	1.000	
	0.06	3.156	1.598	1.280	1.118	1.029	1.000	
	0.02	4.553	1.633	1.289	1.121	1.030	1.000	
0.50	5.00	0.765	0.809	0.862	0.921	0.976	1.000	
	3.00	0.907	0.938	0.969	0.990	1.000	1.000	
	2.40	0.975	1.000	1.012	1.015	1.005	1.000	*
	1.00	1.308	1.236	1.160	1.086	1.025	1.000	
	0.80	1.409	1.293	1.190	1.098	1.028	1.000	
	0.60	1.550	1.364	1.222	1.110	1.030	1.000	
	0.40	1.775	1.452	1.258	1.123	1.033	1.000	
	0.30	1.953	1.506	1.278	1.130	1.035	1.000	
	0.20	2.236	1.569	1.299	1.137	1.036	1.000	
	0.10	2.817	1.645	1.322	1.144	1.038	1.000	
	0.06	3.340	1.679	1.332	1.146	1.038	1.000	
	0.02	4.817	1.717	1.341	1.150	1.039	1.000	
0.60	5.00	0.804	0.847	0.895	0.945	0.984	1.000	
	3.00	0.952	0.983	1.006	1.016	1.009	1.000	
	2.80	0.976	1.002	1.020	1.025	1.011	1.000	*
	1.00	1.375	1.294	1.206	1.115	1.035	1.000	
	0.80	1.481	1.354	1.236	1.127	1.038	1.000	
	0.60	1.630	1.428	1.270	1.140	1.041	1.000	
	0.40	1.866	1.520	1.308	1.153	1.044	1.000	
	0.30	2.054	1.577	1.328	1.160	1.045	1.000	
	0.20	2.352	1.643	1.350	1.167	1.046	1.000	
	0.10	2.963	1.722	1.374	1.174	1.048	1.000	
	0.02	5.066	1.797	1.394	1.180	1.049	1.000	

TABLE 13. VALUES OF ELASTIC PARAMETERS α, β, γ, and δ

For explanatory notes, see p. 417.

Tabular values of parameters are determined by the equations

$$\alpha = \frac{12}{L^3}\int \frac{I_0}{I}\,\frac{(L-x)^2\,dx}{\cos\varphi} \qquad \beta = \frac{12}{L^3}\int \frac{I_0}{I}\,\frac{(L-x)x\,dx}{\cos\varphi}$$

$$\gamma = \frac{12}{Lf^2}\int \frac{I_0}{I}\,\frac{y^2\,dx}{\cos\varphi} \qquad \delta = \frac{12}{Lf^2}\int \frac{I_0}{I}\,\frac{y(f-y)\,dx}{\cos\varphi}$$

Arch k value	PRIME arches				QUADRATIC arches			
	α	β	γ	δ	α	β	γ	δ
5.00	13.000	5.000	14.40	5.60	10.40	3.60	10.060	4.343
4.00	10.750	4.250	12.40	4.60	8.80	3.20	9.143	3.657
3.00	8.500	3.500	10.40	3.60	7.20	2.80	8.229	2.971
2.00	6.250	2.750	8.40	2.60	5.60	2.40	7.314	2.286
1.80	5.800	2.600	8.00	2.40	5.28	2.32	7.131	2.149
1.60	5.350	2.450	7.60	2.20	4.96	2.24	6.949	2.011
1.40	4.900	2.300	7.20	2.00	4.64	2.16	6.766	1.874
1.20	4.450	2.150	6.80	1.80	4.32	2.08	6.583	1.737
1.00	4.000	2.000	6.40	1.60	4.00	2.00	6.400	1.600
0.95	3.887	1.963	6.30	1.55	3.92	1.98	6.354	1.566
0.90	3.775	1.925	6.20	1.50	3.84	1.96	6.309	1.531
0.85	3.662	1.888	6.10	1.45	3.76	1.94	6.263	1.497
0.80	3.550	1.850	6.00	1.40	3.68	1.92	6.217	1.463
0.75	3.437	1.813	5.90	1.35	3.60	1.90	6.171	1.429
0.70	3.325	1.775	5.80	1.30	3.52	1.88	6.126	1.394
0.65	3.212	1.738	5.70	1.25	3.44	1.86	6.080	1.360
0.60	3.100	1.700	5.60	1.20	3.36	1.84	6.034	1.326
0.55	2.987	1.663	5.50	1.15	3.28	1.82	5.989	1.291
0.50	2.875	1.625	5.40	1.10	3.20	1.80	5.943	1.257
0.45	2.762	1.588	5.30	1.05	3.12	1.78	5.897	1.223
0.40	2.650	1.550	5.20	1.00	3.04	1.76	5.851	1.189
0.35	2.537	1.513	5.10	0.95	2.96	1.74	5.806	1.154
0.30	2.425	1.475	5.00	0.90	2.88	1.72	5.760	1.120
0.25	2.312	1.438	4.90	0.85	2.80	1.70	5.714	1.086
0.20	2.200	1.400	4.80	0.80	2.72	1.68	5.669	1.051
0.15	2.087	1.363	4.70	0.75	2.64	1.66	5.623	1.017
0.10	1.975	1.325	4.60	0.70	2.56	1.64	5.577	0.983
0.05	1.862	1.288	4.50	0.65	2.48	1.62	5.531	0.949
0.00	1.750	1.250	4.40	0.60	2.40	1.60	5.486	0.914

TABLE 14a. VALUES OF CONSTANT τ, of constant section arches

For explanatory notes, see p. 417.
Tabular values of constant are determined by the equation

$$\tau = \frac{15}{16}\int \cos^2 \varphi \, ds$$

Arch ratio f/L	0.00	0.05	0.10	0.15	0.20	0.25	0.30
Constant τ	0.937	0.926	0.893	0.845	0.791	0.737	0.684

TABLE 14b. VALUES OF CONSTANT τ, of prime arches

For explanatory notes, see p. 417.
Tabular values of constant are determined by the equation

$$\tau = \frac{12\Theta}{F\gamma}\int \frac{d_0}{d}\cos^2 \varphi \, ds \qquad \text{(A-1)}$$

Arch k value	Arch ratio f/L						
	0.00	0.05	0.10	0.15	0.20	0.25	0.30
5.0	0.417	0.412	0.395	0.372	0.345	0.317	0.292
3.0	0.551	0.541	0.520	0.488	0.454	0.417	0.383
1.0	0.937	0.925	0.883	0.828	0.768	0.701	0.640
0.8	1.033	1.018	0.978	0.915	0.848	0.774	0.704
0.6	1.185	1.160	1.109	1.035	0.957	0.874	0.796
0.4	1.397	1.373	1.316	1.224	1.124	1.029	0.938
0.2	1.804	1.772	1.698	1.575	1.448	1.313	1.190
0.1	2.158	2.121	2.033	1.882	1.725	1.570	1.421
0.06	2.335	2.286	2.182	2.030	1.870	1.698	1.539
0.02	2.404	2.356	2.248	2.084	1.913	1.736	1.571

TABLE 14c. VALUES OF CONSTANT τ, of quadratic arches

For explanatory notes, see p. 417.
Tabular values of constant are determined by Eq. (A-1).

Arch k value	Arch ratio f/L						
	0.00	0.05	0.10	0.15	0.20	0.25	0.30
5.0	0.476	0.468	0.449	0.423	0.395	0.364	0.335
3.0	0.591	0.582	0.559	0.526	0.489	0.450	0.412
1.0	0.937	0.925	0.892	0.831	0.765	0.701	0.640
0.8	1.025	1.010	0.969	0.904	0.834	0.765	0.697
0.6	1.146	1.126	1.080	1.009	0.933	0.851	0.776
0.4	1.319	1.298	1.244	1.160	1.070	0.975	0.888
0.2	1.585	1.559	1.494	1.395	1.279	1.166	1.060
0.1	1.751	1.721	1.650	1.526	1.409	1.281	1.164
0.06	1.795	1.763	1.688	1.568	1.441	1.328	1.190
0.02	1.850	1.816	1.738	1.612	1.479	1.347	1.222

TABLE 15. PRIME ARCHES. VALUES OF LOAD CONSTANTS S, T, and U; vertical uniform and complementary parabolic loads

For explanatory notes, see p. 417.
Tabular values of constants are determined by the equations

$$S = \frac{12}{WL^2 f}\int \frac{I_0}{I}\,\frac{My\,dx}{\cos\varphi} \qquad T = \frac{12}{WL^3}\int \frac{I_0}{I}\,\frac{M(L-x)\,dx}{\cos\varphi}$$

$$U = \frac{12}{WL^3}\int \frac{I_0}{I}\,\frac{Mx\,dx}{\cos\varphi}$$

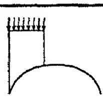
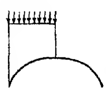
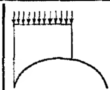
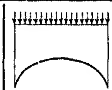
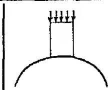
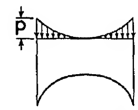
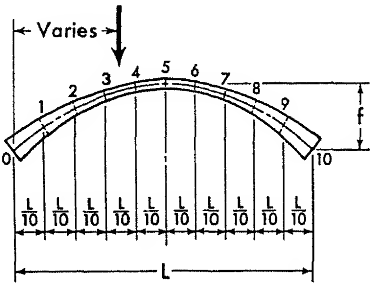
Arch k value	Constant	 Uniform load over 3/8 of the span	 Uniform load over 1/2 of the span	 Uniform load over 5/8 of the span	 Uniform load over the entire span	 Uniform load over the center quarter of the span	 Complementary parabolic load over the entire span $W = \frac{pL}{3}$
5.0	S	1.5300	1.8000	1.9620	1.8000	2.6110	1.0821
	T	1.3420	1.4630	1.4980	1.2500	1.7300	0.8000
	U	0.8373	1.0380	1.1950	1.2500	1.7300	0.8000
3.0	S	1.0950	1.3000	1.4230	1.3000	1.9150	0.7697
	T	0.9188	1.0130	1.0440	0.8750	1.2320	0.5500
	U	0.5930	0.7375	0.8488	0.8750	1.2320	0.5500
2.0	S	0.8777	1.0500	1.1530	1.0500	1.5670	0.6134
	T	0.7069	0.7875	0.8175	0.6875	0.9834	0.4250
	U	0.4708	0.5875	0.6758	0.6875	0.9834	0.4250
1.4	S	0.7473	0.9000	0.9916	0.9000	1.3580	0.5196
	T	0.5798	0.6525	0.6815	0.5750	0.8340	0.3500
	U	0.3975	0.4975	0.5721	0.5750	0.8340	0.3500
1.3	S	0.7255	0.8750	0.9647	0.8750	1.3230	0.5040
	T	0.5587	0.6300	0.6588	0.5563	0.8091	0.3375
	U	0.3853	0.4825	0.5548	0.5563	0.8091	0.3375
1.2	S	0.7038	0.8500	0.9377	0.8500	1.2890	0.4884
	T	0.5375	0.6075	0.6362	0.5375	0.7842	0.3250
	U	0.3731	0.4675	0.5375	0.5375	0.7842	0.3250
1.1	S	0.6821	0.8250	0.9108	0.8250	1.2540	0.4728
	T	0.5163	0.5850	0.6135	0.5188	0.7593	0.3125
	U	0.3609	0.4525	0.5202	0.5188	0.7593	0.3125
1.0	S	0.6604	0.8000	0.8838	0.8000	1.2190	0.4571
	T	0.4951	0.5625	0.5908	0.5000	0.7344	0.3000
	U	0.3486	0.4375	0.5029	0.5000	0.7344	0.3000

TABLE 15 (Continued)

Arch k value	Constant	Uniform load over 3/8 of the span	Uniform load over 1/2 of the span	Uniform load over 5/8 of the span	Uniform load over the entire span	Uniform load over the center quarter of the span	Complementary parabolic load over the entire span $W = \frac{pL}{3}$
0.9	S	0.6386	0.7750	0.8568	0.7750	1.1840	0.4415
	T	0.4739	0.5400	0.5682	0.4813	0.7095	0.2875
	U	0.3364	0.4225	0.4856	0.4813	0.7095	0.2875
0.8	S	0.6169	0.7500	0.8299	0.7500	1.1490	0.4259
	T	0.4528	0.5175	0.5455	0.4625	0.6846	0.2750
	U	0.3242	0.4075	0.4684	0.4625	0.6846	0.2750
0.7	S	0.5952	0.7250	0.8029	0.7250	1.1140	0.4103
	T	0.4316	0.4950	0.5228	0.4438	0.6597	0.2625
	U	0.3120	0.3925	0.4511	0.4438	0.6597	0.2625
0.6	S	0.5734	0.7000	0.7759	0.7000	1.0800	0.3946
	T	0.4104	0.4725	0.5001	0.4250	0.6348	0.2500
	U	0.2998	0.3775	0.4338	0.4250	0.6348	0.2500
0.5	S	0.5517	0.6750	0.7490	0.6750	1.0450	0.3790
	T	0.3892	0.4500	0.4775	0.4063	0.6099	0.2375
	U	0.2876	0.3625	0.4165	0.4063	0.6099	0.2375
0.4	S	0.5300	0.6500	0.7220	0.6500	1.0100	0.3634
	T	0.3680	0.4275	0.4548	0.3875	0.5850	0.2250
	U	0.2753	0.3475	0.3992	0.3875	0.5850	0.2250
0.3	S	0.5082	0.6250	0.6951	0.6250	0.9753	0.3478
	T	0.3468	0.4050	0.4321	0.3688	0.5601	0.2125
	U	0.2631	0.3325	0.3819	0.3688	0.5601	0.2125
0.2	S	0.4865	0.6000	0.6681	0.6000	0.9405	0.3321
	T	0.3257	0.3825	0.4095	0.3500	0.5352	0.2000
	U	0.2509	0.3175	0.3646	0.3500	0.5352	0.2000
0.1	S	0.4648	0.5750	0.6411	0.5750	0.9057	0.3165
	T	0.3045	0.3600	0.3868	0.3313	0.5103	0.1875
	U	0.2387	0.3025	0.3473	0.3313	0.5103	0.1875
0.0	S	0.4430	0.5500	0.6142	0.5500	0.8709	0.3009
	T	0.2833	0.3375	0.3641	0.3125	0.4854	0.1750
	U	0.2265	0.2875	0.3300	0.3125	0.4854	0.1750

TABLE 16. PRIME ARCHES. VALUES OF LOAD CONSTANTS S, T, and U; vertical *concentrated load*

For explanatory notes, see p. 417.
Tabular values of constants are determined by the equations



$$S = \frac{12}{PL^2 f}\int \frac{I_0}{I}\,\frac{My\,dx}{\cos\varphi}$$

$$T = \frac{12}{PL^3}\int \frac{I_0}{I}\,\frac{M(L-x)\,dx}{\cos\varphi}$$

$$U = \frac{12}{PL^3}\int \frac{I_0}{I}\,\frac{Mx\,dx}{\cos\varphi}$$

Arch k Value	Constant	Load at point										
		0	1	2	3	4	5	6	7	8	9	10
5.0	S	0	0.9650	1.7570	2.2950	2.5750	2.6500	2.5750	2.2950	1.7570	0.9650	0
	T	0	1.0250	1.5950	1.8370	1.8590	1.7500	1.5650	1.2950	0.9328	0.4908	0
	U	0	0.4908	0.9328	1.2950	1.5650	1.7500	1.8590	1.8370	1.5950	1.0250	0
3.0	S	0	0.6787	1.2500	1.6560	1.8830	1.9500	1.8830	1.6560	1.2500	0.6787	0
	T	0	0.6836	1.0860	1.2760	1.3140	1.2500	1.1120	0.9204	0.6584	0.3444	0
	U	0	0.3444	0.6584	0.9204	1.1120	1.2500	1.3140	1.2760	1.0860	0.6836	0
2.0	S	0	0.5356	0.9961	1.3360	1.5360	1.6000	1.5360	1.3360	0.9961	0.5356	0
	T	0	0.5128	0.8308	0.9948	1.0410	1.0000	0.8952	0.7332	0.5212	0.2712	0
	U	0	0.2712	0.5212	0.7332	0.8952	1.0000	1.0410	0.9948	0.8308	0.5128	0
1.4	S	0	0.4497	0.8439	1.1440	1.3290	1.3900	1.3290	1.1440	0.8439	0.4497	0
	T	0	0.4103	0.6779	0.8263	0.8771	0.8500	0.7613	0.6209	0.4389	0.2273	0
	U	0	0.2273	0.4389	0.6209	0.7613	0.8500	0.8771	0.8263	0.6779	0.4103	0
1.3	S	0	0.4353	0.8185	1.1120	1.2940	1.3550	1.2940	1.1120	0.8185	0.4353	0
	T	0	0.3932	0.6524	0.7982	0.8498	0.8250	0.7390	0.6022	0.4252	0.2200	0
	U	0	0.2200	0.4252	0.6022	0.7390	0.8250	0.8498	0.7982	0.6524	0.3932	0
1.2	S	0	0.4210	0.7931	1.0800	1.2600	1.3200	1.2600	1.0800	0.7931	0.4210	0
	T	0	0.3762	0.6270	0.7702	0.8226	0.8000	0.7166	0.5834	0.4114	0.2126	0
	U	0	0.2126	0.4114	0.5834	0.7166	0.8000	0.8226	0.7702	0.6270	0.3762	0
1.1	S	0	0.4067	0.7678	1.0480	1.2250	1.2850	1.2250	1.0480	0.7678	0.4067	0
	T	0	0.3591	0.6015	0.7421	0.7953	0.7750	0.6943	0.5647	0.3977	0.2053	0
	U	0	0.2053	0.3977	0.5647	0.6943	0.7750	0.7953	0.7421	0.6015	0.3591	0
1.0	S	0	0.3924	0.7424	1.0160	1.1900	1.2500	1.1900	1.0160	0.7424	0.3924	0
	T	0	0.3420	0.5760	0.7140	0.7680	0.7500	0.6720	0.5460	0.3840	0.1980	0
	U	0	0.1980	0.3840	0.5460	0.6720	0.7500	0.7680	0.7140	0.5760	0.3420	0

TABLE 16 (Continued)

Arch k Value	Constant	Load at point 0	1	2	3	4	5	6	7	8	9	10
0.9	S	0	0.3781	0.7170	0.9844	1.1560	1.2150	1.1560	0.9844	0.7170	0.3781	0
	T	0	0.3249	0.5505	0.6859	0.7407	0.7250	0.6497	0.5273	0.3703	0.1907	0
	U	0	0.1907	0.3703	0.5273	0.6497	0.7250	0.7407	0.6859	0.5505	0.3249	0
0.8	S	0	0.3638	0.6917	0.9525	1.1210	1.1800	1.1210	0.9525	0.6917	0.3638	0
	T	0	0.3078	0.5250	0.6578	0.7134	0.7000	0.6274	0.5086	0.3566	0.1834	0
	U	0	0.1834	0.3566	0.5086	0.6274	0.7000	0.7134	0.6578	0.5250	0.3078	0
0.7	S	0	0.3495	0.6663	0.9205	1.0870	1.1450	1.0870	0.9205	0.6663	0.3495	0
	T	0	0.2908	0.4996	0.6298	0.6862	0.6750	0.6050	0.4898	0.3428	0.1760	0
	U	0	0.1760	0.3428	0.4898	0.6050	0.6750	0.6862	0.6298	0.4996	0.2908	0
0.6	S	0	0.3351	0.6409	0.8886	1.0520	1.1100	1.0520	0.8886	0.6409	0.3351	0
	T	0	0.2737	0.4741	0.6017	0.6589	0.6500	0.5827	0.4711	0.3291	0.1687	0
	U	0	0.1687	0.3291	0.4711	0.5827	0.6500	0.6589	0.6017	0.4741	0.2737	0
0.5	S	0	0.3208	0.6156	0.8566	1.0170	1.0750	1.0170	0.8566	0.6156	0.3208	0
	T	0	0.2566	0.4486	0.5736	0.6316	0.6250	0.5604	0.4524	0.3154	0.1614	0
	U	0	0.1614	0.3154	0.4524	0.5604	0.6250	0.6316	0.5736	0.4486	0.2566	0
0.4	S	0	0.3065	0.5902	0.8247	0.9828	1.0400	0.9828	0.8247	0.5902	0.3065	0
	T	0	0.2395	0.4231	0.5455	0.6043	0.6000	0.5381	0.4337	0.3017	0.1541	0
	U	0	0.1541	0.3017	0.4337	0.5381	0.6000	0.6043	0.5455	0.4231	0.2395	0
0.3	S	0	0.2922	0.5648	0.7927	0.9482	1.0050	0.9482	0.7927	0.5648	0.2922	0
	T	0	0.2224	0.3976	0.5174	0.5770	0.5750	0.5158	0.4150	0.2880	0.1468	0
	U	0	0.1468	0.2880	0.4150	0.5158	0.5750	0.5770	0.5174	0.3976	0.2224	0
0.2	S	0	0.2778	0.5395	0.7608	0.9136	0.9700	0.9136	0.7608	0.5395	0.2778	0
	T	0	0.2054	0.3722	0.4894	0.5498	0.5500	0.4934	0.3962	0.2742	0.1395	0
	U	0	0.1395	0.2742	0.3962	0.4934	0.5500	0.5498	0.4894	0.3722	0.2054	0
0.1	S	0	0.2636	0.5141	0.7288	0.8790	0.9350	0.8790	0.7288	0.5141	0.2636	0
	T	0	0.1883	0.3467	0.4613	0.5225	0.5250	0.4711	0.3775	0.2605	0.1321	0
	U	0	0.1321	0.2605	0.3775	0.4711	0.5250	0.5225	0.4613	0.3467	0.1883	0
0.0	S	0	0.2493	0.4887	0.6968	0.8444	0.9000	0.8444	0.6968	0.4887	0.2493	0
	T	0	0.1712	0.3212	0.4332	0.4952	0.5000	0.4488	0.3588	0.2468	0.1248	0
	U	0	0.1248	0.2468	0.3588	0.4488	0.5000	0.4952	0.4332	0.3212	0.1712	0

TABLE 17. PRIME ARCHES. VALUES OF LOAD CONSTANTS S, T, and U; horizontal loads

For explanatory notes, see p. 417.
Tabular values of constants are determined by the equations

$$S = \frac{12}{PLf^2}\int \frac{I_0}{I}\,\frac{My\,dx}{\cos\varphi} \qquad T = \frac{12}{PL^2f}\int \frac{I_0}{I}\,\frac{M(L-x)\,dx}{\cos\varphi}$$

$$U = \frac{12}{PL^2f}\int \frac{I_0}{I}\,\frac{Mx\,dx}{\cos\varphi}$$

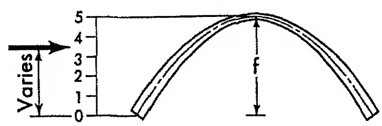
Arch k value	Constant	Concentrated load 						Uniformly distributed load 
		Load at point						
		0	1	2	3	4	5	
5.0	S	0	1.9800	3.8360	5.4370	6.6330	7.2000	4.3290
	T	0	2.3030	4.0240	5.1820	5.7930	5.8500	4.0930
	U	0	0.9948	1.9570	2.8460	3.6100	4.1500	2.3070
3.0	S	0	1.3880	2.7000	3.8530	4.7450	5.2000	3.0790
	T	0	1.5210	2.6880	3.5030	3.9670	4.0500	2.7710
	U	0	0.6969	1.3740	2.0050	2.5550	2.9500	1.6290
2.0	S	0	1.0920	2.1320	3.0610	3.8010	4.2000	2.4540
	T	0	1.1300	2.0190	2.6640	3.0540	3.1500	2.1110
	U	0	0.5479	1.0820	1.5850	2.0280	2.3500	1.2890
1.4	S	0	0.9142	1.7910	2.5860	3.2340	3.6000	2.0790
	T	0	0.8953	1.6180	2.1600	2.5060	2.6100	1.7140
	U	0	0.4585	0.9071	1.3320	1.7120	1.9900	1.0860
1.3	S	0	0.8846	1.7350	2.5070	3.1400	3.5000	2.0160
	T	0	0.8562	1.5510	2.0760	2.4150	2.5200	1.6480
	U	0	0.4436	0.8780	1.2900	1.6590	1.9300	1.0520
1.2	S	0	0.8550	1.6780	2.4270	3.0450	3.4000	1.9540
	T	0	0.8171	1.4840	1.9920	2.3240	2.4300	1.5820
	U	0	0.4287	0.8488	1.2480	1.6070	1.8700	1.0180
1.1	S	0	0.8254	1.6210	2.3480	2.9510	3.3000	1.8910
	T	0	0.7780	1.4170	1.9080	2.2320	2.3400	1.5160
	U	0	0.4138	0.8196	1.2060	1.5540	1.8100	0.9839
1.0	S	0	0.7958	1.5640	2.2690	2.8570	3.2000	1.8290
	T	0	0.7389	1.3510	1.8240	2.1410	2.2500	1.4500
	U	0	0.3989	0.7905	1.1640	1.5010	1.7500	0.9500

TABLE 17 (Continued)

Arch k value	Constant	Load at point 0	1	2	3	4	5	Uniformly distributed load
0.9	S	0	0.7662	1.5070	2.1900	2.7620	3.1000	1.7660
	T	0	0.6998	1.2840	1.7400	2.0500	2.1600	1.3840
	U	0	0.3840	0.7613	1.1220	1.4480	1.6900	0.9161
0.8	S	0	0.7366	1.4510	2.1110	2.6680	3.0000	1.7040
	T	0	0.6607	1.2170	1.6560	1.9590	2.0700	1.3180
	U	0	0.3691	0.7322	1.0800	1.3960	1.6300	0.8821
0.7	S	0	0.7070	1.3940	2.0310	2.5730	2.9000	1.6410
	T	0	0.6216	1.1500	1.5720	1.8670	1.9800	1.2520
	U	0	0.3542	0.7030	1.0380	1.3430	1.5700	0.8483
0.6	S	0	0.6774	1.3370	1.9520	2.4790	2.8000	1.5790
	T	0	0.5825	1.0830	1.4880	1.7760	1.8900	1.1860
	U	0	0.3393	0.6739	0.9958	1.2900	1.5100	0.8143
0.5	S	0	0.6478	1.2800	1.8730	2.3850	2.7000	1.5160
	T	0	0.5434	1.0160	1.4040	1.6850	1.8000	1.1200
	U	0	0.3244	0.6447	0.9538	1.2380	1.4500	0.7804
0.4	S	0	0.6182	1.2230	1.7940	2.2900	2.6000	1.4540
	T	0	0.5043	0.9494	1.3200	1.5930	1.7100	1.0540
	U	0	0.3095	0.6156	0.9117	1.1850	1.3900	0.7464
0.3	S	0	0.5886	1.1660	1.7150	2.1960	2.5000	1.3910
	T	0	0.4652	0.8826	1.2360	1.5020	1.6200	0.9875
	U	0	0.2946	0.5864	0.8697	1.1320	1.3300	0.7125
0.2	S	0	0.5590	1.1100	1.6350	2.1010	2.4000	1.3290
	T	0	0.4261	0.8157	1.1520	1.4110	1.5300	0.9214
	U	0	0.2797	0.5573	0.8277	1.0800	1.2700	0.6786
0.1	S	0	0.5294	1.0530	1.5560	2.0070	2.3000	1.2660
	T	0	0.3870	0.7489	1.0690	1.3200	1.4400	0.8554
	U	0	0.2648	0.5281	0.7856	1.0270	1.2100	0.6446
0.0	S	0	0.4998	0.9961	1.4770	1.9130	2.2000	1.2040
	T	0	0.3479	0.6820	0.9845	1.2280	1.3500	0.7893
	U	0	0.2500	0.4989	0.7436	0.9740	1.1500	0.6107

TABLE 18. QUADRATIC ARCHES. VALUES OF LOAD CONSTANTS S, T, and U; vertical uniform and complementary parabolic loads

For explanatory notes, see p. 417.

Tabular values of constants are determined by the equations

$$S = \frac{12}{WL^2f}\int \frac{I_0}{I}\frac{My\,dx}{\cos\varphi} \qquad T = \frac{12}{WL^3}\int \frac{I_0}{I}\frac{M(L-x)\,dx}{\cos\varphi}$$

$$U = \frac{12}{WL^3}\int \frac{I_0}{I}\frac{Mx\,dx}{\cos\varphi}$$

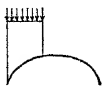
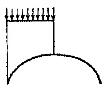
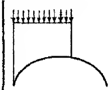
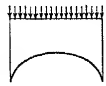
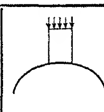
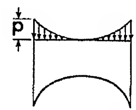
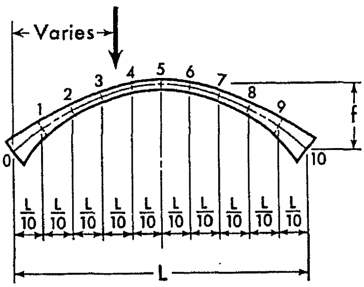
Arch k value	Constant	 Uniform load over 3/8 of the span	 Uniform load over 1/2 of the span	 Uniform load over 5/8 of the span	 Uniform load over the entire span	 Uniform load over the center quarter of the span	 Complementary parabolic load over the entire span $W = \frac{pL}{3}$
5.0	S	1.0700	1.2570	1.3690	1.2570	1.8180	0.7619
	T	0.9828	1.0630	1.0830	0.9000	1.2340	0.5857
	U	0.5946	0.7375	0.8503	0.9000	1.2340	0.5857
3.0	S	0.8652	1.0290	1.1270	1.0290	1.5190	0.6095
	T	0.7389	0.8125	0.8370	0.7000	0.9842	0.4429
	U	0.4716	0.5875	0.6766	0.7000	0.9842	0.4429
2.0	S	0.7628	0.9143	1.0050	0.9143	1.3690	0.5333
	T	0.6170	0.6875	0.7139	0.6000	0.8593	0.3714
	U	0.4101	0.5125	0.5898	0.6000	0.8593	0.3714
1.4	S	0.7013	0.8457	0.9323	0.8457	1.2790	0.4876
	T	0.5439	0.6125	0.6401	0.5400	0.7843	0.3286
	U	0.3732	0.4675	0.5377	0.5400	0.7843	0.3286
1.3	S	0.6911	0.8343	0.9202	0.8343	1.2640	0.4800
	T	0.5317	0.6000	0.6278	0.5300	0.7718	0.3214
	U	0.3671	0.4600	0.5290	0.5300	0.7718	0.3214
1.2	S	0.6808	0.8229	0.9081	0.8229	1.2490	0.4724
	T	0.5195	0.5875	0.6154	0.5200	0.7594	0.3143
	U	0.3609	0.4525	0.5203	0.5200	0.7594	0.3143
1.1	S	0.6706	0.8114	0.8959	0.8114	1.2340	0.4648
	T	0.5073	0.5750	0.6031	0.5100	0.7469	0.3071
	U	0.3548	0.4450	0.5116	0.5100	0.7469	0.3071
1.0	S	0.6604	0.8000	0.8838	0.8000	1.2190	0.4571
	T	0.4951	0.5625	0.5908	0.5000	0.7344	0.3000
	U	0.3486	0.4375	0.5029	0.5000	0.7344	0.3000

TABLE 18 (Continued)

Arch k value	Constant	Uniform load over 3/8 of the span	Uniform load over 1/2 of the span	Uniform load over 5/8 of the span	Uniform load over the entire span	Uniform load over the center quarter of the span	Complementary parabolic load over the entire span $W = \frac{pL}{3}$
0.9	S	0.6501	0.7886	0.8717	0.7886	1.2040	0.4495
	T	0.4829	0.5500	0.5785	0.4900	0.7219	0.2929
	U	0.3425	0.4300	0.4942	0.4900	0.7219	0.2929
0.8	S	0.6399	0.7771	0.8595	0.7771	1.1890	0.4419
	T	0.4707	0.5375	0.5662	0.4800	0.7094	0.2857
	U	0.3363	0.4225	0.4856	0.4800	0.7094	0.2857
0.7	S	0.6296	0.7657	0.8474	0.7657	1.1740	0.4343
	T	0.4585	0.5250	0.5539	0.4700	0.6969	0.2786
	U	0.3302	0.4150	0.4769	0.4700	0.6969	0.2786
0.6	S	0.6194	0.7543	0.8352	0.7543	1.1590	0.4267
	T	0.4464	0.5125	0.5416	0.4600	0.6844	0.2714
	U	0.3240	0.4075	0.4682	0.4600	0.6844	0.2714
0.5	S	0.6091	0.7429	0.8231	0.7429	1.1440	0.4191
	T	0.4342	0.5000	0.5293	0.4500	0.6719	0.2643
	U	0.3179	0.4000	0.4595	0.4500	0.6719	0.2643
0.4	S	0.5989	0.7314	0.8110	0.7314	1.1290	0.4114
	T	0.4220	0.4875	0.5170	0.4400	0.6594	0.2571
	U	0.3117	0.3925	0.4508	0.4400	0.6594	0.2571
0.3	S	0.5886	0.7200	0.7988	0.7200	1.1140	0.4038
	T	0.4098	0.4750	0.5046	0.4300	0.6469	0.2500
	U	0.3056	0.3850	0.4421	0.4300	0.6469	0.2500
0.2	S	0.5784	0.7086	0.7867	0.7086	1.0990	0.3962
	T	0.3976	0.4625	0.4923	0.4200	0.6345	0.2429
	U	0.2994	0.3775	0.4334	0.4200	0.6345	0.2429
0.1	S	0.5682	0.6971	0.7745	0.6971	1.0840	0.3886
	T	0.3854	0.4500	0.4800	0.4100	0.6220	0.2357
	U	0.2933	0.3700	0.4248	0.4100	0.6220	0.2357
0	S	0.5579	0.6857	0.7624	0.6857	1.0690	0.3810
	T	0.3732	0.4375	0.4677	0.4000	0.6095	0.2286
	U	0.2871	0.3625	0.4161	0.4000	0.6095	0.2286

TABLE 19. QUADRATIC ARCHES. VALUES OF LOAD CONSTANTS S, T, and U; vertical concentrated load

For explanatory notes, see p. 417.
Tabular values of constants are determined by the equations



$$S = \frac{12}{PL^2f} \int \frac{I_0}{I} \frac{My\,dx}{\cos\varphi}$$

$$T = \frac{12}{PL^3} \int \frac{I_0}{I} \frac{M(L-x)\,dx}{\cos\varphi}$$

$$U = \frac{12}{PL^3} \int \frac{I_0}{I} \frac{Mx\,dx}{\cos\varphi}$$

Arch k value	Constant	Load at point 0	1	2	3	4	5	6	7	8	9	10
5.0	S	0	0.6877	1.2320	1.5920	1.7890	1.8500	1.7890	1.5920	1.2320	0.6877	0
	T	0	0.7788	1.1680	1.3180	1.3270	1.2500	1.1110	0.9163	0.6625	0.3515	0
	U	0	0.3515	0.6625	0.9163	1.1110	1.2500	1.3270	1.3180	1.1680	0.7788	0
3.0	S	0	0.5400	0.9869	1.3040	1.4900	1.5500	1.4900	1.3040	0.9869	0.5400	0
	T	0	0.5604	0.8720	1.0160	1.0480	1.0000	0.8916	0.7311	0.5233	0.2748	0
	U	0	0.2748	0.5233	0.7311	0.8916	1.0000	1.0480	1.0160	0.8720	0.5604	0
2.0	S	0	0.4662	0.8647	1.1600	1.3400	1.4000	1.3400	1.1600	0.8647	0.4662	0
	T	0	0.4512	0.7240	0.8650	0.9078	0.8750	0.7818	0.6386	0.4536	0.2364	0
	U	0	0.2364	0.4536	0.6386	0.7818	0.8750	0.9078	0.8650	0.7240	0.4512	0
1.4	S	0	0.4219	0.7913	1.0740	1.2500	1.3100	1.2500	1.0740	0.7913	0.4219	0
	T	0	0.3857	0.6352	0.7744	0.8239	0.8000	0.7159	0.5830	0.4119	0.2134	0
	U	0	0.2134	0.4119	0.5830	0.7159	0.8000	0.8239	0.7744	0.6352	0.3857	0
1.3	S	0	0.4145	0.7791	1.0600	1.2350	1.2950	1.2350	1.0600	0.7791	0.4145	0
	T	0	0.3748	0.6204	0.7593	0.8099	0.7875	0.7049	0.5738	0.4049	0.2095	0
	U	0	0.2095	0.4049	0.5738	0.7049	0.7875	0.8099	0.7593	0.6204	0.3748	0
1.2	S	0	0.4072	0.7669	1.0450	1.2200	1.2800	1.2200	1.0450	0.7669	0.4072	0
	T	0	0.3638	0.6056	0.7442	0.7960	0.7750	0.6940	0.5645	0.3979	0.2057	0
	U	0	0.2057	0.3979	0.5645	0.6940	0.7750	0.7960	0.7442	0.6056	0.3638	0
1.1	S	0	0.3998	0.7546	1.0310	1.2050	1.2650	1.2050	1.0310	0.7546	0.3998	0
	T	0	0.3529	0.5908	0.7291	0.7820	0.7625	0.6830	0.5553	0.3910	0.2018	0
	U	0	0.2018	0.3910	0.5553	0.6830	0.7625	0.7820	0.7291	0.5908	0.3529	0
1.0	S	0	0.3924	0.7424	1.0160	1.1900	1.2500	1.1900	1.0160	0.7424	0.3924	0
	T	0	0.3420	0.5760	0.7140	0.7680	0.7500	0.6720	0.5460	0.3840	0.1980	0
	U	0	0.1980	0.3840	0.5460	0.6720	0.7500	0.7680	0.7140	0.5760	0.3420	0

TABLE 19 (Continued)

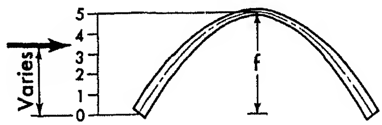
Arch k value	Constant	Load at point 0	1	2	3	4	5	6	7	8	9	10
0.9	S	0	0.3850	0.7302	1.0020	1.1750	1.2350	1.1750	1.0020	0.7302	0.3850	0
	T	0	0.3311	0.5612	0.6989	0.7540	0.7375	0.6610	0.5367	0.3770	0.1942	0
	U	0	0.1942	0.3770	0.5367	0.6610	0.7375	0.7540	0.6989	0.5612	0.3311	0
0.8	S	0	0.3776	0.7179	0.9876	1.1610	1.2200	1.1610	0.9876	0.7179	0.3776	0
	T	0	0.3202	0.5464	0.6838	0.7400	0.7250	0.6500	0.5275	0.3700	0.1903	0
	U	0	0.1903	0.3700	0.5275	0.6500	0.7250	0.7400	0.6838	0.5464	0.3202	0
0.7	S	0	0.3703	0.7057	0.9732	1.1460	1.2050	1.1460	0.9732	0.7057	0.3703	0
	T	0	0.3092	0.5316	0.6687	0.7261	0.7125	0.6391	0.5182	0.3631	0.1865	0
	U	0	0.1865	0.3631	0.5182	0.6391	0.7125	0.7261	0.6687	0.5316	0.3092	0
0.6	S	0	0.3629	0.6935	0.9588	1.1310	1.1900	1.1310	0.9588	0.6935	0.3629	0
	T	0	0.2983	0.5168	0.6536	0.7121	0.7000	0.6281	0.5090	0.3561	0.1826	0
	U	0	0.1826	0.3561	0.5090	0.6281	0.7000	0.7121	0.6536	0.5168	0.2983	0
0.5	S	0	0.3555	0.6813	0.9444	1.1160	1.1750	1.1160	0.9444	0.6813	0.3555	0
	T	0	0.2874	0.5020	0.6385	0.6981	0.6875	0.6171	0.4997	0.3492	0.1788	0
	U	0	0.1788	0.3492	0.4997	0.6171	0.6875	0.6981	0.6385	0.5020	0.2874	0
0.4	S	0	0.3481	0.6690	0.9300	1.1010	1.1600	1.1010	0.9300	0.6690	0.3481	0
	T	0	0.2765	0.4872	0.6234	0.6841	0.6750	0.6061	0.4905	0.3422	0.1750	0
	U	0	0.1750	0.3422	0.4905	0.6061	0.6750	0.6841	0.6234	0.4872	0.2765	0
0.3	S	0	0.3407	0.6568	0.9156	1.0860	1.1450	1.0860	0.9156	0.6568	0.3407	0
	T	0	0.2655	0.4724	0.6083	0.6702	0.6625	0.5951	0.4812	0.3353	0.1711	0
	U	0	0.1711	0.3353	0.4812	0.5951	0.6625	0.6702	0.6083	0.4724	0.2655	0
0.2	S	0	0.3333	0.6446	0.9012	1.0710	1.1300	1.0710	0.9012	0.6446	0.3333	0
	T	0	0.2546	0.4576	0.5932	0.6562	0.6500	0.5841	0.4719	0.3283	0.1673	0
	U	0	0.1673	0.3283	0.4719	0.5841	0.6500	0.6562	0.5932	0.4576	0.2546	0
0.1	S	0	0.3260	0.6324	0.8868	1.0560	1.1150	1.0560	0.8868	0.6324	0.3260	0
	T	0	0.2437	0.4428	0.5781	0.6422	0.6375	0.5732	0.4627	0.3213	0.1635	0
	U	0	0.1635	0.3213	0.4627	0.5732	0.6375	0.6422	0.5781	0.4428	0.2437	0
0.0	S	0	0.3186	0.6201	0.8724	1.0410	1.1000	1.0410	0.8724	0.6201	0.3186	0
	T	0	0.2328	0.4280	0.5630	0.6282	0.6250	0.5622	0.4534	0.3144	0.1596	0
	U	0	0.1596	0.3144	0.4534	0.5622	0.6250	0.6282	0.5630	0.4280	0.2328	0

TABLE 20. QUADRATIC ARCHES. VALUES OF LOAD CONSTANTS S, T, and U; horizontal loads

For explanatory notes, see p. 417.
Tabular values of constants are determined by the equations

$$S = \frac{12}{PLf^2}\int \frac{I_0}{I}\,\frac{My\,dx}{\cos\varphi} \qquad T = \frac{12}{PL^2f}\int \frac{I_0}{I}\,\frac{M(L-x)\,dx}{\cos\varphi}$$

$$U = \frac{12}{PL^2f}\int \frac{I_0}{I}\,\frac{Mx\,dx}{\cos\varphi}$$



Arch k value	Constant	Concentrated load — Load at point 0	1	2	3	4	5	Uniformly distributed load
5.0	S	0	1.4210	2.7230	3.8290	4.6330	5.0290	3.0480
	T	0	1.7910	3.0480	3.8380	4.2240	4.2500	3.0430
	U	0	0.7150	1.4000	2.0260	2.5600	2.9500	1.6430
3.0	S	0	1.1080	2.1470	3.0490	3.7450	4.1140	2.4380
	T	0	1.2650	2.1990	2.8310	3.1830	3.2500	2.2460
	U	0	0.5570	1.0950	1.5950	2.0310	2.3500	1.2960
2.0	S	0	0.9520	1.8560	2.6590	3.3010	3.6570	2.1330
	T	0	1.0020	1.7750	2.3280	2.6620	2.7500	1.8480
	U	0	0.4779	0.9429	1.3800	1.7660	2.0500	1.1230
1.4	S	0	0.8583	1.6810	2.4250	3.0340	3.3830	1.9510
	T	0	0.8441	1.5200	2.0260	2.3490	2.4500	1.6090
	U	0	0.4305	0.8515	1.2500	1.6070	1.8700	1.0190
1.3	S	0	0.8426	1.6520	2.3860	2.9900	3.3370	1.9200
	T	0	0.8178	1.4780	1.9750	2.2980	2.4000	1.5700
	U	0	0.4226	0.8362	1.2290	1.5810	1.8400	1.0020
1.2	S	0	0.8270	1.6220	2.3470	2.9450	3.2910	1.8900
	T	0	0.7915	1.4350	1.9250	2.2450	2.3500	1.5300
	U	0	0.4147	0.8210	1.2070	1.5540	1.8100	0.9846
1.1	S	0	0.8114	1.5930	2.3080	2.9010	3.2460	1.8590
	T	0	0.7652	1.3930	1.8740	2.1930	2.3000	1.4900
	U	0	0.4068	0.8057	1.1860	1.5280	1.7800	0.9673
1.0	S	0	0.7958	1.5640	2.2690	2.8570	3.2000	1.8290
	T	0	0.7389	1.3510	1.8240	2.1410	2.2500	1.4500
	U	0	0.3989	0.7905	1.1640	1.5010	1.7500	0.9500

TABLE 20 (Continued)

Arch k value	Constant	Load at point 0	1	2	3	4	5	Uniformly distributed load
0.9	S	0	0.7802	1.5350	2.2300	2.8120	3.1540	1.7980
	T	0	0.7126	1.3080	1.7740	2.0890	2.2000	1.4100
	U	0	0.3910	0.7752	1.1420	1.4750	1.7200	0.9327
0.8	S	0	0.7645	1.5060	2.1910	2.7680	3.1090	1.7680
	T	0	0.6863	1.2660	1.7230	2.0370	2.1500	1.3700
	U	0	0.3831	0.7600	1.1210	1.4480	1.6900	0.9154
0.7	S	0	0.7489	1.4770	2.1520	2.7230	3.0630	1.7370
	T	0	0.6600	1.2230	1.6730	1.9850	2.1000	1.3310
	U	0	0.3752	0.7447	1.0990	1.4220	1.6600	0.8980
0.6	S	0	0.7333	1.4480	2.1130	2.6790	3.0170	1.7070
	T	0	0.6337	1.1810	1.6230	1.9330	2.0500	1.2910
	U	0	0.3673	0.7295	1.0780	1.3950	1.6300	0.8807
0.5	S	0	0.7177	1.4180	2.0740	2.6350	2.9710	1.6760
	T	0	0.6074	1.1380	1.5720	1.8810	2.0000	1.2510
	U	0	0.3594	0.7143	1.0560	1.3690	1.6000	0.8634
0.4	S	0	0.7021	1.3890	2.0350	2.5900	2.9260	1.6460
	T	0	0.5811	1.0960	1.5220	1.8290	1.9500	1.2110
	U	0	0.3515	0.6990	1.0350	1.3420	1.5700	0.8461
0.3	S	0	0.6864	1.3600	1.9960	2.5460	2.8800	1.6150
	T	0	0.5548	1.0530	1.4720	1.7770	1.9000	1.1710
	U	0	0.3436	0.6838	1.0130	1.3160	1.5400	0.8288
0.2	S	0	0.6708	1.3310	1.9570	2.5010	2.8340	1.5850
	T	0	0.5285	1.0110	1.4210	1.7240	1.8500	1.1310
	U	0	0.3357	0.6685	0.9916	1.2890	1.5100	0.8114
0.1	S	0	0.6552	1.3020	1.9180	2.4570	2.7890	1.5540
	T	0	0.5022	0.9685	1.3710	1.6720	1.8000	1.0920
	U	0	0.3278	0.6533	0.9701	1.2630	1.4800	0.7941
0.0	S	0	0.6396	1.2730	1.8790	2.4130	2.7430	1.5240
	T	0	0.4759	0.9260	1.3210	1.6200	1.7500	1.0520
	U	0	0.3199	0.6380	0.9485	1.2360	1.4500	0.7768

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